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CONNECTED TOTAL DOMINATING SETS AND CONNECTED TOTAL DOMINATION POLYNOMIALS OF THE FAN GRAPH $F_{2,n}$

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Abstract

Let $G = (V, E)$ be a simple graph. A set S of vertices in G is said to be a total dominating set if each vertex $v \in V$ is adjacent to an element of S. A total dominating set S of G is called a connected total dominating set if the induced subgraph $\langle S \rangle$ is connected. In this paper, we study the concept of connected total domination polynomials of the Fan graph $F_{2,n}$. The connected total domination polynomial of a graph G of order n is the polynomial $D_{ct}(G, x) = \sum_{i=\gamma_{ct}(G)}^{n} d_{ct}(G, i)x^i$, where $d_{ct}(G, i)$ is the number of connected total dominating sets of G of size i and $\gamma_{ct}(G)$ is the connected total domination number of G. We obtain some properties of $D_{ct}(F_{2,n}, x)$ and their coefficients. Also, we obtain the recursive formula to derive the connected total dominating sets of the Fan graph $F_{2,n}$.

1. Introduction

Let $G = (V, E)$ be a simple graph of order $|V| = n$. A set S of vertices in a graph G is said to be a dominating set if every vertex $v \in V$ is either an element of S or is adjacent to an element of S. A set S of vertices in a graph G is said to be a total dominating set if every vertex $v \in V$ is adjacent to an element of S. A total dominating set S of G is

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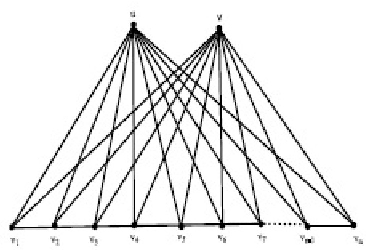
called a connected total dominating set if the induced subgraph $\langle S\rangle$ is connected. The minimum cardinality of a connected total dominating set S of G is called the connected total domination number and is denoted by $\gamma_{ct}(G)$.

Let $F_{2,n}$ be the Fan graph with $n+2$ vertices. In the next section, we construct the families of connected total dominating sets of $F_{2,n}$ by recursive method. In Section 3, we use the results obtained in Section 2 to study the connected total domination polynomials of the Fan graph $F_{2,n}$.

As usual we use $\binom{n}{i}$ for the combination n to i.

2. Connected Total Dominating Sets of the Fan Graph $F_{2,n}$

Fan graph $F_{2,n}$ [5] is a graph obtained by joining two vertices u and v to every vertices of a path graph P_n.



Let $F_{2,n}$ be a Fan graph with $n+2$ vertices. Label the vertices of $F_{2,n}$ as $v_1, v_2, v_3, \cdots,$ v_{n+1}, v_{n+2}. Then, $V(F_{2,n}) = \{v_1, v_2, v_3, \cdots, v_{n+1}, v_{n+2}\}$ and $E(F_{2,n}) =$ $\{(v_1, v_3), (v_1, v_4), (v_1, v_5), \cdots (v_1, v_{n+1}), (v_1, v_{n+2}), (v_2, v_3), (v_2, v_4), (v_2, v_5), \cdots, (v_2, v_{n+1}),$ $(v_2, v_{n+2}), (v_3, v_4), (v_4, v_5), (v_5, v_6), \cdots, (v_n, v_{n+1}), (v_{n+1}, v_{n+2})\}$. Let $d_{ct}(F_{2,n}, i)$ be the number of connected total dominating sets of $F_{2,n}$ with cardinality i.

Lemma 2.1 : The following properties hold for all graph G with $|V(G)| = n+2$ vertices.

(i) $d_{ct}(G, n+2) = 1$.

(ii) $d_{ct}(G, n+1) = n+2$.

(iii) $d_{ct}(G, i) = 0$ if $i > n+2$.

(iv) $d_{ct}(G, 1) = 0$.

Proof : The proof is given in [8].

Lemma 2.2 : For all $n \in Z^{+}$, $\binom{n}{i} = 0$ if $i > n$ or $i < 0$.

Lemma 2.3 : For any path graph P_n with n vertices,

(i) $d_{ct}(P_n, n) = 1$.

(ii) $d_{ct}(P_n, n-1) = 2$.

(iii) $d_{ct}(P_n, n-2) = 1$.

(iv) $d_{ct}(P_n, i) = 0$ if $i < n-2$ or $i > n$.

Theorem 2.4 : For any path graph P_n with n vertices, $D_{ct}(P_n, x) = x^{n-2} + 2x^{n-1} + x^n$.

Proof : The proof is given in [6].

Theorem 2.5 : For a complete bipartite graph $K_{m,n}$, the connected total domination polynomial is $D_{ct}(K_{m,n}, x) = [(1+x)^m - 1][(1+x)^n - 1]$.

Proof : The proof is given in [6].

Theorem 2.6 : Let $K_{2,n}$ be a complete bipartite graph with $n+2$ vertices, then $D_{ct}(K_{2,n}, x) = (x^2 + 2x)[(1+x)^n - 1]$.

Proof : The proof follows from Theorem 2.5 when $m = 2$.

Theorem 2.7 : Let $K_{2,n}$ be a complete bipartite graph with $n+2$ vertices, then,

$$D_{ct}(K_{2,n}, i) = \left\{ \begin{array}{ll} \binom{n+2}{i} - \binom{n}{i} - 1, & \text{if } i = 2 \\ \binom{n+2}{i} - \binom{n}{i} & \text{for all } 3 \le i \le n+2 \end{array} \right\}.$$

Proof : The proof follows from Theorem 2.6.

Theorem 2.8 : Let $F_{2,n}$ be a Fan graph with $n+2$ vertices, then $d_{ct}(F_{2,n}, i) = d_{ct}(K_{2,n}, i) + d_{ct}(P_n, i)$ for all i.

Proof : Let $K_{2,n}$ be a complete bipartite graph with partite sets $V_1 = \{v_1, v_2\}$ and $V_2 = \{v_3, v_4, \cdots, v_{n+2}\}$. Let $v_1, v_2 \in V(K_{2,n})$ are the two vertices adjacent to all the vertices of P_n. Then $K_{2,n}$ be a spanning subgraph of $F_{2,n}$ and since $F_{2,n} - K_{2,n} = P_n$, we have, $K_{2,n} \cup P_n = F_{2,n}$. Therefore, the number of connected total dominating sets of the Fan graph $F_{2,n}$ with cardinality i is the sum of the connected totaldominating sets of the complete bipartite graph $K_{2,n}$ with cardinality i and the number of connected total dominating sets of the path graph P_n with cardinality i. Hence, $d_{ct}(F_{2,n}, i) = d_{ct}(K_{2,n}, i) + d_{ct}(P_n, i)$ for all i.

Theorem 2.9 : Let $F_{2,n}, n \geq 2$ be a Fan graph with $n+2$ vertices, then

(i) $d_{ct}(F_{2,n}, i) = \binom{n+2}{i} - \binom{n}{i}$ for all $3 \leq i \leq n+2$ and $i \neq n-2, n-1, n$.

(ii) $d_{ct}(F_{2,n}, i) = \binom{n+2}{i} - \binom{n}{i} - 1$ if $i = 2$.

(iii) $d_{ct}(F_{2,n}, i) = \binom{n+2}{i} - \binom{n}{i} + 1$, for $i = n-2, n$.

(iv) $d_{ct}(F_{2,n}, i) = \binom{n+2}{i} - \binom{n}{i} + 2$ if $i = n-1$.

Proof : (i) By Theorem 2.8, we have, $d_{ct}(F_{2,n}, i) = d_{ct}(K_{2,n}, i) + d_{ct}(P_n, i)$ for all i. Since, $d_{ct}(P_n, i) = 0$ for all $i < n-2$ and $i > n$ we have,

$$\begin{aligned} d_{ct}(F_{2,n}, i) &= d_{ct}(K_{2,n}, i) \text{ for all } i < n-2 \text{ or} i > n \\ &= \binom{n+2}{i} - \binom{n}{i} \text{ for all } 3 \leq i \leq n+2 \end{aligned}$$

and $i \neq n-2, n-1, n$, by Theorem 2.7.

(ii) When $i = 2$ and $n \geq 5$, $d_{ct}(P_n, i) = 0$. Therefore,

$$\begin{aligned} d_{ct}(F_{2,n}, i) &= d_{ct}(K_{2,n}, i) \\ &= \binom{n+2}{i} - \binom{n}{i} - 1 \text{ by Theorem 2.7.} \end{aligned}$$

(iii) Since, $d_{ct}(P_n, i) = 1$ for $i = n-2, n$, we have,

$$d_{ct}(F_{2,n}, i) = \binom{n+2}{i} - \binom{n}{i} + 1 \text{ if } i = n-2, n.$$

(iv) Since, $d_{ct}(P_n, i) = 2$ if $i = n-1$, we have,

$$d_{ct}(F_{2,n}, i) = \binom{n+2}{i} - \binom{n}{i} + 2 \text{ if } i = n-1.$$

Theorem 2.10 : Let $F_{2,n}, n \geq 6$ be a Fan graph with $n+2$ vertices, then

(i) $d_{ct}(F_{2,n}, i) = d_{ct}(F_{2,n-1}, i) + 2$ if $i = 2$.

(ii) $d_{ct}(F_{2,n}, i) = d_{ct}(F_{2,n-1}, i) + d_{ct}(F_{2,n-1}, i-1) + 1$ if $i = 3$.

(iii) $d_{ct}(F_{2,n}, i) = d_{ct}(F_{2,n-1}, i) + d_{ct}(F_{2,n-1}, i-1) - 1$ for $i = n-1,\ n-3$.

(iv) $d_{ct}(F_{2,n}, i) = d_{ct}(F_{2,n-1}, i) + d_{ct}(F_{2,n-1}, i-1) - 2$ if $i = n-2$.

(v) $d_{ct}(F_{2,n}, i) = d_{ct}(F_{2,n-1}, i) + d_{ct}(F_{2,n-1}, i-1)$ for all $4 \leq i \leq n+2$ and $i \neq n-1, n-2, n-3$.

Proof : (i) When $i = 2$,

$$\begin{aligned} d_{ct}(F_{2,n}, 2) &= \binom{n+2}{2} - \binom{n}{2} - 1, \quad \text{by Theorem 2.9(ii)} \\ &= 2n+1-1 \\ d_{ct}(F_{2,n}, 2) &= 2n. \end{aligned}$$

Consider,

$$\begin{aligned} d_{ct}(F_{2,n-1}, 2) &= \binom{n+2}{2} - \binom{n-1}{2} - 1 \quad \text{by Theorem 2.9(ii)} \\ &= 2n-1-1 \\ &= 2n-2 \\ d_{ct}(F_{2,n-1}, 2) &= d_{ct}(F_{2,n}, 2) - 2. \end{aligned}$$

Therefore, $d_{ct}(F_{2,n}, 2) = d_{ct}(F_{2,n-1}, 2) + 2$.
Hence, $d_{ct}(F_{2,n}, i) = d_{ct}(F_{2,n-1}, i) + 2$ if $i = 2$.
(ii) When $i = 3$, $d_{ct}(F_{2,n}, 3) = \binom{n+2}{3} - \binom{n}{3}$, by Theorem 2.9 (i).
Consider,

$$d_{ct}(F_{2,n-1}, 3) + d_{ct}(F_{2,n-1}, 2) = \binom{n+1}{3} - \binom{n-1}{3} + \binom{n+1}{2} - \binom{n-1}{2} - 1,$$

by Theorem 2.9 (i), (ii).

$$= \binom{n+1}{3} + \binom{n+1}{2} - \left[\binom{n-1}{3} + \binom{n-1}{2}\right] - 1$$

$$= \binom{n+2}{3} - \binom{n}{3} - 1.$$

$$d_{ct}(F_{2,n-1}, 3) + d_{ct}(F_{2,n-1}, 2) = d_{ct}(F_{2,n}, 3) - 1.$$

Therefore, $d_{ct}(F_{2,n}, 3) = d_{ct}(F_{2,n-1}, 3) + d_{ct}(F_{2,n-1}, 2) + 1$.

Hence, $d_{ct}(F_{2,n}, i) = d_{ct}(F_{2,n-1}, i) + d_{ct}(F_{2,n-1}, i-1) + 1$ if $i = 3$.
(iii) When $i = n-1$,

$$\begin{aligned} d_{ct}(F_{2,n}, i) &= \binom{n+2}{i} - \binom{n}{i} + 2, \quad \text{by Theorem 2.9 (iv).} \\ d_{ct}(F_{2,n-1}, i) &= \binom{n+1}{i} - \binom{n-1}{i} + 1, \quad \text{by Theorem 2.9 (iii).} \\ d_{ct}(F_{2,n-1}, i-1) &= \binom{n+1}{i-1} - \binom{n-1}{i-1} + 2, \quad \text{by Theorem 2.9 (iv).} \end{aligned}$$

Consider,

$$\begin{aligned} & d_{ct}(F_{2,n-1}, i) + d_{ct}(F_{2,n-1}, i-1) \\ & = \binom{n+1}{i} - \binom{n-1}{i} + 1 + \binom{n+1}{i-1} - \binom{n-1}{i-1} + 2 \\ & = \binom{n+1}{i} + \binom{n+1}{i-1} - \left[\binom{n-1}{i} + \binom{n-1}{i-1}\right] + 2 + 1 \\ & = \binom{n+2}{i} - \binom{n}{i} + 2 + 1 \\ & = d_{ct}(F_{2,n}, i) + 1. \end{aligned}$$

Therefore, $d_{ct}(F_{2,n}, i) = d_{ct}(F_{2,n-1}, i) + (F_{2,n-1}, i-1) - 1$ if $i = n-1$.
When $i = n-3$,

$$\begin{aligned} d_{ct}(F_{2,n}, i) &= \binom{n+2}{i} - \binom{n}{i}, \quad \text{by Theorem 2.9 (i).} \\ d_{ct}(F_{2,n-1}, i) &= \binom{n+1}{i} - \binom{n-1}{i} + 1, \quad \text{by Theorem 2.9 (iii).} \\ d_{ct}(F_{2,n-1}, i-1) &= \binom{n+1}{i-1} - \binom{n-1}{i-1}, \quad \text{by Theorem 2.9 (i).} \end{aligned}$$

Consider,

$$\begin{aligned} & d_{ct}(F_{2,n-1}, i) + d_{ct}(F_{2,n-1}, i-1) \\ & = \binom{n+1}{i} - \binom{n-1}{i} + 1 + \binom{n+1}{i-1} - \binom{n-1}{i-1} \\ & = \binom{n+1}{i} + \binom{n+1}{i-1} - \left[\binom{n-1}{i} + \binom{n-1}{i-1}\right] + 1 \\ & = \binom{n+2}{i} - (\ n\ ) + 1 \\ & = d_{ct}(F_{2,n}, i) + 1. \end{aligned}$$

Therefore, $d_{ct}(F_{2,n}, i) = d_{ct}(F_{2,n-1}, i) + d_{ct}(F_{2,n-1}, i-1) - 1$ if $i = n-3$.

(iv) When $i = n-2$,

$$\begin{aligned} d_{ct}(F_{2,n}, i) &= \binom{n+2}{i} - \binom{n}{i} + 1 \quad \text{, by Theorem 2.9 (iii).} \\ d_{ct}(F_{2,n} - 1, i) &= \binom{n+1}{i} - \binom{n-1}{i} + 2, \quad \text{by Theorem 2.9 (iv).} \\ d_{ct}(F_{2,n-1}, i-1) &= \binom{n+1}{i-1} - \binom{n-1}{i-1} + 1, \quad \text{by Theorem 2.9 (iii).} \end{aligned}$$

Consider,

$$\begin{aligned} & d_{ct}(F_{2,n-1}, i) + d_{ct}(F_{2,n-1}, i-1) = \binom{n+1}{i} - \binom{n-1}{i} + 2 \\ & + \binom{n+1}{i-1} - \binom{n-1}{i-1} + 1 \\ & = \binom{n+1}{i} + \binom{n+1}{i-1} - \left[\binom{n-1}{i} + \binom{n-1}{i-1}\right] + 1 + 2 \\ & = \binom{n+2}{i} - \binom{n}{i} + 1 + 2 \\ & = d_{ct}(F_{2,n}, i) + 2. \end{aligned}$$

Theorefore, $d_{ct}(F_{2,n}, i) = d_{ct}(F_{2,n-1}, i) + d_{ct}(F_{2,n-1}, i-1) - 2$, if $i = n-2$.

(v) By Theorem 2.9 (i), we have $d_{ct}(F_{2,n}, i) = \binom{n+2}{i} - \binom{n}{i}$ for all $3 \leq i \leq n+2$ and $i \neq n-2, n-1, n$.

$$\begin{aligned} d_{ct}(F_{2,n-1}, i) &= \binom{n+1}{i} - \binom{n-1}{i} \\ d_{ct}(F_{2,n-1}, i-1) &= \binom{n+1}{i-1} - \binom{n-1}{i-1} \end{aligned}$$

Consider,

$$\begin{aligned} & d_{ct}(F_{2,n-1}, i) + d_{ct}(F_{2,n-1}, i-1) = \binom{n+1}{i} - \binom{n-1}{i} + \binom{n+1}{i-1} - \binom{n-1}{i-1} \\ & = \binom{n+1}{i} + \binom{n+1}{i-1} - \left[\binom{n-1}{i} + \binom{n-1}{i-1}\right] \\ & = \binom{n+2}{i} - \binom{n}{i} \\ & = d_{ct}(F_{2,n}, i). \end{aligned}$$

Therefore, $d_{ct}(F_{2,n}, i) = d_{ct}(F_{2,n-1}, i) + d_{ct}(F_{2,n-1}, i-1)$ for all $4 \leq i \leq n+2$ and $i \neq n-1, n-2, n-3$, by (i) (ii) (iii) and (iv).

3. Connected Total Domination Polynomials of the Fan Graph $F_{2,n}$

Definition 3.1 : Let $d_{ct}(F_{2,n}, i)$ be the number of connected total dominating sets of the Fan graph $F_{2,n}$ with cardinality i. Then, the connected total domination polynomial of $F_{2,n}$ is defined as,

$$D_{ct}(F_{2,n}, x) = \sum_{i=\gamma_{ct}(F_{2,n})}^{n+2} d_{ct}(F_{2,n}, i)x^i,$$

where $\gamma_{ct}(F_{2,n})$ is the connected total domination number of $F_{2,n}$.

Remark 3.2 : $\gamma_{ct}(F_{2,n}) = 2$.

Proof : Let $F_{2,n}$ be a Fan graph with $n+2$ vertices. Let $v_1, v_2 \in V(F_{2,n})$ and v_1, v_2 are adjacent to all the other vertices $v_3, v_4, v_5, \cdots, v_{n+1}, v_{n+2}$. The vertex v_1 or v_2 and one more vertex from $v_3, v_4, v_5, \cdots, v_{n+1}, v_{n+2}$ is enough to cover all the other vertices. Therefore, the minimum cardinality is 2.

Hence, $\gamma_{ct}(F_{2,n}) = 2$.

Theorem 3.3 : Let $F_{2,n}$, be a Fan graph with $n+2$ vertices, then

$$D_{ct}(F_{2,n}, x) = D_{ct}(K_{2,n}, x) + D_{ct}(P_n, x).$$

Proof : By the definition of connected total domination polynomial, we have

$$\begin{aligned} D_{ct}(F_{2,n}, x) &= \sum_{i=2}^{n+2} d_{ct}(F_{2,n}, i)x^i. \\ &= \sum_{i=2}^{n+2} [d_{ct}(K_{2,n}, i) + d_{ct}(P_n, i)]x^i, \quad \text{by Theorem 2.8.} \\ &= \sum_{i=2}^{n+2} d_{ct}(K_{2,n}, i)x^i + \sum_{i=2}^{n+2} d_{ct}(P_n, i)x^i. \\ &= D_{ct}(K_{2,n}, x) + D_{ct}(P_n, x). \end{aligned}$$

Therefore, $D_{ct}(F_{2,n}, x) = D_{ct}(K_{2,n}, x) + D_{ct}(P_n, x)$.

Theorem 3.4 : Let $D_{ct}(F_{2,n}, x)$ be the connected total domination polynomial of a Fan graph $F_{2,n}$ with $n+2$ vertices, then

$$D_{ct}(F_{2,n}, x) = \sum_{i=2}^{n+2} \binom{n+2}{i} x^i - \sum_{i=2}^{n+2} \binom{n}{i} x^i - x^2 + x^{n-2} + 2x^{n-1} + x^n.$$

Proof : The proof follows from Theorem 2.4, Theorem 2.7 and Theorem 3.3.

Theorem 3.5 : Let $D_{ct}(F_{2,n}, x)$ be the connected total domination polynomial of a Fan graph $F_{2,n}$ with $n+2$ vertices, then,

$$D_{ct}(F_{2,n}, x) = (1+x)D_{ct}(F_{2,n-1}, x) + 2x^2 + x^3 - x^{n-3} - 2x^{n-2} - x^{n-1}, n \geq 7.$$

Proof : By the definition of connected total domination polynomial, we have,

$$\begin{aligned} D_{ct}, (F_{2,n}, x) &= \sum_{i=2}^{n+2} d_{ct}(F_{2,n}, i)x^i. \\ &= \sum_{i=2}^{n+2} [d_{ct}(F_{2,n-1}, i) + d_{ct}(F_{2,n-1}, i-1)]x^i, \quad \text{by Theorem 2.10 (v)}. \\ &= \sum_{i=2}^{n+2} d_{ct}(F_{2,n-1}, i)x^i + \sum_{i=2}^{n+2} d_{ct}(F_{2,n-1}, i-1)x^i. \\ &= \sum_{i=2}^{n+2} d_{ct}(F_{2,n-1}, i)x^i + x\sum_{i=3}^{n+2} d_{ct}(F_{2,n-1}, i-1)x^{i-1}. \\ &= D_{ct}(F_{2,n-1}, x) + xD_{ct}(F_{2,n-1}, x). \end{aligned}$$

Hence,

$$D_{ct}(F_{2,n}, x) = (1+x)D_{ct}(F_{2,n-1}, x). \tag{1}$$

When $i = 2$,

$$d_{ct}(F_{2,n}, 2)x^2 = [d_{ct}(F_{2,n-1}, 2) + 2]x^2, \quad \text{by Theorem 2.10 (i)}.$$

Hence,

$$d_{ct}(F_{2,n}, 2)x^2 = d_{ct}(F_{2,n-1}, 2)x^2 + 2x^2 \tag{2}$$

When $i = 3$,

$$d_{ct}(F_{2,n}, 3)x^3 = [d_{ct}(F_{2,n-1}, 3) + d_{ct}(F_{2,n-1}, 2) + 1]x^3, \quad \text{by Theorem 2.10 (ii)}.$$

Hence,

$$d_{ct}(F_{2,n}, 3)x^3 = d_{ct}(F_{2,n-1}, 3)x^3 + d_{ct}(F_{2,n-1}, 2)x^3 + x^3. \tag{3}$$

When $i = n-1$,

$$d_{ct}(F_{2,n}, n-1)x^{n-1} = [d_{ct}(F_{2,n-1}, n-1) + d_{ct}(F_{2,n-1}, n-2) - 1]x^{n-1}, \text{ by Theorem 2.10 (iii)}.$$

Hence,

$$d_{ct}(F_{2,n}, n-1)x^{n-1} = d_{ct}(F_{2,n-1}, n-1)x^{n-1} + d_{ct}(F_{2,n-1}, n-2)x^{n-1} - x^{n-1}. \quad (4)$$

When $i = n-2$,

$d_{ct}(F_{2,n}, n-2)n^{-2} = [d_{ct}(F_{2,n-1}, n-2) + d_{ct}(F_{2,n-1}, n-3) - 2]x^{n-2}$, by Theorem 2.10 (iv).

Hence,

$$d_{ct}(F_{2,n}, n-2)x^{n-2} = d_{ct}(F_{2,n-1}, n-2)x^{n-2} + d_{ct}(F_{2,n-1}, n-3)x^{n-2} - 2x^{n-2}. \quad (5)$$

When $i = n-3$,

$d_{ct}(F_{2,n}, n-3)x^{n-3} = [d_{ct}(F_{2,n-1}, n-3) + d_{ct}(F_{2,n-1}, n-4) - 1]x^{n-3}$, by Theorem 2.10 (iii).

Hence,

$$d_{ct}(F_{2,n}, n-3)x^{n-3} = d_{ct}(F_{2,n-1}, n-3)x^{n-3} + d_{ct}(F_{2,n-1}, n-4)x^{n-3} - x^{n-3}. \quad (6)$$

Combining (1), (2), (3), (4), (5) and (6), we get,

$$D_{ct}(F_{2,n}, x) = (1+x)D_{ct}(F_{2,n-1}, x) + 2x^2 + x^3 - x^{n-3} - 2x^{n-2} - x^{n-1}.$$

Example 3.6 :

$$D_{ct}(F_{2,6}, x) = 12x^2 + 36x^3 + 56x^4 + 52x^5 + 28x^6 + 8x^7 + x^8.$$

By Theorem 3.5. we have,

$$\begin{aligned} D_{ct}(F_{2,7}, x) &= (1+x)(12x^2 + 36x^3 + 56x^4 + 52x^5 + 28x^6 + 8x^7 + x^8) \\ &\quad +2x^2 + x^3 - x^4 - 2x^5 - x^6. \\ &= 14x^2 + 49x^3 + 91x^4 + 106x^5 + 79x^6 + 36x^7 + 9x^8 + x^9. \end{aligned}$$

We obtain $d_{ct}(F_{2,n}, i)$ for $2 \leq n \leq 15$ as shown in Table 1.

Table 1

i	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
2	5	4	1													
3	8	10	5	1												
4	9	18	15	6	1											
5	10	26	32	21	7	1										
6	12	36	56	52	28	8	1									
7	14	49	91	106	79	36	9	1								
8	16	64	140	196	183	114	45	10	1							
9	18	81	204	336	378	295	158	55	11	1						
10	20	100	285	540	714	672	451	212	66	12	1					
11	22	121	385	825	1254	1386	1122	661	277	78	13	1				
12	24	144	506	1210	2079	2640	2508	1782	936	354	91	14	1			
13	26	169	650	1716	3289	4719	5148	4290	2717	1288	444	105	15	1		
14	28	196	819	2366	5005	8008	9867	9438	7007	4004	1730	548	120	16	1	
15	30	225	1015	3185	7371	13013	17875	19305	16445	11011	5733	2276	667	136	17	1

Theorem 3.7 : The following properties hold for the coefficients of $D_{ct}(F_{2,n}, x)$ for all n.

(i) $d_{ct}(F_{2,n}, 2) = 2n, n \geq 5.$

(ii) $d_{ct}(F_{2,n}, 3) = n^2, n \geq 6.$

(iii) $d_{ct}(F_{2,n}, i) = 0$ if $i < 2$ or $i > n + 2$.

(iv) $d_{ct}(F_{2,n}, n + 2) = 1.$

(v) $d_{ct}(F_{2,n}, n + 1) = n + 2.$

(vi) $d_{ct}(F_{2,n}, n) = \binom{n+2}{2}, n \geq 3.$

(vii) $d_{ct}(F_{2,n}, n - 1) = \binom{n+2}{3} - \binom{n}{1} + 2, n \geq 4.$

(viii) $d_{ct}(F_{2,n}n - 2) = \binom{n+2}{4} - \binom{n}{2} + 1, n \geq 5.$

(ix) $d_{ct}(F_{2,n}, n - 3) = \binom{n+2}{5} - \binom{n}{3}, n \geq 6.$

(x) $d_{ct}(F_{2,n}, n - 4) = \binom{n+2}{6} - \binom{n}{4}, n \geq 7.$

(xi) $d_{ct}(F_{2,n}, n - i) = \binom{n+2}{i+2} - \binom{n}{i}$ for all $n \geq i + 3$.

Proof : Proof is obvious.

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