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A NEW BELIEF FUNCTION INDUCED BY PROBABILITY MASS FUNCTION

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Abstract

Rarely some of the authors are using approaches from probability to belief functions. In this paper we have defined a basic belief assignment hence belief function (hence Bayesian belief function) using probability mass function. Our definition generalizes Shafer's [6] definition of Bayesian belief function induced by probability density function. Shafer mentions that it is unique but that is not the case. Here we will study various properties of Bayesian belief function defined by us.

1. Introduction

In the world of uncertainty, each and every incidence occurring in our day to day life always follows some known or unknown probability distribution. Therefore choice of appropriate probability distribution plays an important role in decision making. Hence it becomes necessary that we should know common characteristics of all probability distributions.

Key Words : *Belief function, Bayesian belief function, Probability, Probability density function.*

Here we want to find a new transformation which transforms probability mass function into basic belief assignment. While obtaining this new transformation, we concentrate on sufficient axioms of basic belief assignment which must be satisfied by our new transformation. Once we have obtained such new transformation, we are able to find other functions related to belief functions. Also we will check that this new transformation satisfies some more additional properties so that we can recognize the true class of this new transformation.

In this paper, firstly we summarize preliminaries of discrete belief functions and probability functions then we will explain steps in the development of this new transformation. Also we deduce some results of discrete belief function theory in Shafer‘s book [6]. Now we summarize preliminaries of discrete belief functions and probability functions.

2. Preliminaries

2.1 Discrete Belief Function Theory

Frame of Discernment : Dictionary meaning of Frame of Discernment is frame of good judgment insight. The word discern means recognize or find out or hear with difficulty. From Shafer‘s book [6], if frame of discernment Θ is

$$\Theta = \{\theta_1, \theta_2, \ldots, \theta_n\}$$

then every element of Θ is a proposition. The propositions of interest are in one -to -one correspondence with the subsets of Θ. The set of all propositions of interest corresponds to the set of all subsets of Θ, denoted by 2^Θ.

If Θ is *frame of discernment*, then a function $m : 2^\Theta \to [0, 1]$ is called **basic probability assignment** whenever $m(\emptyset) = 0$ and $\sum_{A\subset\Theta} m(A) = 1$. The quantity $m(A)$ is called A's **basic probability number** and it is a measure of the belief committed exactly to A.The *total belief* committed to A is sum of $m(B)$, for all subsets B of A. . A function $Bel : 2^\Theta \to [0, 1]$ is called **belief function** over Θ if it satisfies $Bel(A) = \sum_{B\subset A} m(B)$. If Θ is a frame of discernment, then a function $Bel : 2^\Theta \to [0, 1]$ is *belief function* if and only if it satisfies following conditions

1. $Bel(\emptyset) = 0$.

2. $Bel(\Theta) = 1$.

3. For every positive integer n and every collection $A_1, A_2, \ldots, A_n$ of subsets of Θ

$$Bel(A_1 \cup A_2 \cup \ldots \cup A_n) \geq \sum_{I \subset \{1,2,\cdots,n\}} (-1)^{|I|+1} Bel(\bigcap_{i \in I} A_i). \tag{1}$$

A subset of a frame Θ is called a **focal element** of a belief function Bel over Θ if $m(A) > 0$. The union of all the focal elements of a belief function is called its **core**. The quantity $Q(A) = \sum_{B \subset \Theta, A \subset B} m(B)$ is called **commonality number** for A which measures the total probability mass that can move freely to every point of A. A function $Q : 2^\Theta \to [0,1]$ is called **commonality function** for Bel. Also $Bel(A) = \sum_{B \subset \bar{A}}$ and $Q(A) = \sum_{B \subset A} (-1)^{|B|} Bel(\bar{B})$ for all $A \subset \Theta$.

Degree of doubt :

$$Dou(A) = Bel(\bar{A}) \text{or} Bel(A) = Dou(\bar{A}) \text{and } pl(A) = 1 - Dou(A) = \sum_{A \cap B \neq \emptyset} m(B) \tag{2}$$

which expresses the extent to which one finds A credible or plausible [6]. We have relation between belief function, probability mass (or density) function and plausibility function is $Bel(A) \leq p(A) \leq Pl(A), \quad \forall A \subset \Theta$ [3, 4]. A function $P : \Theta \to [0,1]$ is called probability function if

1 $\forall A \in \Theta, \quad 0 \leq P(A) \leq 1.$

2 $P(\Theta) = 1.$

A set function μ on a frame of discernment Θ is a measure if it satisfies following three conditions:

1. $\mu(A) \in [0, \infty], \quad$ for all $A \in \Theta$.
2. $\mu(\emptyset) = 0$.
3. **Additive Property : For collection** $A_1, A_2, \ldots, A_n, \ldots,$

$$\mu(\cup_{i=1}^{\infty} A_i) = \sum_{\substack{I \subset \{1,2,\ldots,n,\ldots\} \\ I \neq \emptyset}} (-1)^{|A|+1} \mu(\cap_{i=1}^{\infty} A_i). \tag{3}$$

The measure is finite or infinite as $\mu(\Theta) < \infty$ or $\mu(\Theta) = \infty$. It is probability measure if $\mu(\Theta) = 1$ [1]. In Shafer's book [6], we have Bayesian belief function as: If Θ is frame of discernment then a function $Bel : 2^\Theta \to [0,1]$ is called Bayesian belief function if

1 $Bel(\emptyset) = 0$,

2 $Bel(\Theta) = 1$,

3 $Bel(A \cup B) = Bel(A) + Bel(B)$ whenever $A, B \in \Theta$ and $A \cap B = \emptyset$.

Suppose $Bel : 2^{\Theta} \to [0, 1]$ is belief function. Then following statements are equivalent:

1 Bel is Bayesian.

2 All of Bel's focal elements are singletons.

3 Bel awards a zero commonality number to any subset containing more than one element.

4 $Bel(A) = 1 - Bel(\bar{A})$ for all $A \subset \Theta$.

Also we have some other basic belief assignments and we will briefly introduce these bba's.

Classical Pignistic Probability:-

Philippe Smets [8] had given basic idea and implemented in Transferable Belief Model. It transfers positive mass of belief of each non-specific element onto the singletons involved in that element split by the cardinality of the proposition when working with normalized basic belief assignments. In TBM, the classical pignistic probability is
$BetP(\emptyset) = 0$ and $\forall \quad A \in 2^{\Theta} - \{\emptyset\}$

$$BetP(A) = \sum_{\substack{B \in 2^{\Theta}, \\ B \neq \emptyset}} \frac{|A \cap B|}{|B|} \frac{m(B)}{1 - m(\emptyset)}. \tag{4}$$

In shafer [6], $m(\emptyset) = 0$ hence above formula becomes

$$BetP(\theta_i) = m\theta_i + \sum_{\substack{B \in 2^{\Theta}, \\ \theta_i \subset B}} \frac{m(B)}{|B|} \text{and} BetP(A) = \sum_{\theta_i \in A} BetP(\theta_i). \tag{5}$$

Sudano's Probabilities:-

John Sudano [9, 10, 11], had developed transformations which approximates quantative belief mass m by subjective probabilities as follows:

1 $PrPl(.)$ **and** $PrBel(.)$**:-** For all $A \neq \emptyset \in \Theta$,

$$PrPl(A) = Pl(A) \sum_{\substack{B \in 2^{\Theta}, \\ A \subseteq B}} \frac{m(B)}{CS(Pl(B))} \tag{6}$$

and

$$PrBel(A) = Bel(A) \sum_{\substack{B \in 2^{\Theta}, \\ A \subseteq B}} \frac{m(B)}{CS(Bel(B))} \tag{7}$$

where

$$CS(Pl(B)) = \sum_{\substack{B_i \in 2^{\Theta}, \\ |B_i|=1, \\ \cup_i B_i = B}} Pl(B_i) \text{ and } CS(Bel(B)) = \sum_{\substack{B_i \in 2^{\Theta}, \\ |B_i|=1, \\ \cup_i B_i = B}} Bel(B_i)[9]. \tag{8}$$

2 **Transformation Proportional to Normalized Plausibility:-**

$$PrNPl(A) = \frac{1}{\triangle} \sum_{\substack{B \in 2^{\Theta}, \\ A \cap B \neq \emptyset}} Pl(B) = \frac{1}{\triangle} Pl(A) \tag{9}$$

where $\triangle$ is a normalizing factor with $\sum_{A \in \Theta} PrNPl(A) = 1$[9, 11].

3 **Transformation Proportional to all Plausibilities :-**

$$PraPl(A) = Bel(A) + k \cdot Pl(A) \text{where} k = \frac{\sum_{B \in 2^{\Theta}} Bel(B)}{\sum_{B \in 2^{\Theta}} Pl(B)} [9, 11] \tag{10}$$

4 **The Hybrid Pignistic Probability:-**

$$PrHyb(A) = PraPl(A) \cdot \sum_{\substack{B \in 2^{\Theta}, \\ A \subseteq B}} \frac{m(B)}{CS(PraPl(B))} \tag{11}$$

where

$$CS(PraPl(B)) = \sum_{\substack{B_i \in 2^{\Theta}, \\ |B_i|=1, \\ \cup_i B_i = B}} PraPl(B_i)[9, 11]. \tag{12}$$

Cuzzolin's Intersection Probability:-

In [2], Cuzzolin developed transformation $CuzzP(.)$ for any finite and discrete frame of discernment $\Theta \quad n \geq 2$, satisfying Shafer's model as

$$CuzzP(\theta_i) = m(\theta_i) + \frac{\triangle(\theta_i)}{\sum_{j=1}^{n} \triangle(\theta_j)} \times TNSM \tag{13}$$

with $\triangle(\theta_i) = Pl(\theta_i) - m(\theta_i)$ and

$$TNSM = 1 - \sum_{j=1}^{n} m(\theta_j) = \sum_{A \in 2^{\Theta}, |A|>1} m(A). \tag{14}$$

Shannon Entropy;-

Shannon [7] developed transformation discrete probability measure $H(.)$ for discrete frame of discernment Θ by

$$H(P) = -\sum_{i=1}^{n} P(\{\theta_i\}) log_2 P(\{\theta_i\}) \tag{15}$$

$H(P)$ is maximal for uniform probability distribution over Θ i.e. $H_{max} = -\sum_{i=1}^{n}(1/n)log_2(1/n) = log_2(n)$ and $H(P)$ is minimal for a totally deterministic probability measure i.e. $P(\theta_i) = 1$ for some $i \in \{1, 2, \ldots, n\}$ and $P(\theta_j) = 0$ for $j \neq i$.

The Probability Information Content:-

In [10], the probability information content (PIC) of discrete probability $P(.)$ for discrete frame of discernment Θ is

$$PIC(P) = 1 + \frac{1}{H_{max}} \cdot \sum_{i=1}^{n} P(\{\theta_i\}) log_2 P(\{\theta_i\}) \tag{16}$$

Normalized Shannon entropy is dual of PIC metric. Also $PIC(P) = 1 - \dfrac{H(P)}{H_{max}}$. Necessary series results are referred from Hall's book [5].

3. Belief Function by Glenn Shafer

In Shafer‘s book [6], a belief function induced by probability density function is given by following theorem:

Theorem 3.1 : A function $Bel : 2^{\Theta} \to [0, 1]$ is a Bayesian belief function if and only if there exists a function $p : \Theta \to [0, 1]$ such that $\sum_{\theta \in \Theta} p(\theta) = 1$ and $Bel(A) = \sum_{\theta \in A} p(\theta)$ for all $A \subset \Theta$

Note:- If above such Bayesian belief function exists then it is unique and is given by $p(\theta) = m(\theta)$. If $\Theta = \theta_1, \theta_2, \ldots, \theta_n$ is a frame of discernment (in discrete probability distributions Θ is sample space or population space) then

$$m(A) = \begin{cases} p(A) & \text{If } A \text{ is singleton} \\ 0 & \text{otherwise} \end{cases} \tag{17}$$

$$\text{with } m(\theta_i) = p(\theta_i), \qquad i = 1, 2, \ldots, n$$

Here $\sum_{i=1}^{n} p(\theta_i) = 1$.

3.1 Belief of Subset of Θ

Let $A \subseteq \Theta$, then belief of set A by (17) is

$$Bel(A) = \sum_{B \subseteq A} m(B) = p(A). \tag{18}$$

Therefore $Bel(A) = p(A), \forall A \subseteq \Theta$.

Remarks :

1 $Bel(\emptyset) = p(\emptyset) = 0$.

2 $Bel(\Theta) = p(\Theta) = 1$.

3 $Bel(\theta_i) = m(\theta_i), \quad \forall \theta_i \in \Theta$.

4 For any subset $A \subseteq \Theta, 0 \leq p(A) \leq 1 \Rightarrow 0 \leq Bel(A) \leq 1$.

5 Bel is a probability measure.

6 $\sum_{A \subseteq \Theta} Bel(A) = \sum_{A \subseteq \Theta} p(A) = 2^{n-1}$.

3.2 Commonality of Subset of Θ

Let $A \subseteq \Theta$. Then commonality of subset A by (17) is

$$Q(A) = \sum_{B \supseteq A} m(B)$$

$$\text{where } Q(A) = \begin{cases} m(A) & \text{if } A \text{ is singleton set.} \\ 1 & \text{if } A = \emptyset. \\ 0 & \text{otherwise.} \end{cases} \tag{19}$$

which is clear by definition of m, since *bba* of non-singleton set, including $\emptyset$ is 0. For any non-empty subset $A \subseteq \Theta$, consider

$$\begin{aligned} \sum_{A \subseteq \Theta} Q(A) &= \sum_{A \subseteq \Theta} m(A), \text{if } A \text{ is singleton set} \\ &= \sum_{\theta_i \subseteq \Theta} m(\theta_i \\ &= 1. \end{aligned} \tag{20}$$

Also, $Q(\emptyset) = 1$.
Therefore, $\sum_{A\subseteq\Theta} Q(A) = \sum_{\emptyset = A\subseteq\Theta} Q(A) + \sum_{\emptyset\neq A\subseteq\Theta} Q(A) = 1 + 1 = 2$.

3.3 Plausibility of Subset of Θ

Let $A \subseteq \Theta$. Then plausibility of subset A by (17) is

$$\begin{aligned} Pl(A) &= \sum_{A\cap B\neq\emptyset} m(B) \\ &= \sum_{A\cap B\neq\emptyset} p(B), \text{ if } B \text{ is singleton set (by definition of } m \text{)} \\ &= p(A) \end{aligned} \tag{21}$$

Remarks :

1 For any subset $A \subseteq \Theta, Bel(A) = Pl(A)$.

2 $\sum_{A\subseteq\Theta} Pl(A) = \sum_{A\subseteq\Theta} Bel(A) = 2^{n-1}$.

3 In Shafer's book [6], We have

$$Q(A) = \kappa q(A)$$

$$\begin{aligned} \text{where } Q(A) &= \text{commonality of } A \\ \kappa &= constant \\ q(A) &= \text{function } q : \Theta \to [0, \infty]. \end{aligned} \tag{22}$$

Here we can take $\kappa = 1 =$ constant and $q(A) = m(A)$ where $m : \Theta \to [0,1] \subset [0, \infty]$ with A maps $m(A)$.

3.4 Some Statistical Concepts

In statistics, some concepts viz. distribution function, expectation, variance, standard deviation, skewness and kurtosis play important roles in determination of nature of probability distributions. As we have result about belief function, probability function and plausibility function as : $Bel(A) \leq p(A) \leq Pl(A), \quad \forall A \subseteq \Theta$ [3, 4]. Now we introduce variable V whose values corresponds to subsets of Θ. Here we can apply above these concepts for basic belief assignments induced by probability mass function.

Values of above statistical quantities can be obtained similar to concepts in statistics. Finally we draw conclusions in terms of subsets of Θ instead of values of variable V. Here fortunately the quantities obtained with help of basic belief assignment $m(V)$ are equal to quantities obtained with help of probability mass function $p(V)$ for probability distributions by definition of $m(A)$ where A is subset of Θ .

Thus the results obtained by Bayesian belief function (in Shafer‘s book [6]) and probability mass function of probability distribution are same.

4. A New Basic Belief Assignment induced by Probability Mass Function

In this section, we obtain new basic belief assignment induced by probability mass function of discrete probability distribution.

4.1 Prerequisites in obtaining a new transformation

When $|\Theta| > 1$ then we observe that sum of probabilities of all subsets is not equal to 1. But if we find a generalized formula about repeation of singleton set of Θ in all subsets of Θ then we have following result as:

Theorem 4.1 : If $|\Theta| = n$ then every element in frame of discernment Θ is repeated exactly 2^{n-1} number of times and sum of probabilities of all subsets of Θ is 2^{n-1}.

Proof:- We prove it in two parts.

Part I:

In this part, we show that every element in frame of discernment Θ is repeated exactly 2^{n-1} number of times. we prove it by principle of mathematical induction on n i.e. number of exhaustive elements of frame of discernment Θ.

Step 1 : claim: Result is true for $n = 1$.

Let $|\Theta| = 1$. $\therefore$ Consider $\Theta = \{a\}$,

$\Rightarrow \mathcal{P}(\Theta) = \{\emptyset, \{a\}\}$.

i.e. $\{a\}$ is repeated exactly $1 = 2^{1-1}$ times.

Step 2 : Assume that result is true for $n = k$ number of exhaustive elements frame of discernment Θ.

i.e. every element in frame of discernment Θ is repeated exactly 2^{k-1} number of times.

Step 3 : claim: Result true for $n = k + 1$ number of exhaustive elements frame of discernment Θ.

Let $\Theta = \{\theta_1, \theta 2, \ldots, \theta_k\}$
Now we obtain frame of discernment $\Theta^{`}$ from Θ by adjoining element θ_{k+1}. Therefore

$$\Theta^{`} = \{\theta_1, \theta 2, \ldots, \theta_k, \theta_{k+1}\}$$

$$\text{and} \qquad |\Theta^{`}| = k+1$$

Partitioning power set of $\Theta^{`}$ into X and Y such that

$$X = \{ \text{ subsets of } \Theta^{`} \text{ not containing } \theta_{k+1} \ \} = \Theta \text{ and}$$

$$Y = \{ \text{ subsets of } \Theta^{`} \text{ containing } \theta_{k+1} \ \}$$

$$= \{ \text{ Element of } X \cup \theta_{k+1}\}$$

Here number of subsets in $X =$ Number of subsets in Y.
By inspecting set Y, we notice that element θ_{k+1} lies in every subset in Y. As every non-empty subset of frame of discernment is expressed as union of its singleton subsets i.e. exhaustive elements of frame of discernment. Hence element θ_{k+1} is repeated exactly 2^k times.

Here every element of frame of discernment $\Theta^{`}$ except element $\theta_{k+1}$, is repeated same number of times in $X$ and $Y$. By step 2 of principle of mathematical induction, every element of frame of discernment $\Theta$ is repeated exactly $2^{k-1}$ times. In all every element of frame of discernment $\Theta^{`}$ except θ_{k-1} is repeated exactly $2^{k-1} + 2^{k-1} = 2^k$ times.
Thus every element of frame of discernment $\Theta^{`}$ is repeated exactly 2^k times. i.e. result is true for all frame of discernments having $n = k+1$ exhaustive elements.
By principle of mathematical induction, every element of frame of discernment Θ having n exhaustive elements is repeated exactly 2^{n-1} times.

Part II:
In this part we will prove that sum of probabilities of all subsets of frame of discernment Θ is 2^{n-1}.
Since the elements θ_i of frame of discernment Θ are exhaustive i.e. $\forall\, \theta_i, \theta_j \in \Theta \quad \theta_i \cap \theta_j = \emptyset$,

$$p(\theta_i \cap \theta_j) = 0, \quad \forall\, \theta_i, \theta_j \in \Theta$$

By repeated application of above result, we get

$$p(\theta_i \cap \theta_j \cap \cdots \cap \theta_k) = 0, \quad \forall\, \theta_i, \theta_j, \ldots, \theta_k \in \Theta$$

Hence

$$p(\theta_i \cup \theta_j) = p(\theta_i) + p(\theta_j), \quad \forall\, \theta_i, \theta_j$$

i.e. In general,

$$p(\theta_i \cup \theta_j \cup \cdots \cup \theta_k) = p(\theta_i) + p(\theta_j) + \cdots + p(\theta_k), \quad \forall\, \theta_i, \theta_j$$

i.e.

$$p(\cup_i \theta_i) = \sum_i p(\theta_i), \forall\, \theta_i$$

i.e.

$$\text{for} \quad A \in \Theta, p(A) = p(\cup_{\theta_i \in A} \theta_i) = \sum_{\theta_i \in A} p(\theta_i).$$

i.e. Probability of any subset A of Θ is summation of probabilities of exhaustive elements θ_i of frame of discernment contained in A. Thus, while adding probabilities all subsets of frame of discernment Θ, probability of exhaustive element goes on adding equal to number of times concerned exhaustive element is repeated.

By Part I, every exhaustive element $\theta_i \in \Theta$ is repeated 2^{n-1} times. Therefore sum of probabilities due to single exhaustive element is $2^{n-1}p(\theta_i)$.

By applying same criteria for all exhaustive elements of frame of discernment Θ, we get

$$\begin{aligned}\sum_{A \in \Theta} p(A) &= \sum_{A \in \Theta} \sum_{\theta_i \in \Theta} p(\theta_i), \\ &= \sum_{\theta_i \in \Theta} 2^{n-1} p(\theta_i) \\ &= 2^{n-1} \sum_{\theta_i \in \Theta} p(\theta_i) \\ &= 2^{n-1}.\end{aligned} \tag{23}$$

$\therefore$ Sum of probabilities of all subsets of frame of discernment Θ is 2^{n-1}.

Remark 1 : If $|\Theta| = n$ then by counting $\sum_{\theta_i \in \Theta} p(\theta_i) = \sum_{i=1}^{n} p(\theta_i)$.

$$\begin{aligned} \text{Now consider } \sum_{A \subseteq \Theta} p(A) &= \sum_{A \subseteq \Theta} \sum_{\theta_i \in A} p(\theta_i) \\ 2^{n-1} &= \sum_{A \subseteq \Theta} \sum_{\theta_i \in A} p(\theta_i) \\ \text{i.e. } \quad \sum_{A \subseteq \Theta} \sum_{\theta_i \in A} p(\theta_i) &= 2^{n-1}. \end{aligned} \tag{24}$$

Remark 2 : Also, we observe that if $|\Theta| = n$ then any $\{\theta_i\} \in \Theta$ is repeated 2^{n-1} times i.e. $\{\theta_1\}$ appears 2^{n-1} times in subsets of Θ. Therefore the probability corresponding to $\{\theta_i\} \in \Theta$ is added 2^{n-1} times. Also, if $|\Theta| = n$ then $\sum_{A \in \Theta} p(A) = 2^{n-1}$. Hence in order to get $\sum_{A \in \Theta} p(A) = 1$, we have to divide each probability entry by a quantity 2^{n-1}.

$$\therefore m(\{\theta_i\}) = \frac{p(\{\theta_i\})}{2^{n-1}}, \quad \forall \theta_i \in \Theta. \tag{25}$$

Remark 3 : Now, let $A = \{\{\theta_1\}, \{\theta_2\}, \ldots, \{\theta_k\}\} \subseteq \Theta$. In discrete space, since singletons are disjoint hence the intersection of any number of singleton subsets of Θ is always empty set. Therefore

$$\begin{aligned} p(A) &= p(\{\theta_1\} \cup \{\theta_2\} \cup \cdots \cup \{\theta_k\}) \\ &= p(\{\theta_1\}) + p(\{\theta_2\}) + \cdots + p(\{\theta_k\}) \\ &= 2^{n-1} * m(A), \quad \text{where} \quad m(A) = \sum_{\{\theta\} \in A} m(\{\theta\}) \end{aligned} \tag{26}$$

Therefore, we get

$$m(A) = \frac{p(A)}{2^{n-1}}, \forall A \subseteq \Theta. \tag{27}$$

Now we check for some properties which are satisfied by this new transformation:

Theorem 4.2 : The function $m : 2^{\Theta} \to [0, 1]$ defined by (27), $m(A) = \dfrac{P(A)}{2^{n-1}}$ is a basic probability assignment.

Proof : As $0 \le p(A) \le 1,$ and for $\ n \ge 1, \quad 2^{n-1} \ge 0.$
$\Rightarrow 0 \le m(A) = \dfrac{p(A)}{2^{n-1}} \le 1.$

As $p(\emptyset) = 0,$ and for$n \ge 1, \quad 2^{n-1} \ge 0.$
$\Rightarrow m(\emptyset) = \dfrac{p(\emptyset)}{2^{n-1}} = 0.$

$$\begin{aligned} \text{Consider } \sum_{A\subset\Theta} m(A) &= \sum_{A\subset\Theta} \frac{p(A)}{2^{n-1}} \\ &= \frac{1}{2^{n-1}} \sum_{A\subset\Theta} p(A) = 1. \end{aligned} \tag{28}$$

Therefore, $m(A) = \dfrac{p(A)}{2^{n-1}}$ is a basic probability assignment.

By using above *bba* (27), we have $m(A) = \dfrac{p(A)}{2^{n-1}}$, the belief function $Bel : 2^{\Theta} \to [0,1]$ becomes

$$\begin{aligned} Bel(A) &= \sum_{B\subset A} m(B) \\ &= \sum_{B\subset A} \frac{p(B)}{2^{n-1}}. \end{aligned} \tag{29}$$

Result : In [6], If A is a finite set then

$$\sum_{B\subset A} (-1)^{|B|} = \begin{cases} 1 & \text{if } A = \emptyset \\ 0 & \text{otherwise.} \end{cases} \tag{30}$$

Here, we give some deductions of some theorems in Shafer‘s book [6] by using (27).

Theorem 4.3 : The belief function $Bel : 2^{\Theta} \to [0,1]$ defined by (27), $Bel(A) = \sum_{B\subset A} \dfrac{p(B)}{2^{n-1}}$ satisfies,

1. $Bel(\emptyset) = 0$.
2. $Bel(\Theta) = 1$.
3. **Sub-additive Property :-** For collection $A_1, A_2, \ldots, A_n$,

$$Bel(\cup_{i=1}^{n} A_i) \geq \sum_{\substack{I\subset 1,2,\ldots,n \\ I\neq\emptyset}} (-1)^{|A|+1} Bel(\cap_{i=1}^{n} A_i).$$

Proof : By definition, $Bel(A) = \sum_{B\subset A} \dfrac{p(B)}{2^{n-1}}$, we have

$$\begin{aligned} Bel(\emptyset) &= \sum_{B\subset A=\emptyset} m(B) \\ &= \sum_{B\subset\emptyset} \frac{p(B)}{2^{n-1}} = 0. \end{aligned} \tag{31}$$

$$\begin{aligned} Bel(\Theta) &= \sum_{B\subset A} m(B) \\ &= \sum_{B\subset\Theta} \frac{p(B)}{2^{n-1}} \\ &= \frac{1}{2^{n-1}} \sum_{B\subset\Theta} p(B) = 1. \end{aligned} \tag{32}$$

For a collection $A_1, A_2, \ldots, A_n$ of subsets of Θ. Let $I(B) = \{i|1 \leq i \leq n, B \subset A_i\}$, for each $B \subset \Theta$.

$$\begin{aligned} Consider & \sum_{\substack{I\subset\{1,2,\ldots,n\}\\ I\neq\emptyset}} (-1)^{|I|+1} Bel(\cap_{i=1}^{n} A_i) \\ &= \sum_{\substack{I\subset\{1,2,\ldots,n\}\\ I\neq\emptyset}} (-1)^{|I|+1} \sum_{\substack{B\subset\cap A_i\\ i\in A_i}} m(B) \\ &= \sum_{\substack{B\subset\Theta\\ I(B)\neq\emptyset}} m(B) \sum_{\substack{I\subset I(B)\\ I\neq\emptyset}} (-1)^{|I|+1} \\ &= \sum_{\substack{B\subset\Theta\\ I(B)\neq\emptyset}} m(B)(1 - \sum_{I\subset I(B)} (-1)^{|I|}) \\ &= \sum_{\substack{B\subset\Theta\\ I(B)\neq\emptyset}} m(B) \text{ .... By result above} \\ &= \sum_{\substack{B\subset\Theta\\ B\subset A_i,\text{ for some } i}} m(B) \\ &= \sum_{\substack{B\subset\Theta\\ B\subset A_i,\text{ for some } i}} \frac{p(B)}{2^{n-1}} \\ &\leq \sum_{B\subset\cup A_i} \frac{p(B)}{2^{n-1}} \\ &= \sum_{B\subset\cup A_i} m(B) \\ &= Bel(\cup A_i). \end{aligned}$$

Theorem 4.4 : Suppose $Bel : 2^{\Theta} \to [0,1]$ is the belief function given by basic probability assignment $m : 2^{\Theta} \to [0,1]$. Then

$$m(A) = \sum_{B\subset A} (-1)^{|A-B|} Bel(B); \text{ for all } A \subset \Theta.$$

is a basic probability assignment.

Proof : Consider a function $Bel : 2^{\Theta} \to [0,1]$ satisfying conditions in above theorem and define a function m on 2^{Θ} by

$$\begin{aligned} m(A) &= \sum_{B \subset A} (-1)^{|A-B|} Bel(B) \\ &= \sum_{B \subset A} (-1)^{|A-B|} \sum_{C \subset B} m(C) \\ &= \sum_{B \subset A} (-1)^{|A-B|} \sum_{C \subset B} \frac{p(C)}{2^{n-1}} \end{aligned} \tag{33}$$

Claim : $m(A)$ is a basic probability assignment.

$$\begin{aligned} \text{Consider } m(\emptyset) &= \sum_{B \subset A = \emptyset} (-1)^{|A-B|} \sum_{C \subset B} \frac{p(C)}{2^{n-1}} \\ &= \sum_{B \subset A = \emptyset} (-1)^{|A-B|} \sum_{C \subset B = \emptyset} \frac{p(C)}{2^{n-1}} \\ &= \sum_{B \subset A = \emptyset} (-1)^{|A-B|} \sum_{C \subset B = \emptyset} \frac{p(\emptyset)}{2^{n-1}} \quad \text{since } C \subset B \subset A = \emptyset \Rightarrow C = \emptyset. \\ &= \sum_{B \subset A = \emptyset} (-1)^{|A-B|} \sum_{C \subset B = \emptyset} \frac{0}{2^{n-1}} \quad \text{Since } p(\emptyset) = 0 \\ &= \sum_{B \subset A = \emptyset} (-1)^{|A-B|} 0. \\ &= 0. \end{aligned}$$

$$\begin{aligned} \text{We have } \sum_{A \subset \Theta} m(A) &= Bel(\Theta) \\ &= 1. \end{aligned}$$

Let $A = \{\theta_1, \theta_2, \ldots, \theta_n\}$, where $n \geq 1$ and θ_i are distinct.
Let $A_i = A - \theta_i$.
$\Rightarrow A_1, A_2, \ldots, A_n$ are subsets of A which excludes only one element of A.
$\Rightarrow$ Every proper subset B of A can be uniquely expressed as an intersection of A_i.

i.e. If $A - B = \{\theta_{i_1}, \theta_{i_2}, \ldots, \theta_{i_k}\}$ then $B = A_{i_1} \cap A_{i_2} \cap \cdots, A_{i_k}$.

$$\begin{aligned}
\text{Therefore } m(A) &= \sum_{B \subset A} (-1)^{|A-B|} Bel(B) \\
&= Bel(A) + \sum_{\substack{I \subset 1,2,\ldots,n \\ I \neq \emptyset}} (-1)^{|I|} Bel(\cap_{i \in I} A_i) \\
&= Bel(A) - \sum_{\substack{I \subset 1,2,\ldots,n \\ I \neq \emptyset}} (-1)^{|I|+1} Bel(\cap_{i \in I} A_i) \\
&\geq 0.
\end{aligned}$$

Here $A = A_1 \cup A_2, \cup \cdots \cup A_n$ and by sub-additivity property of belief functions. $\therefore m(A) \geq 0$.

$\therefore$, By above three claims 1, 2 and 3, $m(A)$ is a basic probability assignment.

The basic probability assignment function defined by (27), $m(A) = \dfrac{p(A)}{2^{n-1}}$, for any subset $A \in \Theta$, is a non-decreasing function. Also The belief function defined by basic probability assignment (27), $m(A) = \dfrac{1}{2^{n-1}} p(A)$, for any subset $A \in \Theta$, is a non-decreasing function. $Bel(\theta) = 1$, $Bel(\phi) = 0$ and if $A \neq \Theta, A \neq \emptyset$ i. e. $\emptyset \subseteq A \subseteq \Theta$, then $0 < Bel(A) < 1$. In all, $0 \leq Bel(A) \leq 1, \forall A \subseteq \Theta$. The belief function defined by basic belief assignment (27), $m(A) = \dfrac{p(A)}{2^{n-1}}$, for any subset A of Θ, is a probability measure.

Note : We have result in [6] as:

Suppose Θ is a finite set and f and g are functions on 2^{Θ}. Then

$$f(A) = \sum_{B \subset A} g(B)$$

for all $A \subset \Theta$ if and only if

$$g(A) = \sum_{B \subset A} (-1)^{|A-B|} f(B), \text{ for all } A \subset \Theta.$$

In this result, we can substitute $f \equiv Bel$ and $g \equiv m$ on 2^{Θ}. Suppose $\mathcal{C}$ is the core of a belief function Bel over Θ. Then a subset $B \subset \Theta$ satisfies $Bel(B) = 1$ if and only if $\mathcal{C} \subset B$. Let $\mathcal{C}$ be collection of subsets of Θ having non-zero probability. Then a subset $B \subset \Theta$ satisfies $Bel(B) = 1$ if and only if $\mathcal{C} \subset B$.

With our new basic belief assignment, relation between belief function and commonality function by using (27), reduces to

$$\sum_{C\subset A} p(C) = \sum_{B\subset \bar{(A)}} (-1)^{|B|} \sum_{B\subset C} p(C)$$
$$\text{and } \sum_{A\subset C} p(C) = \sum_{B\subset A} (-1)^{|B|} \sum_{C\subset \bar{(B)}} p(C), \text{ for all } A\subset\Theta.$$

Notes :

1. If $A=\Theta$ then $p(A)=1$.
   $\therefore \quad Q(A) = \frac{1}{2^{n-1}} \sum_{B\supseteq A} p(B) = m(\Theta)$.

2. If $A=\emptyset$ then $p(A)=0$.
   $\therefore \quad Q(A) = \frac{1}{2^{n-1}} \sum_{B\supseteq A} p(B) = 1$.

3. If $A\neq\Theta, A\neq\emptyset$ i. e. $\emptyset\subset A\subset\Theta$, then

$$Q(A) = \frac{1}{2^{n-1}} \sum_{B\supseteq A} \sum_{\{b\}\in B} p(\{b\}).$$

In all, $\frac{1}{2^{n-1}} \leq Q(A) \leq 1, \forall A\subseteq\Theta$.
With our new basic belief assignment, theorem (27) in Shafer‘s book [6] becomes

Theorem 4.5 : Suppose $\mathcal{C}$ is a collection of subsets of Θ having non-zero probability and Q is a commonality function of a belief function over Θ. Then an element θ is in $\mathcal{C}$ if and only if $m(\{\theta\})>0$ i.e $p(\{\theta\})>0$.

Remark : In necessary and sufficient condition for subset of Θ to lie in core $\mathcal{C}$, we can not get focal element in core having zero commonality number because non-zero probability give rise to non-zero commonality number by the transformation (27), $m(A) = \frac{p(A)}{2^{n-1}}$.

Determination of value K :-

We have

$$K = \Big(\sum_{\substack{A\subset\Theta \\ A\neq\emptyset}} (-1)^{|A|+1} q(A)\Big)^{-1}$$

with $Q(A) = Kq(A)$ where $q : (2^{\Theta}-\emptyset) \to [0,\infty)$ is a known function. By (27), we have

$Q(A) = \sum_{A\subset B} \dfrac{p(B)}{2^{n-1}}$,

$$\begin{aligned} & Q(A) = Kq(A) \\ & \Rightarrow \sum_{A\subset B} \frac{p(B)}{2^{n-1}} = Kq(A) \\ & \Rightarrow K = \frac{\sum_{A\subset B} \frac{p(B)}{2^{n-1}}}{q(A)} \end{aligned} \tag{34}$$

Here we get two values of K and both are equal. Therefore

$$K = \Big(\sum_{\substack{A\subset\Theta \\ A\neq\emptyset}} (-1)^{|A|+1} q(A)\Big)^{-1} = \frac{\sum_{A\subset B} \frac{p(B)}{2^{n-1}}}{q(A)}. \tag{35}$$

Theorem 4.6 : A belief function $Bel : 2^{\Theta} \to [0,1]$ obtained by *bba* (27), $m(A) = \dfrac{p(A)}{2^{n-1}}$, is a Bayesian belief function.

Proof : We have $Bel(\emptyset) = 0$ and $Bel(\Theta) = 1$. Now it is enough to show that $Bel(A \cup B) = Bel(A) + Bel(B)$, if $A \cap B = \emptyset$.

Consider two distinct subsets $A, B \in \Theta$, such that $p(A) \neq 0$ and $p(B) \neq 0$.
By *bba* $m(A) = \dfrac{p(A)}{2^{n-1}}$, we get

$$m(A) = \frac{p(A)}{2^{n-1}} \neq 0, \text{ and } m(B) = \frac{p(B)}{2^{n-1}} \neq 0.$$

We have $p(A \cup B) = p(A) + p(B) - p(A \cap B)$. Since A and B are disjoint subsets of Θ

then $A \cap B = \emptyset$ hence $p(A \cap B) = p(\emptyset) = 0$. $\therefore m(A \cap B) = 0$ hence $Bel(A \cap B) = 0$.

$$\begin{aligned}
\text{Consider } Bel(A \cup B) &= \sum_{C \subset A \cup B} m(C) \\
&= \sum_{C \subset A \cup B} \frac{p(C)}{2^{n-1}} \\
&= \frac{1}{2^{n-1}} \sum_{C \subset A \cup B} p(C) \\
&= \frac{1}{2^{n-1}} [\sum_{C \subset A} p(C) + \sum_{C \subset B} p(C) - \sum_{C \subset A \cap B} p(C)] \\
&\text{Since } A \text{ and } B \text{ are disjoint subsets in } \Theta \\
&= \sum_{C \subset A} \frac{p(C)}{2^{n-1}} + \sum_{C \subset B} \frac{p(C)}{2^{n-1}} - \sum_{C \subset A \cap B} \frac{p(C)}{2^{n-1}} \\
&= \sum_{C \subset A} \frac{p(C)}{2^{n-1}} + \sum_{C \subset B} \frac{p(C)}{2^{n-1}}, \\
&\text{since } p(A \cap B) = 0
\end{aligned} \tag{36}$$

hence for any subset $C \subset A \cap B$.

$$\begin{aligned}
&= \sum_{C \subset A} m(C) + \sum_{C \subset B} m(C) \\
&= Bel(A) + Bel(B).
\end{aligned}$$

Hence for disjoint subsets $A, B \in \Theta, Bel(A \cup B) = Bel(A) + Bel(B)$.
Therefore, belief function $Bel : 2^{\Theta} \to [0, 1]$ obtained by *bba*
$m(A) = \dfrac{p(A)}{2^{n-1}}$, is Bayesian belief function.
In Shafer's book [6], we have following theorem and note as:

Theorem 4.7 : A belief function $Bel : 2^{\Theta} \to [0, 1]$ is Bayesian if and only if its basic probability assignment m is given by

$$m(\{\theta\}) = Bel(\{\theta\})$$
$$\text{and } m(A) = 0, \text{ for all non-singleton subsets } A \text{ of } \Theta .$$

Note : If Bel is Bayesian belief function then the function m is unique.
But belief function Bel defined by our transformation (27), $m(A) = \dfrac{p(A)}{2^{n-1}}$ is a Bayesian belief function hence uniqueness of Bayesian belief function is not true. Also it is generalization of basic probability assignment in above theorem.

Plausibility Function : by (27), we have, for any $A \subseteq \Theta$,

$$\begin{aligned} Pl(A) &= \sum_{B\cap A\neq\emptyset} m(B) \\ &= \frac{1}{2^{n-1}} \sum_{B\cap A\neq\emptyset} \sum_{\{b\}\in B} p(\{b\}). \end{aligned} \tag{37}$$

Notes :

1. If $A = \Theta$ then $p(A) = 1$.
   $\therefore Pl(A) = \frac{1}{2^{n-1}} \sum_{B\cap A\neq\emptyset} p(B) = 1$

2. If $A = \emptyset$ then $p(A) = 0$.
   $\therefore Pl(A) = \frac{1}{2^{n-1}} \sum_{B\cap A\neq\emptyset} p(B) = 0$

3. If $A \neq \Theta, A \neq \emptyset$ i. e. $\emptyset \subset A \subset \Theta$, then

$$Pl(\emptyset) < Pl(A) = \frac{1}{2^{n-1}} \sum_{B\cap A\neq\emptyset} \sum_{\{b\}\in B} p(\{b\}) < Pl(\Theta).$$
$$\Rightarrow 0 < Pl(A) < 1$$

In all, $0 \leq Pl(A) \leq 1, \forall A \subseteq \Theta$.

Theorem 4.8 : Suppose $Bel : 2^{\Theta} \to [0,1]$ obtained by *bba* (27), $m(A) = \frac{p(A)}{2^{n-1}}$, is Belief function with plausibility function Pl. Then following assertions are equivalent:

1 Bel is Bayesian.

2 $Bel(A) + Bel(\bar{A}) = 1, \quad \forall \quad A \subseteq \Theta$.

Proof : As we know that $A \cap \bar{A} = \emptyset$ and $A \cup \bar{A} = \Theta$. Consider

$$A \cup \bar{A} = \Theta$$
$$\Rightarrow Bel(A) + Bel(\bar{A}) = 1, \text{by Bayesian belief function } Bel.$$

Conversely, assume that $Bel(A) + Bel(\bar{A}) = 1$. Consider

$$Bel(A) + Bel(\bar{A}) = Bel(A \cup \bar{A}) - Bel(A \cap \bar{A})$$
$$\Rightarrow Bel(\emptyset) = 0$$

By combining it with $Bel(A \cup B) = Bel(A) + Bel(B) - Bel(A \cap B)$ we get

$$\Rightarrow Bel(A \cup B) = Bel(A) + Bel(B) \text{ if } A \cap B = \emptyset \text{ and } Bel(\emptyset) = 0.$$

Therefore Bel is a Bayesian.

Theorem 4.9 : For all $A \subset \Theta$,

$$Bel(A) = \frac{p(A)}{2^{n-k}} \text{ and } Pl(A) = p(A) + \frac{(2^k - 1)}{2^k} p(\bar{A}). \tag{38}$$

where $n = |\Theta|$ and $k = |A|$.

Proof : Let $n = |\Theta|$ and $k = |A|$, for some $A \subset \Theta$. WOLOG, we assume that $A = \{A_1, A_2, \ldots, A_k\}$, where $A_j, j = 1, 2, \ldots, k$, are singlton sets. Let $p(A_j) = s_j, \quad j = 1, 2, \ldots, k$ and $p(A) = \sum_{i=1}^{k} p(A_j) = \sum_{i=1}^{k} s_j$. Therefore by (27), $m(A_j) = \frac{s_j}{2^{n-1}}$ and $m(A) = \frac{\sum_{i=1}^{k} p(A_j)}{2^{n-1}}$. Consider

$$\begin{aligned}
Bel(A) &= \sum_{B \subseteq A} m(B) \\
&= \frac{1}{2^{n-1}} \{ \sum_{B \subseteq A} p(B) \} \\
&= \frac{1}{2^{n-1}} \{ s_1 + s_2 + \cdots + s_k + s_1 + s_2 + s_1 + s_3 + \cdots + s_1 + s_k \\
&+ s_2 + s_3 + s_2 + s_4 + \cdots + s_2 + s_k + \cdots + s_{k-1} + s_k \\
&+ \cdots + s_1 + s_2 + \cdots + s_k \} \qquad (39) \\
&= \frac{1}{2^{n-1}} \{ 2^{k-1} (s_1 + s_2 + \cdots + s_k) \} \\
&= \frac{1}{2^{n-k}} \sum_{A_j \in A} p(A_j) \\
&= \frac{1}{2^{n-k}} p(A) \\
&= \frac{p(A)}{2^{n-k}}.
\end{aligned}$$

$$\begin{aligned}
\text{Consider } Pl(A) &= 1 - Bel(\bar{A}) \\
&= 1 - \frac{p(\bar{A})}{2^{n-(n-k)}} \\
&= \frac{2^k - p(\bar{A})}{2^k} \\
&= \frac{2^k(p(A) + p(\bar{A})) - p(\bar{A})}{2^k} \\
&= \frac{2^k p(A) + (2^k - 1)p(\bar{A})}{2^k} \\
&= p(A) + \frac{(2^k - 1)}{2^k} p(\bar{A}).
\end{aligned} \tag{40}$$

Notes :

1 By above theorem, we have $Bel(A) \leq p(A) \leq Pl(A), \quad \forall A \subseteq \Theta$.

2 If Bel is a Bayesian belief function then the function p is given by $p(\theta) = 2^{n-1}m(\theta)$.

we have pignistic probability function by using (4), (5) and (27) as:

$$\begin{aligned}
Bet \quad p(x) &= \sum_{\substack{A \subseteq \mathcal{A}, \\ A \neq \emptyset}} \frac{m(A)}{1 - m(\emptyset)} \frac{I_A(x)}{|A|}, \quad \forall x \in \mathcal{A}. \\
&= \frac{1}{1 - m(\emptyset)} \sum_{\substack{A \subseteq \mathcal{A}, \\ A \neq \emptyset}} m(A) \frac{I_A(x)}{|A|}. \\
&= \frac{1}{1 - m(\emptyset)} \sum_{\substack{A \subseteq \mathcal{A}, \\ A \neq \emptyset}} \frac{p(A)}{2^{n-1}} \frac{I_A(x)}{|A|}. \\
&= \frac{1}{1 - m(\emptyset)} \sum_{\substack{A \subseteq \mathcal{A}, \\ A \neq \emptyset}} \frac{\sum_{\{a\} \in A} p(\{a\})}{2^{n-1}} \frac{I_A(x)}{|A|}.
\end{aligned} \tag{41}$$

Also we have $Bet \quad p(A) = \sum_{x \in A} Bet \quad p(x)$.

5. Some Results about Finding Belief Functions, Commonality Functions and Plausibility Functions

Theorem 5.1 : If $|\Theta| = n \quad 0 \leq r \leq n$ then the no. of subsets of Θ containing r elements of Θ are 2^{n-r}.

Theorem 5.2 : If $|\Theta| = n$ and $|A| = r$, then number of subsets B of Θ such that $A \cap B \neq \emptyset$ are $2^{n-r}(2^r - 1)$.

Here we use result that singletons in discrete space, are disjoint therefore for any subsets A and B of Θ, $\quad p(A \cup B) = p(A) + p(B)$.

Theorem 5.3 : For any subset A of

$$\Theta, \quad Bel(A) = \frac{p(A)}{2^{n-r}} \text{ where } |\Theta| = n \text{ and } |A| = r. \tag{42}$$

Theorem 5.4 : For any $A \subset \Theta$ with $|\Theta| = n, |A| = r$ then

$$Q(A) == 2^{1-r} * \sum_{\{a\} \in A} p(\{a\}) + \frac{1}{2^r)} * \sum_{\{a\} \notin A} p(\{a\}). \tag{43}$$

Theorem 5.5 : For any subset A of Θ with $|\Theta| = n$ and $|A| = r$,

$$Pl(A) = \sum_{\{a\} \in A} p(\{a\}) + \sum_{\{a\} \notin A} (1 - \frac{1}{2^r}) p(\{a\}). \tag{44}$$

In general, observing carefully, we noticed that number of repetitions of element of Θ is as follows:

Let $|A| = r$ and $|\Theta| = n$ if $\{a\} \in \Theta$ then $\{a\}$ appears 2^{n-1} times and if $\{a\} \notin \Theta$ then $\{a\}$ appears $2^{n-2} + 2^{n-3} + 2^{n-4} + \cdots + 2^{n-(r+1)}$ times hence $2^{n-(r+1)}(2^r - 1)$ times. Therefore formula for plausibility function becomes

$$\begin{aligned} Pl(A) &= \sum_{A \cap B \neq \emptyset} m(B) \\ &= 2^{n-1} \sum_{\{a\} \in A} \frac{p(\{a\})}{2^{n-1}} + 2^{n-(r+1)}(2^r - 1) \sum_{\{a\} \notin A} \frac{p(\{a\})}{2^{n-1}} \\ &= \sum_{\{a\} \in A} p(\{a\}) + 2^{n-(r+1)-n+1}(2^r - 1) \sum_{\{a\} \notin A} p(\{a\}) \\ &= \sum_{\{a\} \in A} p(\{a\}) + 2^{-r}(2^r - 1) \sum_{\{a\} \notin A} p(\{a\}) \\ &= \sum_{\{a\} \in A} p(\{a\}) + (1 - \frac{1}{2^r}) \sum_{\{a\} \notin A} p(\{a\}). \end{aligned} \tag{45}$$

6. Conclusion

In this paper, we have defined new basic belief assignment induced by probability mass function of discrete probability distribution. With this, we have deduced some theorems about basic belief assignment in Shafer book [6]. Also we have obtained formulae to calculate belief, commonality and plausibility functions in this regard.

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