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# DARK ENERGY Λ- COSMOLOGY

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**Abstract**

In this paper, We have revived FRW (Friedmann Robertson Walker) cosmology of spatially homogenous and isotropic universe by considering cosmological constant term in the Einstein's field equations in view of the latest developments in cosmology initialed by Saul Perlmutter, Adam G.Riess etc. The model exhibits the feature that at present we are living in the accelerated phase of expanding universe. Various cosmological implications such as Hubble's constant, Present dust and dark energy densities, Acceleration of the universe, luminosity distance & Apparent magnitudes versus red shift relation for SN Ia Supernova are discussed.

## 1. Introduction

Last two decades had been the most striking and exploring tenures for Cosmology. The centers of attraction are the low and high red shift supernova's which are found to be most reliable standard candles . In fact in cosmology, our sophisticated observatories receive radiations of all sorts from near to very large distance emitting sources up to the distance of order 3000 Mps. The Astronomers measure absolute and apparent magnitudes of standard candles which are present everywhere in the universe. These two determines the luminosity distances of the objects from us. The cosmologists also

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measures red shifts of the objects. Then comes the role of theoretical cosmologists who explore the wonderful ghosts in the universe in the form of dark energy, dark matter( baryons as well as non baryons)etc. WMAP [1](Wilkinson Microwave Anisotropy Probe 2001-12) and HKP[2](Hubble Key Project) explored that our universe is nearly flat. This has given concept of two component density parameters $\Omega_m$ and $\Omega_\Lambda$, which are related through

$$\Omega_\Lambda + \Omega_m = 1 \tag{1}$$

Equation (1) is obtained by solving Einstein's Field Equations(E.F.E.) with cosmological constant for Friedmann Robertson Walker metric [eqn(2) given below] which represent a spatially homogenous and isotropic universe and by taking Energy momentum tensor as that of perfect fluid. For low and high red shift supernova's, the luminosity distances on the basis of FRW cosmology are given by following equations respectively.

$$D_L = \frac{cz}{H_0} \;\&\; D_L = \frac{c(1+z)}{H_0} \int_0^z \frac{dz}{\sqrt{[(\Omega_m)_0(1+z)^3 + (\Omega_\Lambda)_0]}}$$

The luminosity distance $D_L$ is associated to absolute and apparent magnitudes by the following equation.

$$m - M = 5log_{10}(\frac{D_L}{Mpc}) + 25$$

Imposing SN Ia supernova data's of magnitude and red shift $(m, z)$ on the corresponding theoretical values obtained from FRW $\Lambda$-cosmology, the SRP(Supernova Research Project) team headed by Perlmutter and Riess[3-7] found that the present values of $\Omega_m$ and $\Omega_\Lambda$are nearly .29 and .71 respectively.

In this paper, we have revived the FRW cosmology from the point of view of the latest developments exhibiting the importance of cosmological constant in the E.F.E. The first section describes the latest development in cosmology.The second section develops infra structure for $\Lambda-$cosmology. It begins with the salient features of FRW space - time.We have incorporated $\Lambda$ and curvature term $k$ into the energy part of E.F.E. and established equation of states for various form of energies. We have also presented an expression for Hubble's constant in term of observed parameters like $(\Omega_m)_0, (\Omega_\Lambda)_0, (\Omega_k)_0$ and the red shift $z$. We obtained expressions for the densities of the dust and dark energies, the evolution of scale function $S(t)$ over time which represents expansion of the universe, the present age of the universe and the deceleration parameter which is at present an acceleration parameter. These are supported by the graphical representation and the present accelerated phase of the universe is justified.It is found that the present age of the universe obtained on the basis of the model comes close to the recent WMAP3 data,s. and the model shows that we are running in the acceleration phase of the universe.We have also worked out numerically the present densities of dark(lambda) and matter energies. In the last two sections, we have obtained expressions for luminosity distance and the apparent magnitude for SN Ia supernova's and we have compared observed values with the theoretical values with the help of $\chi^2$. It is verified that the model on the basic of $\Omega_m = .3, \Omega_\Lambda = .7$ fits best on the observational basic.

## 2. Latest Developments in Cosmology

Cosmology is the study of the Universe at large. During the present century, a large number of observational results reveal the overall regularities, which are global rather than local. The basic purpose of the cosmological studies is to construct a coherent picture of the Universe based on these data and build up theoretical models which fit them best and tell us about the past and future of the Universe. During the 1980, it was realized that type Ia supernova can serve as standard candles in the following way. The light curve of such a supernova has an approximately symmetric rise and fall over 30 days, followed by a much slower decline. Because of their peak luminosity they can be spotted very distant galaxies. Their red shift is found to be in the range z $\geq 1$. In the year 1999 Perl mutter[5] et al. used 60 supernovas to draw up the Hubble plot. Their investigations have completely changed the cosmological scenario. It is now believed that in the present age galaxies are accelerating instead of deceleration. Now the question arises that who is responsible and what is the cause of this acceleration. Various theories have been put forward by cosmologists and it is now firmly believed that there is a mysterious dark energy prevailing all over the universe. It is this energy which is responsible for accelerating universe. It is estimated that baryonic matter (composed of electrons, protons and neutrons) is only 4%, dark matter including black holes are 23% (major non baryonic part) and the rest of 73% is dark energy. In this new development, cosmological constant appearing in the Einstein's field equation of gravitation is playing a very important role which was at a time considered as a blunder in the life of Einstein. One half of the 2011 Nobel Prize in Physics has been awarded to Saul Perlmutter of the

Lawrence Berkeley National Laboratory in Berkeley, California; the other half jointly to Brian P. Schmidt of the Australian National University in Weston Creek, Australia and to Adam G. Riess of Johns Hopkins University and the Space Telescope Science Institute in Baltimore, Maryland "for the discovery of the accelerating expansion of the Universe through observations of distant supernovas".

## 3. Foundation of Λ- Cosmology

So far the most successful model of the physical universe which fits best on empirical basis is the hot big bang FRW model[8,9]. This model succeeded well in explaing.the observation of expansion of universe by Edwin Hubble[10] in 1929, the development of the theory of Big-Bang nucleosynthesis by Ralph Alpher, George Gamow and Robert Herman[11] in the early 1050 and the discovery of cosmic background radiation by Arno Penzias and Robert Wilson[12] in 1960. The metric of the FRW universe is given as follows.

$$ds^2 = c^2dt^2 - S(t)^2[dr^2/(1+kr^2) + r^2(d\theta^2 + sin^2\theta d\phi^2)] \tag{2}$$

Where k=-1 for closed universe, k=1 for open universe and k=0 for spatially flat universe. It is convenient to write the metric(2) in the following form

$$ds^2 = c^2dt^2 - S(t)^2\left[d\chi^2 + f_k^2(\chi)\left(d\theta^2 + sin^2\theta d\phi^2\right)\right] \tag{3}$$

where

$$\begin{aligned} f_k(\chi) &= sin\chi, k=-1, (closed\ universe) \\ &= \chi, k=0, (flat) \\ &= sinh\chi, k=1 (open\ universe) \end{aligned}$$

This metric is expressed in a co-moving coordinate system. This has following features.

- There is a proper cosmic time. Observers all over space-time read same time.
- All galaxies have stationary space coordinates $(r, \theta, \phi)$ but the proper distance between any two galaxies varies in natural way through expansion factor S(t) for example the proper distance between $(r_1, \theta, \phi)$ and $(r_2, \theta, \phi)$ is $D = (r_2 - r_1)\, S\,(t_0)$ at present. $t_0$ is present proper time ie $D \propto S(t)$.

Same way the three spatial volume $V \propto S^3(t)$. The relative velocity of recession of galaxies is determined by Hubble's constant $H = \frac{\dot{S}}{S}$ . As stated in section (2), for small red shifts z and accordingly for small distances, $v = cz = H_0 D$, $(H)_0 = \left(\frac{\dot{S}}{S}\right)_0$ ,suffix 0 means present value.

- The expansion factor S(t) is slowly increasing function of time. its past value is determined by the red shift z through the following equation.

$$S(t) = \frac{S(t_0)}{1+z} \tag{4}$$

- The evolution of the scale function is determined by solving Einstein's Field Equations for FRW metric(2).

$$R_{ij} - \frac{1}{2}Rg_{ij} + \Lambda g_{ij} = -\frac{8\pi G}{c^4}T_{ij} \tag{5}$$

- The energy momentum tensor of the contents of the universe is assumed to be that of perfect fluid. This is given by

$$T_{ij} = (\rho + p)\, u_i u_j - p g_{ij} \tag{6}$$

- The co-moving co ordinate system has feature that all points keeps the space coordinates same at all the time. So

$$u^\alpha = 0, \alpha = 1, 2, 3$$

- the mass energy of the universe has so many individual components(see Box)

**The Various Components Of The Mass Energy**

(a)Radiation $\rho_{rad}$ : composed of massless or nearly massless particles that move at the speed of light. Known examples include photons (light) and neutrinos. This form of matter is characterized by having a large positive pressure.
(b)Baryonic matter $\rho_{bar}$ : In this cosmological context, this is "ordinary matter" composed primarily of protons, neutrons and electrons. This form of matter has essentially no pressure of cosmological importance.
(c)Dark matter $\rho_{dm}$ : this generally refers to "exotic" non-baryonic matter that interacts only weakly with ordinary matter. While no such matter has ever been directly observed in the laboratory, its existence has long been suspected for reasons discussed in a subsequent page. This form of matter also has no cosmologically significant pressure.
(d) dark energy $\rho_{de}$ : this is a truly bizarre form of matter, or perhaps a property of the vacuum itself, that is characterized by a large, negative pressure (repelling force). This is the only form of matter that can cause the expansion of the universe to accelerate, or speed up.

- Solving Equations (5) and (6) for metric (2) we get the following two Friedmann equations of Dynamic cosmology

$$2\frac{\ddot{S}}{S} + H^2 - \frac{kc^2}{S^2} - \Lambda c^2 = -\frac{8\pi G}{c^2}p \tag{7}$$

$$H^2 - \frac{kc^2}{S^2} - \frac{\Lambda c^2}{3} = \frac{8\pi G}{3c^2}\rho \tag{8}$$

We assume that cosmological constant $\Lambda$ and curvature term $k$ also acts like an energy with density

$$\rho_\Lambda = \frac{c^4\Lambda}{8\pi G}, \rho_k = \frac{3c^4 k}{8\pi G S^2} \ \& \ \rho_c = \frac{3c^2H^2}{8\pi G} \tag{9}$$

$$\Omega_m = \frac{\rho}{\rho_c}, \Omega_\Lambda = \frac{\rho_\Lambda}{\rho_c} \ \& \ \Omega_k = \frac{\rho_k}{\rho_c} \tag{10}$$

So that from eqn(8) to (10)

$$\Omega_m + \Omega_\Lambda + \Omega_k = 1 \tag{11}$$

We also assume that

$$p_\Lambda = -\frac{c^4\Lambda}{8\pi G} \ \& \ p_k = -\frac{c^4 k}{8\pi G S^2} \tag{12}$$

So that, we get the followings from equations (9) and (12).

$$\rho_\Lambda + p_\Lambda = 0 \ \& \ \rho_k + 3p_k = 0 \tag{13}$$

Eqns (7) and (8) become

$$2\frac{\ddot{S}}{S} + H^2 = -\frac{8\pi G}{c^2}\left(p + p_\Lambda + p_k\right)$$

$$H^2 = \frac{8\pi G}{3c^2}\left(\rho + \rho_\Lambda + \rho_k\right)$$

From these two we get one more following equation which describe conservation of energy

$$\dot{\rho} + 3H(p + \rho) = 0 \tag{14}$$

It is interesting to see here that equation (14) is satisfied by all the three components of density and pressure.ie following equations are satisfied simultaneously

$$\dot{\rho_m} + 3H(p_m + \rho_m) = 0, \dot{\rho_\Lambda} + 3H(p_\Lambda + \rho_\Lambda) = 0 \ \& \ \dot{\rho_k} + 3H(p_k + \rho_k) = 0 \tag{15}$$

The equation (15) is integrable when suitable function relates pressure to density. This function is known as equation of state and is as follows

$$p_m = \omega_m \rho_m, \omega_m = \omega_m\left(\rho_m, T\right)(\omega \ may \ or \ may \ not \ be \ constant)$$

In fact equation of state describes the various cosmological epochs. In the early part of the universe it was radiation dominated.

$$\omega_m = \frac{1}{3}, \rho_{rad} \propto S^{-4}, \rho_{rad} \propto T_{rad}^4 \because S \propto T^{-1}$$

Later on when radiation decoupled from baryons,matter dominated era began, In this epoch,

$$\omega_m = 0, p_{bar} = 0, \rho_{bar} \propto S^{-3}$$

The equation(13) gives equations of states for energies corresponding to $\Lambda$ and $k$ as follows

$$p_\Lambda = \omega_\Lambda \rho_\Lambda \ \& \ p_k = \omega_k \rho_k \ where \ \omega_\Lambda = -1 \ \& \ \omega_k = -3 \tag{16}$$

In fact at present cosmological constant dominates over barionic matter. It acts as a fluid(dark energy) with pressure related to density through (15). For flat universe in absence of barionic matter

$$\ddot{S} = \frac{8\pi G}{3c^2}\rho_\Lambda S > 0$$

This means that in this phase **universe accelerates.**

### 3.1 Density of the Universe and Hubble's Constant

As stated in the Box mass energy has so many components, accordingly

$$\rho = \rho_{bar} + \rho_{rad} + \rho_{dm} + \rho_{de}$$

As $\Lambda$ is estimated for the best candidates for dark energy, we also write $\rho_{de}$ for $\rho_\Lambda$. From equations (4), (15) and subsequent equations, energy density $\rho$ is given by

$$\rho = \sum_i (\rho_i)_0 (1+z)^{3(1+\omega_i)} \tag{17}$$

Where

$$(\rho_i)_0 = \frac{3c^2 H_0^2}{8\pi G} (\Omega_i)_0 \tag{18}$$

Here $(\rho_i)_0$ are the present energy density of various components. Taking, $(\Omega_m)_0$=.3, $H_0 = 72km/sec./Mpc$, $G = 6.6720e - 008cm^3/sec^2/gm$ The current value of dust energy $(\rho_m)_0$ for flat universe is given as

$$(\rho_m)_0 = 0.6h_0^2 \times 10^{-29} gm/cm^3 \tag{19}$$

And the current value of dark energy $(\rho_\Lambda)_0$ is given as

$$(\rho_\Lambda)_0 = \frac{(\Omega_\Lambda)_0}{(\Omega_m)_0} (\rho_m)_0 = 1.4h_0^2 \times 10^{-29} gm/cm^3 \tag{20}$$

From (8), we get expression for Hubble's Constant

$$H^2 = H_0^2 \sum_i (\Omega_i)_0 (1+z)^{3(1+w_i)}$$

,

$$H^2 = H_0^2[(1 + (\Omega_m)_0 z)(1+z)^2 - (2+z)z(\Omega_\Lambda)_0] \, when \, \Omega_k \neq 0, \tag{21}$$

$$H^2 = H_0^2[(\Omega_m)_0(1+z)^3 + (\Omega_\Lambda)_0] \, when \, \Omega_k = 0, \tag{22}$$

We use equation(4) to get relationship between scale function and Hubble's constant.

$$H^2 = H_0^2[(\Omega_m)_0 \left(\frac{S_0}{S}\right)^3 + (\Omega_\Lambda)_0] \tag{23}$$

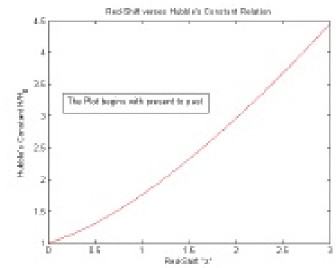


Figure 1: [based on equation(22) for $(\Omega_\Lambda)_0 = .7$, & $(\Omega_m)_0 = .3$]

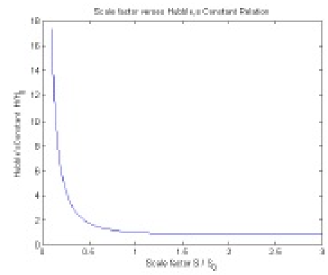


Figure 2: [based on equation(23) for $(\Omega_\Lambda)_0 = .7$, & $(\Omega_m)_0 = .3$]

**3.2 Evolution of Scale Function S(t) Over Time**

We begin with the integral

$$t = \int_0^t dt = \int_0^S \frac{dS}{SH}$$

We consider the case k=0 and use the equation (23), we get

$$t = \int_0^S \frac{dS}{SH_0\sqrt{[(\Omega_m)_0(\frac{S_0}{S})^3 + (\Omega_\Lambda)_0]}}$$

Integrating it, we get

$$H_0 t = \frac{2}{3\sqrt{(\Omega_\Lambda)_0}} log\left\{\sqrt{\frac{(\Omega_\Lambda)_0 S^3}{(\Omega_m)_0 S_0^3}} + \sqrt{1 + \frac{(\Omega_\Lambda)_0 S^3}{(\Omega_m)_0 S_0^3}}\right\} \tag{24}$$

In term of red-shift z, the expression is as follows

$$H_0 t = \frac{2}{3\sqrt{(\Omega_\Lambda)_0}} log\left\{\sqrt{\frac{(\Omega_\Lambda)_0}{(\Omega_m)_0(1+z)^3}} + \sqrt{1 + \frac{(\Omega_\Lambda)_0}{(\Omega_m)_0(1+z)^3}}\right\} \tag{25}$$

**3.3 The Present Age Of The Universe**

For present age $t_0$ of the universe $S = S_0$, or $z = 0$.

$$\therefore H_0 t_0 = \frac{2}{3\sqrt{(\Omega_\Lambda)_0}} log\left\{\frac{1 + \sqrt{(\Omega_\Lambda)_0}}{\sqrt{(\Omega_m)_0}}\right\} \tag{26}$$

we see that $H_0 t_0 = .964$ when $(\Omega_\Lambda)_0 = .7 and (\Omega_m)_0 = .3$

$$\because H_0^{-1} = 1.3574e + 010 \;\; \therefore \;\; t_0 = 1.3085e + 010 yrs = 13.1 Gyrs$$

This is consistence with most recent WMAP3 data

$$t_0 = 13.73^{+.13}_{-.17}$$

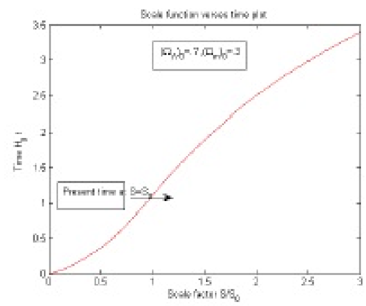



Figure 3: [based on equation(24) for $(\Omega_\Lambda)_0 = .7$, & $(\Omega_m)_0 = .3$]

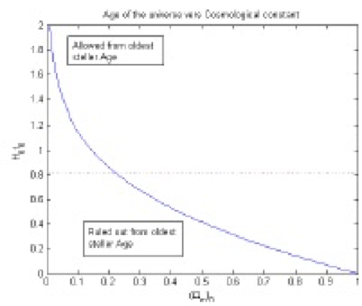



Figure 4: [based on equation(26) for $(\Omega_\Lambda)_0 = .7$, & $(\Omega_m)_0 = .3$]

### 3.4 Deceleration Parameter q

The deceleration parameter $q$ is given by

$$q = -S\ddot{S}/\dot{S}^2$$

The equation(12) gives

$$-2q + 1 = \frac{3H_0^2(\Omega_\Lambda)_0}{H^2} \tag{27}$$

As the universe at present is $\Lambda$ dominated and the observations shows that it is accelerated,the present value of q must be -ve. This put restriction on lambda as

$$-2\,(q)_0 = 3(\Omega_\Lambda)_0 - 1 > 0 \Rightarrow (\Omega_\Lambda)_0 > .33 \tag{28}$$

which is consistent with the present observes value of lambda $(\Omega_\Lambda)_0 = .7$. For this value

$$q_0 = -.55$$

Now

$$H^2 = H_0^2\left\{(\Omega_m)_0(1+z)^3 + (\Omega_\Lambda)_0\right\}$$

$$q(z) = \frac{3}{2}\frac{(\Omega_m)_0(1+z)^3}{(\Omega_m)_0(1+z)^3 + (\Omega_\Lambda)_0} - 1$$

From this, the universe reaches accelerating phase when $q < 0$ and

$$z < z_c \cong \left(\frac{2(\Omega_\Lambda)_0}{(\Omega_m)_0}\right)^{1/3} - 1$$

$(\Omega_\Lambda)_0 = .7$and $(\Omega_m)_0 = .3$ gives $z_c = .6711$. Thus the acceleration must begin in the past at $z_c = .6711$.

## 4. Proper and Luminosity Distance in Λ-Universe

The FRW universe represented by metric(2) has a very interesting intrinsic and astonishing geometric feature that the universe is expressed in such a fashioning co-ordinate system that its various constituents in form of cluster of galaxies of scale of order 100 Mpc are points and the observed portion of it (nearly 3000 Mpc) is spatially homogeneous and isotropic.It is the beauty of the geometry as well as the co-ordinate system that the points which are dynamic bodies maintain their position but the separation between any two point geometrically change through a scale factor S(t) which is function of the time.This separation is called **proper distance** between two points and is given by $D(t) = S(t)r$ where $r$ is the radial coordinate of one point and the other one is assumed as origin.In FRW universe there is no privileged position.There is one more cosmological distance known as **Luminosity distance** $D_L$which appears in the expression of the flux of a luminous object received by an observer at the origin.It is given by

$$F = \frac{L}{4\pi D_L^2}, D_L = S_0 r(1+z) \tag{29}$$

where L is the absolute luminosity of the object(energy flux at the source) and F is the apparent luminosity(energy flux received by the observer).The presence of factor $(1+z)$in $D_L$is the relativistic contribution due to stretching of photon wave length during the journey.The luminosity distance in metric (3) is as follows

$$D_L = S_0 f_k(\chi)(1+z)$$

To get expression for luminosity distance we consider light travailing along radial direction$\chi$.It satisfies null geodesic given by

$$ds^2 = c^2dt^2 - S(t)^2 d\chi^2 = 0$$

From this we obtain

$$\chi = \int_0^{\chi_s} d\chi = \int_{t_1}^{t_0} \frac{dt}{S(t)} = \frac{1}{S_0 H_0}\int_0^z \frac{dz}{h(z)} = \frac{1}{S_0 H_0}\int_0^z \frac{dz}{\sqrt{\sum_i (\Omega_i)_0 (1+z)^{3(1+w_i)}}}$$

where we have used

$$\dot{z} = \left(\frac{\dot{S}_0}{S}\right) = -H(1+z) \ \& \ h(z) = \frac{H}{H_0}$$

So the luminosity distance is given

$$D_L = \frac{c(1+z)}{H_0\sqrt{(\Omega_k)_0}} f_k(\sqrt{(\Omega_k)_0} \int_0^z \frac{dz}{\sqrt{[(1+(\Omega_m)_0 z)(1+z)^2 - (2+z)z(\Omega_\Lambda)_0]}}) \ \Omega_k \neq 0 \tag{30}$$

$$D_L = \frac{c(1+z)}{H_0} \int_0^z \frac{dz}{\sqrt{[(\Omega_m)_0(1+z)^3 + (\Omega_\Lambda)_0]}} \ \Omega_k = 0 \tag{31}$$

It should be noted that when $z << 1, 1+z \sim 1$ integral in (31) is z, so luminosity distance is

$$D_L \simeq \frac{cz}{H_0} \tag{32}$$

Now we present the following table which displays luminosity distance versus red shift relation for three cases

$$\Omega_m = 1, \Omega_\Lambda = 0, \Omega_m = 0, \Omega_\Lambda = 1 \ \& \ \Omega_m = .3, \Omega_\Lambda = .7$$

**[Table 1:Red shift versus luminosity distance relation]**

| **Cases** | $\Omega_i's, w_m = 0, w_\Lambda = -1$ | $H_0 D_L$ | $z = [0, .5, 1, 1.5, 2, 2.5, 3];$ |
|---|---|---|---|
| **I** | $\Omega_m = 1, \Omega_\Lambda = 0$ | $(1+z)\int_0^z \frac{dz}{(1+z)^{3/2}}$ | $= 2\{1 + z - \sqrt{1+z}\}$ |
| **II** | $\Omega_m = 0, \Omega_\Lambda = 1$ | $(1+z)\int_0^z dz$ | $z(1+z)$ |
| **III** | $\Omega_m = .3, \Omega_\Lambda = .7$ | $(1+z)\int_0^z \frac{dz}{\sqrt{0.3(1+z)^3+0.7}} \Longrightarrow$ | $\Longrightarrow [0, 0.6615, 1.5429, 2.5473, 3.6284, 4.7625, 5.9361];$ |

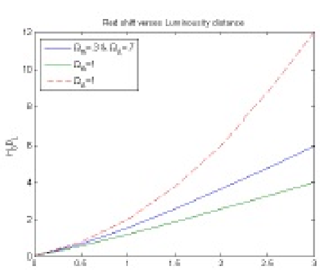



Figure 5: [based on Table:1]

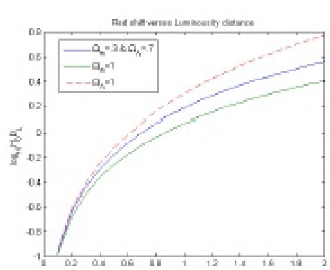



Figure 6: [based on Table:1]

## 5. Type Ia supernova (SN Ia) Observations and Accelerated Universe at Present

The direct evidence for the current acceleration of the universe is related to the observation of luminosity distances of high red shift supernovas. The apparent magnitude m of the source with an absolute magnitude M is related to the luminosity distance $D_L$ via following relation.

$$m - M = 5log_{10}(\frac{D_L}{Mpc}) + 25 \tag{33}$$

The Type Ia supernova (SN Ia) can be observed when white dwarf stars exceed the mass of the Chandrasekhar limit and explode. The belief is that SN Ia are formed in the same way irrespective of where they are in the universe, which means that they have a common absolute magnitude M independent of the red shift z. Thus they can be treated as an ideal standard candle. We can measure the apparent magnitude m and the red shift z observationally, which of course depends upon the objects we observe. To get absolute Magnitude M of a supernova, we consider supernova at very small red shift. Let us consider a supernova 1992P at low-red shift z = 0.026 with m = 16.08.For low red shift supernova

$$D_L = \frac{cz}{H}$$

So from Eqn(33)

$$M = 16.08 - 25 + 5log_{10}\frac{H_0}{.026c} \tag{34}$$

This gives the following expression for luminosity distance verses apparent magnitude relation

$$m = 16.08 + 5log_{10}\frac{H_0 D_L}{.026cMpc} \tag{35}$$

$$log_{10}(H_0 D_L) = (m - 16.08)/5 + log_{10}(.026cMpc) \tag{36}$$

From (30),(31) and (36) we get expression for m in term of z.

$$m = 16.08 + 5log_{10}(\frac{(1+z)}{.026\sqrt{(\Omega_k)_0}} f_k(\sqrt{(\Omega_k)_0}\int_0^z \frac{dz}{\sqrt{[(1+(\Omega_m)_0 z)(1+z)^2 - (2+z)z(\Omega_\Lambda)_0]}})) \tag{37}$$

$$m = 16.08 + 5log_{10}(\frac{(1+z)}{.026}\int_0^z \frac{dz}{\sqrt{[(\Omega_m)_0(1+z)^3 + (\Omega_\Lambda)_0]}})), k = 0 \tag{38}$$

Now we present a Table [See Appendix in the end] which contains red- shifts and apparent magnitudes of 60 SN Ia supernova's.This Table also contains the corresponding theoretical values of apparent magnitudes as per equation (46) for $\Omega_m = .3, 1$and 0 respectively. In order to see that out of the three sets of theoretical data's of apparent magnitudes corresponding to $\Omega_m = .3$,1 and 0 which one is close to the observational values, we calculate $\chi^2$ using following formula due to R. Amanullah[13]

$$\chi^2_{SN} = A - \frac{B^2}{C} + log_{10}(\frac{C}{2\pi})$$

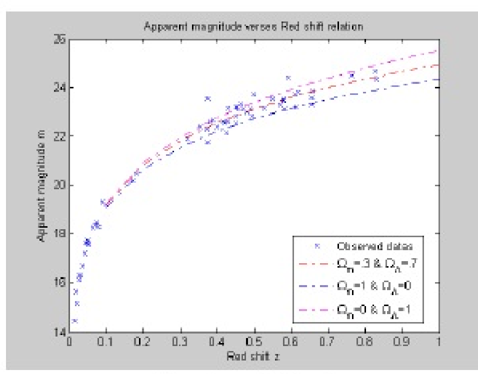
Where

$$A = \sum_{i=1}^{60} \frac{[(m_B)_{ob} - (m_B)_{th}]^2}{\sigma_i^2}$$

$$B = \sum_{i=1}^{60} \frac{[(m_B)_{ob} - (m_B)_{th}]}{\sigma_i^2} \ \& \ C = \sum_{i=1}^{60} \frac{1}{\sigma_i^2}$$

| $\Omega_m$ | $\chi^2_{SN}$ | $\chi^2_{SN}/59$ |
|---|---|---|
| .3 | 106.4719 | 1.8046 |
| 1 | 140.0639 | 2.3739 |
| 0 | 129.9575 | 2.2027 |

The above table shows that the best fit value of $\Omega_m$ is .3.




[based on Table:2]

## 6. Conclusion

We summarize our work by presenting the following table which displays the values of cosmological parameters at present obtained by us.

| Cosmological Parameters | Values at Present |
|---|---|
| $(\Omega_\Lambda)_0$ | .7 |
| $(\Omega_m)_0$ | .3 |
| $(q)_0$ | -.55 |
| $(\rho_m)_0$ | $0.6h_0^2 \times 10^{-29} gm/cm^3$ |
| $(\rho_\Lambda)_0$ | $1.4h_0^2 \times 10^{-29} gm/cm^3$ |
| Acceleration begun | At z=.6711 |
| Present Age of the universe | $13.1 \times 10^9 years$ |

We are sincerely hopeful that this work is going to be very much useful for the beginners in the field of cosmology. We tried to present the basics of the new developments in cosmology which are taking place during the last two decades in a very simple and elegant manner.

## Appendix

**[Table2: SN Ia observed apparent magnitude and the theoretical magnitude ]**

**(observed data's are taken from Perlmutter)**

| $z$ | $(m_B)_{ob} \pm \sigma_{m_\mathbf{B}}$ | $(m_B)_{\Omega_m=.3}$ | $(m_B)_{\Omega_m=1}$ | $(m_B)_{\Omega_m=0}$ |
|---|---|---|---|---|
| .030 | $16.26 \pm .20$ | 16.4401 | 16.4067 | 16.4549 |
| .050 | $17.63 \pm .18$ | 17.5811 | 17.5263 | 17.6059 |
| .026 | $16.08 \pm .24$ | 16.1229 | 16.0939 | 16.1357 |
| .075 | $18.43 \pm .20$ | 18.5000 | 18.4193 | 18.5375 |
| .026 | $16.28 \pm .20$ | 16.1229 | 16.0939 | 16.1357 |
| .014 | $14.47 \pm .23$ | 14.7591 | 14.7433 | 14.7660 |
| .101 | $19.16 \pm .23$ | 19.1848 | 19.0783 | 19.2357 |
| .020 | $15.18 \pm .20$ | 15.5434 | 15.5210 | 15.5533 |
| .036 | $16.66 \pm .21$ | 16.8457 | 16.8058 | 16.8634 |
| .045 | $17.61 \pm .19$ | 17.3445 | 17.2950 | 17.3668 |
| .043 | $17.19 \pm .18$ | 17.2426 | 17.1952 | 17.2639 |
| .018 | $15.61 \pm .21$ | 15.3114 | 15.2912 | 15.3202 |
| .079 | $18.27 \pm .18$ | 18.6188 | 18.5342 | 18.6584 |
| .088 | $19.28 \pm .18$ | 18.8665 | 18.7729 | 18.9107 |
| .063 | $18.24 \pm .18$ | 18.1031 | 18.0347 | 18.1345 |
| .071 | $18.33 \pm .20$ | 18.3749 | 18.2983 | 18.4104 |

| | | | | |
|---|---|---|---|---|
| .052 | $17.54 \pm .18$ | 17.6694 | 17.6125 | 17.6952 |
| .050 | $17.69 \pm .20$ | 17.5811 | 17.5263 | 17.6059 |
| .458 | $23.11 \pm .46$ | 22.8798 | 22.5045 | 23.1282 |
| .354 | $22.38 \pm .33$ | 22.2194 | 21.9084 | 22.4082 |
| .425 | $22.13 \pm .49$ | 22.6867 | 22.3308 | 22.9162 |
| .374 | $21.72 \pm .22$ | 22.3592 | 22.0351 | 22.5594 |
| .420 | $22.55 \pm .25$ | 22.6563 | 22.3034 | 22.8828 |
| .372 | $22.26 \pm .20$ | 22.3455 | 22.0228 | 22.5446 |
| .378 | $22.58 \pm .37$ | 22.3863 | 22.0597 | 22.5888 |
| .453 | $23.17 \pm .25$ | 22.8514 | 22.4790 | 23.0970 |
| .465 | $23.33 \pm .30$ | 22.9191 | 22.5398 | 23.1716 |
| .498 | $23.71 \pm .25$ | 23.0973 | 22.6996 | 23.3688 |
| .655 | $23.27 \pm .21$ | 23.8172 | 23.3427 | 24.1803 |
| .400 | $22.36 \pm .19$ | 22.5310 | 22.1904 | 22.7461 |
| .615 | $23.19 \pm .25$ | 23.6506 | 23.1942 | 23.9904 |
| .480 | $22.96 \pm .24$ | 23.0015 | 22.6138 | 23.2626 |
| .450 | $22.51 \pm .23$ | 22.8342 | 22.4636 | 23.0780 |
| .388 | $22.65 \pm .20$ | 22.4530 | 22.1200 | 22.6612 |
| .570 | $23.27 \pm .22$ | 23.4505 | 23.0156 | 23.7640 |
| .490 | $23.10 \pm .20$ | 23.0551 | 22.6618 | 23.3220 |
| .495 | $22.83 \pm .19$ | 23.0815 | 22.6855 | 23.3514 |
| .656 | $23.57 \pm .28$ | 23.8212 | 23.3463 | 24.1850 |
| .828 | $24.65 \pm .54$ | 14.4410 | 23.8982 | 24.9052 |
| .450 | $23.17 \pm .23$ | 22.8342 | 22.4636 | 23.0780 |
| .430 | $23.13 \pm .22$ | 22.7169 | 22.3580 | 22.9492 |
| .580 | $23.46 \pm .23$ | 23.4963 | 23.0564 | 23.8156 |
| .763 | $24.47 \pm .53$ | 24.2227 | 23.7038 | 24.6490 |
| .526 | $23.15 \pm .20$ | 23.2400 | 22.8274 | 23.5278 |
| .172 | $20.17 \pm .18$ | 20.4391 | 20.2672 | 20.5274 |
| .619 | $23.80 \pm .28$ | 23.6677 | 23.2094 | 24.0098 |
| .592 | $24.42 \pm .37$ | 23.5502 | 23.1045 | 23.8765 |
| .550 | $23.51 \pm .25$ | 23.3568 | 22.9319 | 23.6586 |
| .180 | $20.43 \pm .17$ | 20.5483 | 20.3695 | 20.6409 |
| .374 | $23.52 \pm .24$ | 22.3592 | 22.0351 | 22.5594 |
| .472 | $23.11 \pm .19$ | 22.9579 | 22.5746 | 23.2144 |
| .430 | $22.57 \pm .18$ | 22.7169 | 22.3580 | 22.9492 |
| .657 | $23.83 \pm .23$ | 23.8253 | 23.3499 | 24.1896 |
| .612 | $23.69 \pm .21$ | 23.6377 | 23.1827 | 23.9757 |
| .320 | $21.86 \pm .18$ | 21.9641 | 21.6764 | 22.1338 |
| .579 | $23.48 \pm .22$ | 23.4917 | 23.0524 | 23.8104 |
| .450 | $22.83 \pm .30$ | 22.8342 | 22.4636 | 23.0780 |
| .581 | $22.09 \pm .22$ | 23.5008 | 23.0605 | 23.8207 |
| .416 | $22.57 \pm .20$ | 22.6317 | 22.2812 | 22.8559 |
| .830 | $24.32 \pm .22$ | 24.4475 | 23.9039 | 24.9128 |

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