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SPAN METHOD FOR SOLVING THE TRANSSHIPMENT PROBLEM WITH MIXED CONSTRAINTS

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Abstract

A simple method has been developed to find the Basic feasible solution of the transshipment problem with mixed constraints. Transshipment problem is converted into an equivalent transportation problem. We search the Span of the elements of each row and each column separately and assignment is made in the cell having the least element corresponding to the maximum span of row and column. The crux of the method is to search the span value. Method is simple, easy to understand and apply for any OR practitioner.

1. Introduction

In a transportation problem shipment of commodity takes place among sources and destinations. But instead of direct shipments to destinations, the commodity can be transported to a particular destination through one or more intermediate or transshipment points. Each of these points in turn supply to other points. Thus when the

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shipments also pass from destination to destination and from source to source, the transportation problem is termed as the transport problem.

Since transshipment problem is a particular case of transportation problem hence to solve transshipment problem, we firstly convert transshipment problem into equivalent transportation problem and then solve it to obtain basic feasible solution using span method.

Bridgen (1974)[1], considered the transportation problem with mixed constraints. He solved this problem by considering a related standard transportation problem having two additional supply points and two additional destinations.

Khurana, Arora(2011)[2], considered the transshipment problem with mixed constraints. They change it to transportation problem with mixed constraints.

We propose a method for getting the basic feasible solution for the transshipment problem with mixed constraints.

2. Mathematical Formulation of the Transshipment Problem

To formulate the transshipment problem we consider a transportation table given below: Transportation Table



In the transportation table $O_1, O_2, \dots, O_i, \dots O_m$ are sources from where goods are to be transported to destinations $D_1, D_2, \dots, D_j, \dots, D_n$. Any of the sources can transport to any of the destinations. C_{ij} is per unit transporting cost of goods from i^{th} source O_i to j^{th} destination D_j for all $i = 1, 2, \dots, m$ and $j = 1, 2, \dots, n$. x_{ij} is the amount of goods transporting from i^{th} source O_i to j^{th} destination D_j . a_i be the amount of goods available at the origins O_i and b_j the demand at the destinations D_j . The corresponding transportation problem is

$$\min z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_i, x_{ij}$$

s.t.
$$\sum_{j=1}^{n} x_{ij} = a_i, \quad i = 1, 2, \cdots, m$$
$$\sum_{i=1}^{m} x_{ij} = b_j, \quad j = 1, 2, \cdots, n$$
$$x_{ij} \ge 0, \quad \forall \ i = 1, 2, \cdots, m \quad \& \ j = 1, 2, \cdots, m.$$

Since in a transshipment problem, any origin or destination can ship to any other origin or destination it would be convenient to number them successively so that the origins are numbered from 1 to m and the destinations from m + 1 to m + n.

We now extend this transportation problem to permit transshipment with the additional feature that shipments may go via any sequence of points rather than being restricted to direct connections from one origin to one of the destination. The unit cost of shipment from a point considered as a shipper to the same point considered as receiver is set equal to zero.

$$\begin{aligned} x_{i1} + x_{i2} + \dots + x_{i} \ _{m+n} &= a_i + (x_{1i} + x_{2i} + \dots + x_{m+ni}) \\ x_{i1} + x_{i2} + \dots + x_{i,j-1}, \dots + x_{i,j+1} + \dots + x_{i,m+n} \\ &= a_i + (x_{1i} + x_{2i} + \dots + x_{i-1,i} + x_{i+1,j} + \dots + x_{m+n,i}) \\ i.e. \quad \sum_{j=1, j \neq i}^n x_{ij} &= a_i + \sum_{j=1, j \neq i}^{m+n} x_{ji}, \quad i = 1, 2, \dots, m \\ i.e. \quad \sum_{j=1, j \neq i}^{m+n} x_{ij} - \sum_{j=1, j \neq i}^{m+n} x_{ji} = a_i, \quad i = 1, 2, \dots, m. \end{aligned}$$

Similarly the total amount received at a destination D_j must be equal to its demand

plus what it transships.

$$\begin{aligned} x_{1,m+j} + x_{2,m+j} + \cdots + x_{m+j-1,m+j} + x_{m+j+1,m+j} + \cdots + x_{m+n,m+j} \\ &= b_{m+j} + (x_{m+j,1} + x_{m+j,2} + \cdots + x_{m+j,m+j-1} + x_{m+j,m+j+1} + \cdots + x_{m+j,m+n}) \\ i.e. \quad x_{1\ m+j} + x_{2\ m+j} + \cdots + x_{m+n\ m+j} \\ &= b_{m+j} + (x_{m+1j\ 1} + x - m + j\ 2 + \cdots + x_{m+j\ m} + x_{m+j\ m+j} + \cdots + x_{m+j\ m+n}) \\ i.e. \quad \sum_{i=1, i \neq j}^{m+n} x_{i\ m+j} = b_{m+j} + \sum_{i=1, i \neq j}^{m+n} x_{m+j\ i}, \quad j = 1, 2, \cdots, n \\ i.e. \quad \sum_{i=1, i \neq j}^{m+n} x_{i\ m+j} - \sum_{i=1, i \neq j}^{m+n} x_{m+j\ i}, \quad j = 1, 2, \cdots, n \\ i.e. \quad \sum_{i=1, i \neq j}^{m+n} x_{i\ m+j} - \sum_{i=1, i \neq j}^{m+n} x_{m+j\ i}, \quad j = 1, 2, \cdots, n \\ i.e. \quad \sum_{i=1, i \neq j}^{m+n} x_{i\ j} - \sum_{i=1, i \neq j}^{m+n} x_{j\ i} = b_j, j = m+1, m+2, \cdots, m+n \\ and \quad x_{i\ j} \ge 0, i = 1, 2, \cdots, m+n, j \neq i. \end{aligned}$$

Thus the transshipment problem may be written as

$$\begin{aligned} \text{Minimize} \quad z &= \sum_{i=1}^{m+n} \sum_{j=1, j \neq i}^{m+n} c_{ij} x_{ij} \\ \text{subject to constraints} \\ &\sum_{j=1, j \neq i}^{m+n} x_{ij} - \sum_{i=1, j \neq i}^{m+n} x_{ji} = a_i, \quad i = 1, 2, \cdots, m \\ &\sum_{i=1, i \neq j}^{m+n} x_{ij} - \sum_{i=1, i \neq j}^{m+n} x_{ji} = b_j, \quad j = m+1, m+2, \cdots, m+n \\ &x_{ij} \geq 0, \text{`} i = 1, 2, \cdots, m+n, j \neq i. \end{aligned}$$

The above formulation is a linear programming problem, which is similar to a transportation problem but not exactly since the coefficients of x_{ij} 's are -1. The problem however easily be converted to a standard transportation problem.

$$t_{i} = \sum_{j=1, j \neq i}^{m+n} x_{ji}, i = 1, 2, \cdots, m$$

i.e. $t_{i} + x_{ii} = \sum_{j=1}^{m+n} x_{ji}, i = 1, 2, \cdots, m$

and
$$t_j = \sum_{i=1, i \neq j}^{m+n} x_{ji}, \quad j = m+1, m+2, \cdots, m+n$$

i.e.
$$t_j + x_{jj} = \sum_{i=1}^{m+n} x_{ji}, \quad j = m+1, m+2, \cdots, m+n$$

where t_i represents the total amount of transshipment through the i^{th} origin and t_j represents the total amount shipped put from the j^{th} destination as transshipment.

Let T > 0 be sufficiently large number so that $t_i \leq T$ for all i and $t_j \leq T$ for all j. We now write $t_i + x_{ii} = T$, then the non negative slack variable x_{ii} represents the difference between T and the actual amount of transshipment through the i^{th} origin. Similarly, if we let $t_j + x_{jj} = T$, then the non negative slack variable x_{jj} represents the difference between T and the actual amount of transshipment through the j^{th} destination.

Note that T can be interpreted as a buffer stock at each origin and destination. Since we assume that any amount of goods can be transshipped at each point, T should be large enough to take care of all transshipments. It is clear that the volume of goods transshipped at any point cannot exceed the amount produced or received and hence we take $T = \sum_{i=1}^{m} a_i$. The transshipment problem then reduces to

$$\min z = \sum_{i=1}^{m+n} \sum_{j=1, j \neq i}^{m+n} c_{ij} x_{ij}$$

s.t.
$$\sum_{j=1}^{m+n} x_{ij} = a_i + T, \quad i = 1, 2, \cdots, m,$$
$$\sum_{j=1}^{m+n} x_{ij} = T, \quad i = m+1, m+2, \cdots, m+n$$
$$\sum_{i=1}^{m+n} x_{ij} = T, \quad j = 1, 2, \cdots, m,$$
$$\sum_{i=1}^{m+n} x_{ij} = b_j + T, j = m+1, m+2, \cdots, m+n,$$
$$x_{ij} \ge 0, i = 1, 2, \cdots, m+n \text{ and } j = 1, 2, \cdots, m+n,$$

where $c_{ii} = 0, i = 1, 2, \cdots, m + n.$

The above mathematical model represents a standard transportation problem with (m+n) origins and (m+n) destinations.

The solution of the problem contains 2m + 2n - 1 basic variables. However, m + n of these variables appearing in the diagonal cells represent the remaining buffer stock and if they are omitted. We have m + n - 1 basic variables of our intrest.

3. Transshipment Problem with Mixed Constraints

The substantially increase or decrease of the capacity of a factory will affect the overall production and transportation cost.

Similarly, the substantially increase or decrease of the demand of a destination will affect the overall production and transportation cost.

Suppose that the source $O_i, i \in \alpha_1$ supplies exactly fixed amount a_i , source $O_i, i \in \alpha_2$ supplies at least amount a_i and source $O_i, i \in \alpha_3$ supplies at most an amount a_i . Similarly, the destination $D_j, j \in \beta_1$ demands exactly the fixed amount b_j , the destination $D_j, j \in \beta_2$ demands at least an amount b_j , the destination $D_j, j \in \beta_2$ demands at least an amount b_j , the destination $D_j, j \in \beta_3$ demands at most an amount b_j .

Considering this fact, the standard transportation problem may be written as

$$\min \quad z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

$$s.t. \quad \sum_{j=1}^{n} x_{ij} = a_i \quad \forall \quad i \in \alpha_1$$

$$\sum_{j=1}^{n} x_{ij} \ge a_i \quad \forall \quad i \in \alpha_2$$

$$\sum_{j=1}^{n} x_{ij} \le a_i \quad \forall \quad i \in \alpha_3$$

$$\sum_{i=1}^{m} x_{ij} \le b_j \quad \forall \quad j \in \beta_1$$

$$\sum_{i=1}^{m} x_{ij} \ge b_j \quad \forall \quad j \in \beta_2$$

$$\sum_{i=1}^{m} x_{ij} \le b_j \quad \forall \quad j \in \beta_3$$

where

$$I_1 = \{1, 2, \cdots, m\} = \alpha_1 \cup \alpha_2 \cup \alpha_3$$
$$I_2 = \{1, 2, \cdots, n\} = \beta + 1 \cup \beta_2 \cup \beta_3.$$

The corresponding transshipment problem then according is as follows. Find the values of x_{ij} such that

Minimize
$$z = \sum_{i=1}^{m+n} \sum_{j=1, j \neq i}^{m+n} c_{ij} x_{ij}$$

Subject to constraints

$$\begin{split} \sum_{i=1, i \neq j}^{m+n} x_{ij} &- \sum_{i=1, i \neq j}^{m+n} x_{ji} = a_i, \quad \forall \quad i \in \alpha_1 \\ \sum_{i=1, i \neq j}^{m+n} x_{ij} &- \sum_{i=1, i \neq j}^{m+n} x_{ji} \geq a_i, \quad \forall \quad i \in \alpha_2 \\ \sum_{j=1, j \neq i}^{m+n} x_{ij} &- \sum_{j=1, j \neq i}^{m+n} x_{ji} \leq a_i, \quad \forall \quad i \in \alpha_3 \\ \sum_{i=1, i \neq j}^{m+n} x_{ij} &- \sum_{i=1, i \neq j}^{m+n} x_{ji} = b_j, \quad \forall \quad j \in \beta_1 \\ \sum_{i=1, i \neq j}^{m+n} x_{ij} &- \sum_{i=1, i \neq j}^{m+n} x_{ji} \geq b_j, \quad \forall \quad j \in \beta_2 \\ \sum_{i=1, i \neq j}^{m+n} x_{ij} &- \sum_{i=1, i \neq j}^{m+n} x_{ji} \leq b_j, \quad \forall \quad j \in \beta_3 \\ x_{ij} \geq 0, \quad i, j = 1, 2, \cdots, m+n, \quad i \neq j \\ where \quad \alpha_1 \cup \alpha_2 \cup \alpha_3 = I_1 = \{1, 2, \cdots, m+n\}, \\ \beta_1 \cup \beta_2 \cup \beta_3 = I_2 = \{m+1, m+2, \cdots, m+n\}. \end{split}$$

The problem is said to be the transshipment problem with mixed constraints. If $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$ then the problem is said to be a balanced transshipment problem with mixed constraints. If $\sum_{i=1}^{m} a_i \neq \sum_{j=1}^{n} b_j$ then the problem is said to be an unbalanced transshipment problem with mixed constraints. In this case a dummy origin/destination can be introduced to make it a balanced transshipment problem with mixed constraints. Now supposing large number T for $\sum_{j=1}^{m+n} x_{ji}$, $\forall i \in I_1$ and also for $\sum_{i=1}^{m+n} x_{ij}$, $\forall j \in I_2$, the above transshipment problem can be reduced to

Minimize
$$z = \sum_{i=1}^{m+n} \sum_{j=1, j \neq i}^{m+n} c_{ij} x_{ij}$$

Subject to constraints

$$\sum_{i=1, i \neq j}^{m+n} x_{ij} = a_i + T, \quad \forall \quad i \in \alpha_1$$

$$\sum_{i=1, i \neq j}^{m+n} x_{ij} \ge a_i + T, \quad \forall \quad i \in \alpha_2$$

$$\sum_{j=1, j \neq i}^{m+n} x_{ij} \le a_i + T, \quad \forall \quad i \in \alpha_3$$

$$\sum_{j=1, j \neq i}^{m+n} x_{ij} \le a_i + T, \quad \forall \quad i \in \alpha_3$$

$$\sum_{i=1, i \neq j}^{m+n} x_{ij} = T, \quad \forall \quad i \in \alpha_1 \cup \alpha_2 \cup \alpha_3$$

Which comes out to be the transportation problem with mixed constraints.

The idea given by Bridgen [1], to convert the transportation problem with mixed constraints to an equivalent standard transportation problem is used to convert the transshipment problem with mixed constraints into standard transportation problem as follows. m+n+1=m+n+1

min
$$z = \sum_{i=1}^{m+n+1} \sum_{j=1}^{m+n+1} c'_{ij} x_{ij}$$

$$s.t. \begin{cases} \sum_{j=1}^{m+n+1} x_{ij} = a'_i, \ \forall i \in I_1, \\ a'_i = a_i + T, \ \forall i \in I_2, \\ \sum_{j=1}^{m+n+1} x_{ij} = T, \ \forall i \in I_2, \\ b'_j = b_j + T, \ \forall j \in I_2, \\ a'_i = T, \ \forall i \in I_2, \\ b'_j = T, \ \forall i \in I_2, \\ b'_j = T, \ \forall j \in I_1, \\ a'_{m+n+1} = \left(\sum_{i=1}^m a'_i + nT\right), \\ b'_{m+n+1} = \left(\sum_{i=1}^m a'_i + nT\right), \\ b'_{m+n+1} = \left(\sum_{i=1}^{m+n} bj' + mT\right), \\ \left. \begin{array}{l} c'_{i,m+n+1} = \min_{j \in \beta_2} \{c_{ij}\}, \ \forall i \in \alpha_1 \cup \alpha_2, \\ = 0, \ \forall i \in \alpha_3 \\ c'_{i,m+n+1} = \min_{j \in \beta_2} \{c_{ij}\}, \ \forall i \in \beta_1 \cup \beta_2, \\ = 0, \ \forall i \in \beta_3 \\ c'_{m+n+1,j} = \min_{i \in \beta_2} \{c_{ij}\}, \ \forall j \in \alpha_1 \cup \alpha_2, \\ = 0, \ \forall i \in \beta_3 \\ c'_{m+n+1,j} = \min_{i \in \beta_2} \{c_{ij}\}, \ \forall j \in \beta_1 \cup \beta_2, \\ = 0, \ \forall j \in \beta_3 \\ c'_{m+n+1,j} = \min_{i \in \alpha_2} \{c_{ij}\}, \ \forall j \in \beta_1 \cup \beta_2, \\ = 0, \ \forall j \in \beta_3 \\ c'_{m+n+1,j} = \min_{i \in \alpha_2} \{c_{ij}\}, \ \forall j \in \beta_1 \cup \beta_2, \\ = 0, \ \forall j \in \beta_3 \\ c'_{m+n+1,m+n+1} = 0, \\ x_i \ge 0, i = 1, 2, \cdots, m+n. \end{cases} \end{cases}$$

where A is the inlarged cost matrix.

Method for Changing Transshipment Problem into Transportation Problem Step 1 : Given the linear transshipment problem. If $\sum_{i=1}^{m} a_i = \sum_{j=m+1}^{m+n} b_j$, then the transshipment problem is balanced, take $T = \sum_{i=1}^{m} a_i$ else take $T = \max\left(\sum_{i=1}^{m} a_i, \sum_{j=1}^{m+n} b_j\right)$ and go to step 2.

Step 2: Construct a transportation tableau as follows. A row in the tableau will be needed for each supply point and transshipment point, and a column will be needed for each demand point and transshipment point.

Step 3 : Add a dummy demand point/column with a demand $= \sum_{i=1}^{m} a'_i + nT$ or a dummy supply point/row with a supply $= \sum_{j=m+1}^{m+n} b'_j + mT$ having costs as defined in (A) of equation Shipments to the dummy and from a point to itself are taken as zero. **Step 4** : Each transhipment point will have a supply equal to its original supply a_i $(i = 1, 2, \dots, m) + T$ and will have a demand equal to its original demand b_j $(j = m+1, m+2, \dots, m+n) + T$. Also, each supply point will have supply equal to original supply, T (for $i = m+1, m+2, \dots, m+n$) and each demand point will have its demand equal to original demand, T (for $j = 1, 2, \dots, m$). This ensures that any transshipment point that is a net supplier will have a net outflow equal to points original supply and a net demander will have a net inflow equal to points original demand. Although we don't know how much will be shipped through each transshipment point, we can be sure that the total amount will not exceed T.

4. Span Algorithm

Step 1 : Change transshipment problem into transportation problem.

Step 2 : Write the Span (difference of the greatest and the least element) of each row and each column along the side of the table against the corresponding row and column.
Step 3 : Identify the maximum Spanned row and column and assigned the cell having the least element in it. If the Span corresponding to two or more rows or columns are equal, select the cell where allocation is to be taken maximum.

Step 4 : Ignoring the row/column whose supply/demand is fulfilled.

Step 5 : Calculate fresh span for the remaining sub-matrix and then go to Step 2.

Step 6 : Continue the process from Step 2 to Step 5 until all the demand and supply are fulfilled.

5. Numerical Example

Balanced transhipment problem with mixed constraints.

To illustrate the span method we consider the balanced transhipment problem involving threeorigins and three destinations. The availabilities at the origins, the requirements at the destinations and the costs transportation are given below in the Table.

	$O_1(j=1)$	$O_2(j=2)$	$O_3(j=3)$	$O_4(j=4)$	$O_5(j=5)$	$O_6(j=6)$	a_i
$O_1(j=1)$	0	1	1	5	4	7	= 4
$O_2(j=2)$	1	0	1	2	6	5	≥ 6
$O_3(j=3)$	1	1	0	4	8	3	≤ 5
$O_4(j=4)$	5	2	4	0	2	2	
$O_5(j=5)$	4	6	8	2	0	2	
$O_6(j=6)$	7	5	3	2	2	2	
b_i				=5	≥ 6	≤ 4	

Since $T = \sum_{i=1}^{3} a_i = \sum_{j=4}^{6} b_j = 15$, we convert the problem into a linear transportation problem by adding 15 units to each a_i and b_j .

	$O_1(j=1)$	$O_2(j=2)$	$O_3(j=3)$	$O_4(j=4)$	$O_5(j=5)$	$O_6(j=6)$	a_i
$O_1(j=1)$	0	1	1	5	4	7	= 19
$O_2(j=2)$	1	0	1	2	6	5	21
$O_3(j=3)$	1	1	0	4	8	3	20
$O_4(j=4)$	5	2	4	0	2	2	15
$O_5(j=5)$	4	6	8	2	0	2	15
$O_6(j=6)$	7	5	3	2	2	2	15
b_j	15	15	15	20	21	19	

Also, we add a dummy column D_4 with demand equal to $\sum_{i=1}^{m} a'_i + nT = 105$ and a dummy row O_4 with availability equal to $\sum_{j=m+1}^{m+n} b'_j + mT = 105$. We have the following table

	01	O ₂	O ₃	D1	D ₂	D₃	dD₄	Supply
01	0	1	1	5	4	7	4	19
O ₂	1	0	1	2	6	5	6	21
O₃	1	1	0	4	8	3	0	20
D1	5	2	4	0	2	2	2	15
D ₂	4	6	8	2	0	2	6	15
D ₃	7	5	3	2	0	2	6	15
dO ₄	4	6	0	2	6	0	0	105
Demand	15	15	15	20	21	19	105	

Us	ing a	a sp	an n	neth	od											
	01		02		0 ₃	8	D1		D ₂		D₃		dD.	1		Row Span
01	0	15	1		1	4	5		4		7		4		19	(7)(7)(7)(5)(5)(4)(4)(4)(4)
02	1		0	15	1	6	2		6		5		6		21	(6)(6)(6)(6)(6)(5)(1)(1)(1)(1)
O3	1		1		0		4		8		3		0	20	20	(8)
D1	5		2		4		0	15	2		2		2		15	(5)(5)(5)(5)(4)(4)(4)(4)
D ₂	4		6		8		2		0	15	2		6		15	(8)(8)
D ₃	7		5		3	4	2	5	0	6	2		6		15	(7)(7)(7)(7)(6)(5)(3)(1)(1)(1)(1)
dO ₄	4		6		0	1	2		6		0	19	0	85	105	(6)(6)(6)(6)(6)(6)(2)(2)(2)
	15		15		15		20		21		19		105	5		
Column Span	(7)		(6)		(8)		(5)		(8)		(7)		(6)			
	(7)		(6)		(8)		(5)		(6)		(7)		(6)			
	(7)		(5)		(4)		(5)		(6)		(7)		(6)			
	(7)		(5)		(4)		(5)		(6)				(6)			
			(5)		(4)		(5)		(6)				(6)			
			(6)		(4)		(5)		(6)							
					(4)		(5)		(6)							
					(4)		(5)									
					(3)		(3)									
					(3)		(2)									
					(2)		(0)									

Total transshipment cost by span method is given by

$$Z = 0^{*}15 + 1^{*}4 + 0^{*}15 + 1^{*}6 + 0^{*}20 + 0^{*}15 + 0^{*}15 + 3^{*}4 + 2^{*}5$$
$$+0^{*}6 + 0^{*}1 + 0^{*}19 + 0^{*}85 = 32.$$

Similar illustration can be made to solve the transshipment problem with mixed constraints using span method.

Conclusion: In this paper, a new and simple method Span method for solving Transshipment problem with mixed constraints is proposed. This method is useful for all types of Transshipment problems- maximization or minimization, balanced or unbalanced and restricted. The algorithm of the method has been presented.

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