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A POLAR CONTINUUM THEORY FOR FLUENT CONTINUA

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Abstract

Polar decomposition of the changing velocity gradient tensor in a deforming fluent continua into pure stretch rates and rates of rotations shows that a location and its neighboring locations can experience different rates of rotation during evolution. Alternatively, we can also consider decomposition of the velocity gradient tensor into symmetric and skew symmetric tensors. The skew symmetric tensor is also a measure of pure rates of rotations whereas the symmetric tensor is a measure of strain rates. The measures of the internal rates of rotation due to deformation in the two approaches describe the same physics but in different forms. Polar decomposition gives the rate of rotation matrix and not the rates of rotation angles whereas the skew symmetric part of the velocity gradient tensor yields rates of rotation angles that are explicitly defined in terms of velocity gradients. These varying rates of rotation arise due to varying deformation of the continua, hence are internal to the volume of matter and are explicitly defined by deformation, hence do not require additional rotational degrees of freedom. If the internal varying rates of rotations are resisted by the continua, then there must exist internal moments corresponding to these. The internal rates of rotations and the corresponding moments can result in additional rate of energy storage or rate of dissipation.

Key Words : *Fluent polar continua, Polar decomposition, Rates of rotations, Internal polar theory for fluent continua.*

This physics is all internal to the deforming continua (hence does not require consideration of additional external rotational degrees of freedom and associated external moments) and is neglected in the presently used continuum theories for fluent continua. The continuum theory presented here considers internal varying rates of rotations and the associated conjugate moments in the derivation of conservation and balance laws, thus the theory presented in this paper is "a polar continuum theory" but is different than micropolar continuum theories published currently in which material points have six external degrees of freedom i.e. the rotation rates are additional external degrees of freedom. This polar continuum theory only accounts for internal rotation rates and associated moments that exist as a consequence of deformation but are neglected in the present theories. We call this theory "a polar continuum theory" as it considers rates of rotations and moments in a deforming fluent continua though these are internal, hence are purely related to the deformation of the fluid. In this theory we have no concept of 'stress couples' at the onset of the derivation of the theory. At this stage all we know is that internal varying rotation rates must be accounted for in a continuum theory. It is shown that the polar continuum theory presented in this paper is not the same as the strain rate gradient theories for fluent continua and others. The differences are obvious in the physics described by them and the mathematical details associated with the conservation and balance laws. The polar continuum theory presented in this paper is suitable for compressible as well as incompressible thermoviscous fluent continua such as Newtonian, Power law, Carreau-Yasuda fluids etc. and thermoviscoelastic fluent continua such as Maxwell, Oldroyd-B, Giesekus etc. The thermodynamic framework presented here is applicable to all isotropic, homogeneous fluent continua. Obviously the constitutive theories will vary depending upon the choice of physics. These are considered in subsequent papers.