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ENERGY OF A COMPLEX FUZZY GRAPH

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Abstract

In this paper, the concept of fuzzy graph is extended to a complex fuzzy graph. Complex fuzzy graphs are encountered in complex fuzzy set theory. The novelty of the complex fuzzy set lies in the range of values its membership function may attain. In contrast to a traditional fuzzy membership function, this range is not limited to [0, 1], but extended to the unit circle in the complex plane. Thus, the complex fuzzy set provides a mathematical framework for describing membership in a set in terms of a complex number. We defined energy of a complex fuzzy graph , it is an extension of energy of a fuzzy graph. We also defined lower bound and upper bound of energy of a complex fuzzy graph. These concepts provided with numerical example.

Key Words : *Complex fuzzy set, Complex fuzzy graph, Energy of a graph, Energy of a complex fuzzy Graph, Lower bound, Upper bound.*

1. Introduction

Fuzzy set was introduced by Zadeh [8] whose basic component is only a membership function. In Zadehs fuzzy set, the sum of membership degree and a non-membership degree is equal to one. Complex fuzzy set (CFS) [5]-[6] is a new development in the theory of fuzzy systems in [8]. The concept of CFS is an extension of fuzzy set, by which the membership for each element of a complex fuzzy set is extended to complex-valued state.

The first definition of fuzzy graphs was proposed by Kauffmann [3] in 1973, from the Zadeh's fuzzy relations [8] [9] [10]. The first definition of a complex fuzzy graph defined in [7]. The energy of a fuzzy graph defined in [1] and the energy of an intuitionistic fuzzy graph defined in [4].

In this paper, we extend the concept of energy of fuzzy graph extends to the energy of a complex fuzzy graph. This paper is organized as follows. In Section 2, we give all the basic definitions related to complex fuzzy sets and complex fuzzy graph. In Section 3, we define the innovative concept of energy of a complex fuzzy graph with its lower bound and upper bound. In Section 4, we illustrate these concepts with numerical example. In Section 5, we give the conclusion.

2. Preliminaries

Definition: 2.1 : Ramot et al. [5] proposed an important extension of these ideas, the Complex Fuzzy Sets (CFS), where the membership function of a CFS is complex-valued, different from fuzzy complex numbers developed in [2] . The membership function to characterize a CFS consists of an amplitude function and a phase function. In other words, the membership of a CFS is in the two-dimensional complex-valued unit disc space, instead of in the one-dimensional real-valued unit interval space. Thus, CFS can be much richer in membership description than traditional fuzzy set. Assume there is a complex fuzzy set S whose membership function $\mu_S(h)$ is given as follows.

$$\begin{aligned}\mu_S(h) = r_S(h)e^{j\omega_S(h)} &= Re(\mu_S(h)) + j\ Im(\mu_S(h)) \\ &= r_S(h)\cos(\omega_S(h)) + jr_S(h)\sin(\omega_S(h))\end{aligned}$$

where $j = \sqrt{-1}$, h is the base variable for the complex fuzzy set, $r_s(h)$ is the amplitude function of the complex membership, $\omega_s(h)$ is the phase function of the complex membership function. The property of sinusoidal waves appears obviously in the definition of complex fuzzy set. In the case that $\omega_s(h)$ equals to 0, a traditional fuzzy set is regarded as a special case of a complex fuzzy set.

Definition 2.2 : Let V be a nonempty set. A fuzzy graph is a pair of functions $G : (\sigma, \mu)$ where σ is a fuzzy subset of +V and μ is a symmetric fuzzy relation on σ. i.e. $\sigma : V \to [0, 1]$ and $\mu : V \times V \to [0, 1]$ such that $\mu(u, v) \leq \sigma(u) \wedge \sigma(v)$ for all u, v in V.

Definition 2.3 : A complex fuzzy directed graph G is a quadruple of the form $G = (V, \sigma, E, \phi)$, where V is a complex fuzzy set referred to as the set of vertices and $E \subseteq VXV$ is a complex fuzzy set of edges. $\sigma : V \to [0, 1] \times [0, 1]$ is a mapping from V to $[0, 1] \times [0, 1]$. i.e. σ is the assignment of the complex degrees of membership to members of V, $\varphi : E \to [0, 1] \times [0, 1]$ is a function that maps elements of the form $e \in E : (u, v)$ to $[0, 1] \times [0, 1]$, where $u \in V, v \in V$, we assume that $V \cap E = \phi$. In general, we use the form $e = (a, b)$ to denote a specific edge that is said to connect the vertices a and b. For an undirected graph both $e_1 = (u, v)$ and $e_2 = (v, u)$ are in the domain of φ.

Example 2.1 :

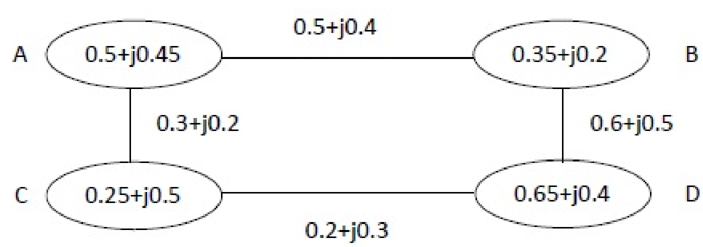

Fig.1. Complex Fuzzy Graph

Definition 2.4 : Alternative definition of complex fuzzy graph given as follows. An complex fuzzy graph is defined as $CG = (V, E, r, \omega)$ where V is the set of vertices and E is the set of edges. r represents a amplitude membership function defined on $V \times V$ and ω represents a phase membership function defined on $V \times V$. We denote $r(v_i, v_j)$ by r_{ij} and $\omega(v_i, v_j)$ by ω_{ij} such that (i) $0 \leq r_{ij}, \omega_{ij} \leq 1$. Hence $(V \times V, r, \omega)$ is an complex fuzzy set.

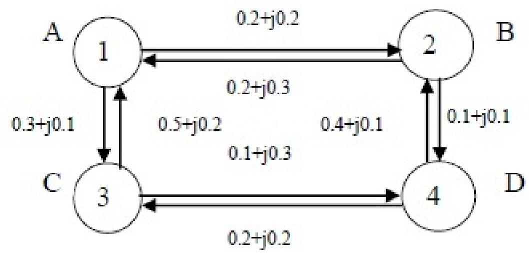

Fig.2. Directed complex fuzzy graph

3. Energy of a Complex Fuzzy Graph

Definition 3.1 [1] : Let $G = (V, \sigma, \mu)$ be a fuzzy graph and A be its adjacency matrix. The eigen values of A are called eigen values of G. The spectrum of A is called the spectrum of G. It is denoted by Spec G.

Definition 3.2 [1] : Let $G = (V, \sigma, \mu)$ be a fuzzy graph and A be its adjacency matrix. Energy of G is defined as the sum of the absolute values of the Eigen values of A.

Definition 3.3 : The adjacency matrix of a complex fuzzy graph can be written as two adjacent matrices, one containing the entries as membership values of amplitude function and the other containing the entries as membership values of phase function. i.e., $A(CG) = (A(r_{ij}), A(\omega_{ij}))$.

Definition 3.4 : The eigen value of an adjacency matrix of complex fuzzy graph $A(CG)$ is defined as (X, Y), where X is the set of eigen values of $A(r_{ij})$ and Y is the set of eigen values of $A(\omega_{ij})$.

Definition 3.5 : The energy of an complex fuzzy graph $CG = (V, E, r, \omega)$ is defined as $\left(\sum_{\alpha_i \in X} |\alpha_i|, \sum_{\beta_i \in Y} |\beta_i|\right)$ where $\sum_{\alpha_i \in X} |\alpha_i|$ is defined as an energy of the Amplitude matrix denoted by $E(r_{ij}(CG))$ and $\sum_{\beta_i \in Y} |\beta_i|$ is an energy of the Phase matrix denoted by $E(\omega_{ij}(CG))$.

Result [1] : Let CG is an complex fuzzy directed graph(without loops) with $|V| = n$ and $|E| = m$ and $A(CG) = (A(r_{ij}), A(\omega_{ij}))$ be an adjacency matrix of complex fuzzy graph of CG then Upper bound and Lower bound of the Energy of the Complex fuzzy graph is

(i) $$\sqrt{2 \sum_{1 \le i < j \le n} r_{ij} r_{ji} + n(n-1)|A|^{\frac{2}{n}}} \le E(r_{ij}(CG)) \le \sqrt{2n \sum_{1 \le i < j \le n} r_{ij} r_{ji}}$$

where $|A|$ is the determinant of $A(r_{ij})$ and

$$\text{(ii)} \qquad \sqrt{2\sum_{1\leq i<j\leq n}\omega_{ij}\omega_{ji}+n(n-1)|B|^{\frac{2}{n}}}\leq E(\omega_{ij}(CG))\leq\sqrt{2n\sum_{1\leq i<j\leq n}\omega_{ij}\omega_{ji}}$$

where $|B|$ is the determinant of $A(\omega_{ij})$.

4. Numerical Example

For a complex fuzzy graph in Fig.2, the adjacency matrix is $A(CG)=(A(r_{ij}),A(\omega_{ij}))$ where

$$A(r_{ij})=\begin{pmatrix}0&0.2&0.3&0\\0.2&0&0&0.1\\0.5&0&0&0.1\\0&0.4&0.2&0\end{pmatrix} \text{ and } A(\omega_{ij})=\begin{pmatrix}0&0.2&0.1&0\\0.3&0&0&0.1\\0.2&0&0&0.3\\0&0.1&0.2&0\end{pmatrix}$$

Eigen values of Amplitude matrix $=\{0.110132,-0.110132,0.444827,-0.444827\}$

Eigen values of Phase matrix $=\{0.121622,-0.121622,0.367706,-0.367706\}$

Energy of Amplitude matrix $=1.10992$

Energy of Phase matrix $=0.97865$

Lower bound and upper bound of Amplitude matrix $=\{1.043,1.414\}$

Lower bound and upper bound of Phase matrix $=\{1.0376,1.4697\}$

5. Conclusion

Energy of a graph : This concept is of great interest in a vast range of fields, especially in chemistry since it can be used to approximate the total π-electron energy of molecule. The use of a complex fuzzy graph was advantageous because it provided a method for describing the time frame of the relation. Thus in all, the energy of a complex fuzzy graph seems to be a promising new concept, paving the way to numerous possibilities for future research.

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