International J. of Math. Sci. & Engg. Appls. (IJMSEA)
ISSN 0973-9424, Vol. 10 No. I (April, 2016), pp. 23-33

PERISTALTIC TRANSPORT OF A COUPLE STRESS FLUID IN AN INCLINED PLANAR CHANNEL THROUGH POROUS MEDIUM UNDER THE EFFECT OF MAGNETIC FIELD

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Abstract

The present paper investigates the peristaltic transport of a couple stress fluid in an inclined planar channel through porous medium under the effect of magnetic field. The effects of various physical parameters on velocity, pressure gradient and friction force have been discussed and computed numerically. The effects of various key parameters are discussed with the help of graphs.

1. Introduction

Peristalsis is known to be one of the main mechanisms of transport for many

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Key Words : *Peristaltic transport, Couple stress fluid, Porous medium, Magnetic field and inclined channel.*

physiological fluids, which is achieved by the passage of progressive waves of area contraction and expansion over flexible walls of a tube containing fluid. Various studies on peristaltic transport, experimental as well as theoretical, have been carried out by many researchers to explain peristaltic pumping in physiological systems. The study of couple stress fluid is very useful in understanding various physical problems because it possesses the mechanism to describe rheological complex fluids such as liquid crystals and human blood. By couple stress fluid, we mean a fluid whose particles sizes are taken into account, a special case of non-Newtonian fluids. Srivastava et.al., [1] peristaltic transport of a physiological fluid: part I flow in non- uniform geometry. Latham [2] investigated the fluid mechanics of peristaltic pump and science. Mekhemier [3] studied non-linear peristaltic transport a porous medium in an inclined planar channel. Srivastava and Srivastava [4] studied peristaltic transport of a non-newtonian fluid: applications to the vas deferens and small intestine. El-dabe and El-Mohandis [5] have studied magneto hydrodynamic flow of second order fluid through a porous medium on an inclined porous plane. Rathod and Asha [6] effects of magnetic field and an endoscope on peristaltic motion. Rathod and Mahadev [7] studied effect of magnetic field on ureteral peristalsis in cylindrical tube. Rathod and Pallavi [8] studied the influence of wall properties on MHD peristaltic transport of dusty fluid. Rathod and Pallavi [9] studied the effect of slip condition and heat transfer on MHD peristaltic transport through a porous medium with complaint wall. Rathod and Mahadev [10] studied slip effects and heat transfer on MHD peristaltic flow of Jeffrey fluid in an inclined channel. Rathod and Laxmi [11] investigated effects of heat transfer on the peristaltic MHD flow of a Bingham fluid through a porous medium in an inclined channel. Rathod and Laxmi [12] studied effects of heat transfer on the peristaltic MHD flow of a Bingham fluid through a porous medium in a channel. Jayarami Reddy et.al., [13] studied peristaltic flow of a Williamson fluid in an inclined planar channel under the effect of a magnetic field. Ramana Kumari and Radhakrishnamacharya [14] studied effect of slip on peristaltic transport in an inclined channel with wall effects. Reddappa et.al., [15] studied the peristaltic transport of a Jeffrey fluid in an inclined planar channel with variable viscosity under the effect of a magnetic field. Rathod et.al., [16] studied peristaltic transport of a conducting couple stress fluid through a porous medium in a channel. Rathod and Sridhar [17] studied peristaltic flow of a couple stress fluid in an inclined channel. Rathod et.al., [18] studied peristaltic flow of a couple stress fluid in an inclined channel under the effect of magnetic field.

The present research aim is to investigate the peristaltic transport of a couple stress fluid in a two dimensional inclined planar channel through porous medium under the effect of magnetic field. The computational analysis has been carried out for drawing velocity profiles, pressure gradient and frictional force.

2. Formulation of the Problem

We consider a peristaltic flow of a Couple stress fluids through two-dimensional channel of width 2a and inclined at an angle α to the horizontal symmetric with respect to its axis. The walls of the channel are assumed to be flexible.

The wall deformation is

$$H(x,t) = a + b\cos\left(\frac{2\pi}{\lambda}(X - ct)\right) \tag{1}$$

where 'b' is the amplitude of the peristaltic wave, 'c' is the wave velocity, 'λ' is the wave length, t is the time and X is the direction of wave propagation.

The equations governing the flow for present problem are given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2}$$

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \mu\nabla^2 u - \eta^*\nabla^4 u - \frac{\mu}{K^*}u - \sigma B_0^2 u + \rho g\sin\alpha \tag{3}$$

$$\rho\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{\partial p}{\partial y} + \mu\nabla^2 v - \eta^*\nabla^4 v - \frac{\mu}{K^*}v - \sigma^2 B_0^2 v - \rho g\cos\alpha \tag{4}$$

where, $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$.

u and v are velocity components, 'p' is the fluid pressure, 'ρ' is the density of the fluid, 'μ' is the coefficient of viscosity, 'η^*' is the coefficient of couple stress, 'g' is the gravity due to acceleration, 'α' angle of inclination, 'σ' is electric conductivity, 'B_0' is applied magnetic field and 'K' is porous media.

Introducing a wave frame (x, y) moving with velocity c away from the fixed frame (X, Y) by the transformation

$$x = X - ct, \quad y = Y, \quad u = U, \quad v = V, \quad p = P(X,t) \tag{5}$$

We introduce the non-dimensional variables:

$$x^* = \tfrac{x}{\lambda}, \quad y^* = \tfrac{u}{c}, \quad v^* = \tfrac{v}{c\delta}, \quad t^* = \tfrac{tc}{\lambda}, \quad G = \tfrac{\rho g a^2}{\mu c},$$
$$p = \tfrac{a^2}{\lambda \mu c} p^*(z^*), \quad K = \tfrac{K^*}{\lambda}, \quad M = B_0\sqrt{\tfrac{\sigma}{\mu a^2}}, \quad \phi = \tfrac{b}{a} \tag{6}$$

Equation of motion and boundary conditions in dimensionless form becomes

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{7}$$

$$Re\delta\left(u\tfrac{\partial u}{\partial x} + v\tfrac{\partial u}{\partial y}\right) = -\tfrac{\partial p}{\partial x} + \left(\delta^2\tfrac{\partial^2 u}{\partial x^2} + \tfrac{\partial^2 u}{\partial y^2}\right) - \tfrac{1}{\gamma^2}\left(\delta^2\tfrac{\partial^2}{\partial x^2} + \tfrac{\partial^2}{\partial y^2}\right)\left(\delta^2\tfrac{\partial^2 u}{\partial x^2} + \tfrac{\partial^2 u}{\partial y^2}\right)$$
$$M^2 u - K^2 u + G\sin\alpha \tag{8}$$

$$Re\delta^3\left(u\tfrac{\partial v}{\partial x} + v\tfrac{\partial v}{\partial y}\right) = -\tfrac{\partial p}{\partial y} + \delta^2\left(\delta^2\tfrac{\partial^2 v}{\partial x^2} + \tfrac{\partial^2 v}{\partial y^2}\right) - \tfrac{1}{\gamma^2}\delta^2\left(\delta^2\tfrac{\partial^2}{\partial x^2} + \tfrac{\partial^2}{\partial y^2}\right)\left(\delta^2\tfrac{\partial^2 v}{\partial x^2} + \tfrac{\partial^2 v}{\partial y^2}\right)$$
$$M^2\delta^2 v - K^2\delta^2 v - G\delta\cos\alpha \tag{9}$$

where, $\gamma^2 = \frac{\eta^*}{\mu a^2}$ couple-stress parameter, $k^2 = \frac{a}{K}$ porous media and $M^2 = B_0^2\frac{\sigma}{\mu a^2}$ Hartmann number.

The dimensionless boundary conditions are:

$$\frac{\partial u}{\partial y} = 0; \quad \frac{\partial^2 u}{\partial y^2} = 0 \quad \text{at} \quad y = 0$$

$$u = -1; \quad \frac{\partial^2 u}{\partial y^2} \ \text{finite at} \ \ y = \pm h = 1 + \phi\cos[2\pi x]. \tag{10}$$

Using long wavelength approximation and neglecting the wave number δ, one can reduce governing equations:

$$\frac{\partial p}{\partial y} = 0 \tag{11}$$

$$\frac{\lambda p}{\partial x} = \frac{\partial^2 u}{\partial y^2} - \frac{1}{\gamma^2}\frac{\partial^4 u}{\partial y^4} - M^2 u - K^2 u + G\sin\alpha. \tag{12}$$

Solving the Eq. (12) with the boundary conditions (10), we get

$$u = \tfrac{\partial p}{\partial x}\left(\left(1 + \tfrac{h^2}{\gamma^2}\right)\tfrac{y^2}{2} - \tfrac{h^2}{2} - \tfrac{1}{\gamma^2}\left(\tfrac{y^4}{4} + \tfrac{h^4}{4}\right) - \tfrac{1}{(M^2+K^2)}\right)$$
$$+\tfrac{G\sin\alpha}{2}(h^2 - y^2) + \tfrac{(y^2-h^2)}{2\gamma^2} + \tfrac{1}{\gamma^2} - 1. \tag{13}$$

The volumetric flow rate in the wave frame is defined by

$$q = \int_0^h u dy = \frac{h^3}{3}\left(G\sin\alpha - \frac{1}{\gamma^2}\right) + h\left(\frac{1}{\gamma^2} - 1\right) - \frac{\partial p}{\partial x}\left(\frac{h^3}{3} + \frac{2h^5}{15\gamma^2} - \frac{h}{(M^2+K^2)}\right). \tag{14}$$

The expression for pressure gradient from Eq. (14) is given by

$$\frac{\partial p}{\partial x} = \frac{\left(\frac{h^3}{3}\left(G\sin\alpha - \frac{1}{\gamma^2}\right) h\left(\frac{1}{\gamma^2} - 1\right) - q\right)}{\left(\frac{h^3}{3} + \frac{2h^5}{15\gamma^2} + \frac{h}{(M^2+K^2)}\right)}. \tag{15}$$

The instantaneous flux $Q(x,t)$ in the laboratory frame is

$$Q(x,t) = \int_0^h (u+1)dy = q + h. \tag{16}$$

The average flux over one period of peristaltic wane is $\overline{Q}$

$$\overline{Q} = \frac{1}{T}\int_0^T Q dt = q + 1. \tag{17}$$

From equations (15) and (17), the pressure gradient is obtained as

$$\frac{\partial p}{\partial x} = \frac{\left(\frac{h^3}{3}\left(G\sin\alpha - \frac{1}{\gamma^2}\right) + h\left(\frac{1}{\gamma^2} - 1\right) - (\overline{Q} - 1)\right)}{\left(\frac{h^3}{3} + \frac{2h^5}{15\gamma^2} + \frac{h}{(M^2+K^2)}\right)}. \tag{18}$$

The pressure rise (drop) over one cycle of the wave can be obtained as

$$\Delta P = \int_0^1 \left(\frac{dp}{dx}\right) dx. \tag{19}$$

The dimensionless frictional force F at the wall across one wavelength is given by

$$F = \int_0^1 h\left(-\frac{dp}{dx}\right) dx. \tag{20}$$

3. Results and Discussions

In this section we have presented the graphical results of the solutions axial velocity u, pressure rise ΔP, friction force F for the different values of couple stress (γ), porous medium (K), angle of inclination (α) and magnetic field (M). The axial velocity is shown in Figs. (1 to 4).

The Variation of u with γ, we find that u depreciates with increase in γ (Fig. 1). The Variation of u with porous medium K shows that for u decreases with increasing in K (fig. 2). The Variation of u with angle of inclination α shows that for u increases with increasing in α (Fig. 3). The Variation of u with magnetic field M shows that for u decreases with increasing in M (Fig 4).

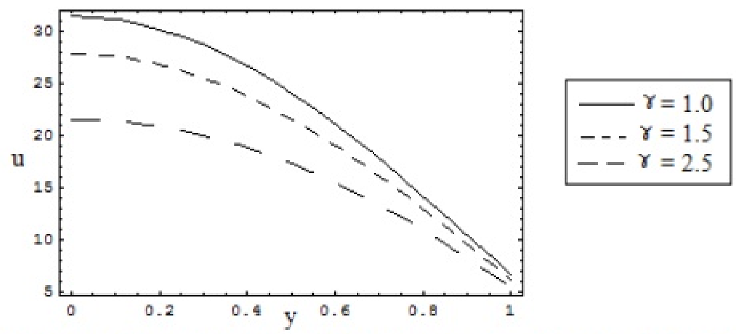

Fig. 1: Effect of γ on u, when $\phi = 0.2$, x= 0.1, p=−25, M=1, G= 6, K=5 & $\alpha = \pi/4$.

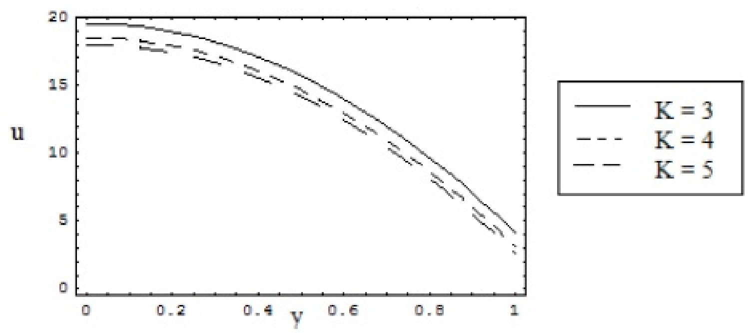

Fig. 2: Effect of **K** on u, when $\gamma = 8$, $\phi = 0.1$, x=0.1, p=−25, M=1, G=8 & $\alpha = \pi/4$.

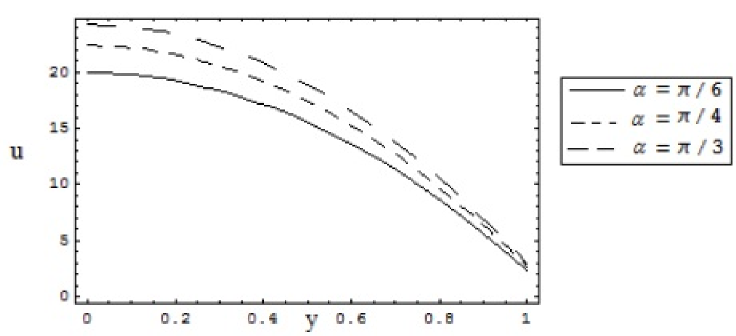

Fig. 3: Effect of α on u, when $\gamma = 8$, $\phi = 0.1$, x= 0.1, p =−25, M=3, **K** = 8 & **G** =20.

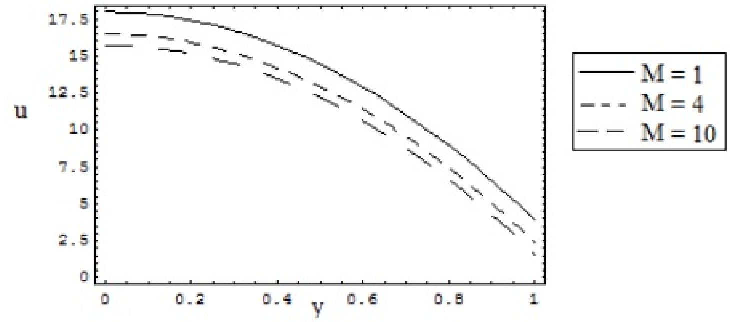

Fig. 4: Effect of M on u, when γ =6,ϕ=.1,x=0.1, p=-25, G=4, K = 3 & $\alpha = \pi/4$.

The variation of pressure rise ΔP is shown in Figs (5 to 8) for a different values of γ, K, α and M. We find that ΔP depreciates with increase in γ (Fig. 5). The Variation of ΔP with porous medium K shows that for ΔP increases with increasing in K (Fig 6). The Variation of ΔP with angle of inclination α shows that for ΔP increases with increasing in α (Fig 7). The Variation of ΔP with magnetic field M shows that for ΔP increases with increasing in M (Fig 8).

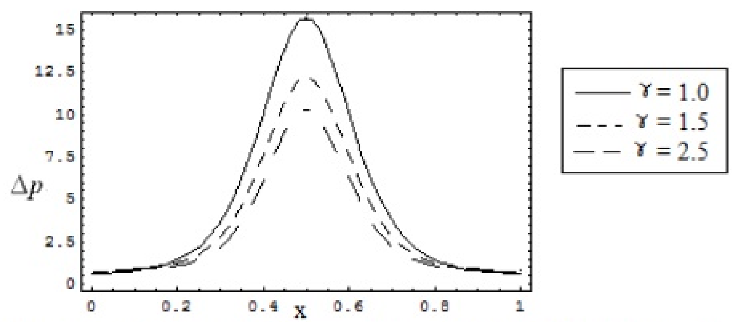

Fig. 5: Effect of γ on Δp ,when ϕ=0.5, G = 2, $\alpha = \pi/4$, Q = 0, M=1 & K = 5.

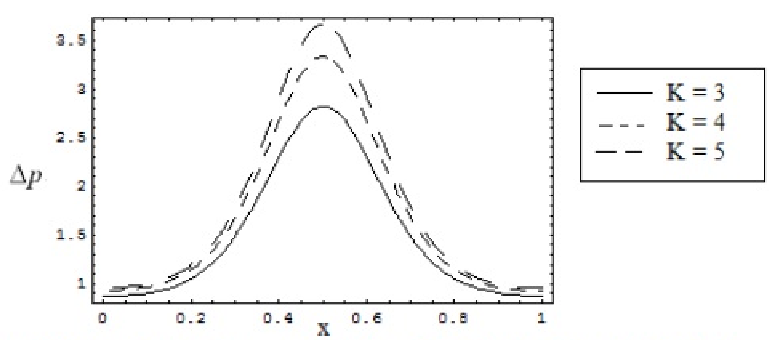

Fig. 6: Effect of **K** on Δp ,when γ =3,ϕ=0.3, G=2, M=1, $\alpha = \pi/4$ & Q = 0.

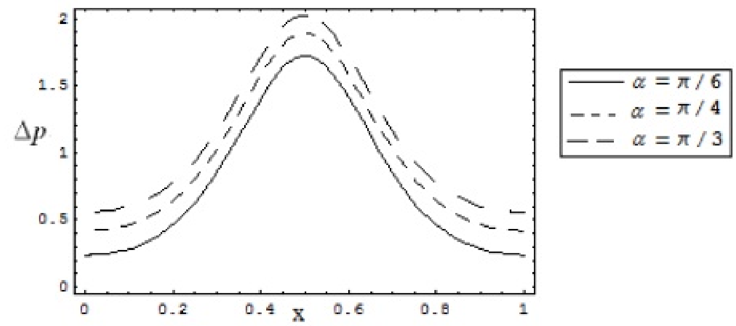

Fig. 7: Effect of α on Δp ,when γ =3, ϕ =0.2,M=1, K = 5, Q=0 & G = 1.

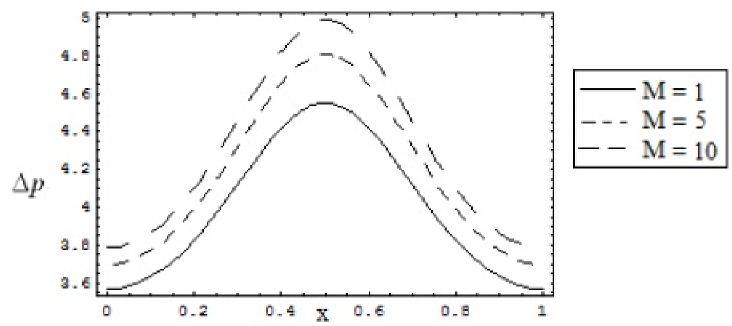

Fig. 8: Effect of M on Δp ,when γ =1.5, ϕ =0.1,G=6, K = 5, Q=0 & α =π/4.

The variation of friction force F is shown in Figs. (9 to 12) for a different values of γ, K, α and M. Here, it is observed that the effect of all the parameters on friction force are opposite behavior as to the effects on pressure with time average mean flow rate is observed.

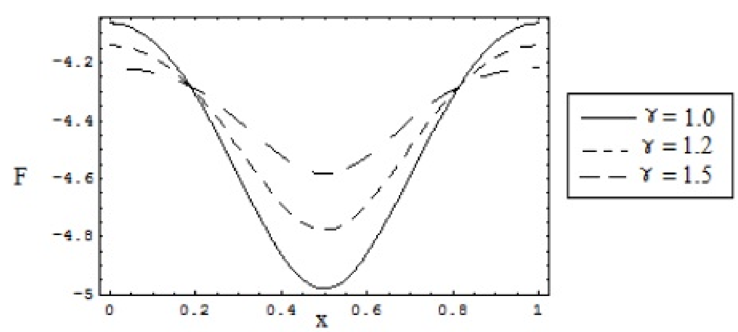

Fig. 9: Effect of γ on F,when G = 6, ϕ =0.1,α = π/4, Q = 0, M=25 & K = 5.

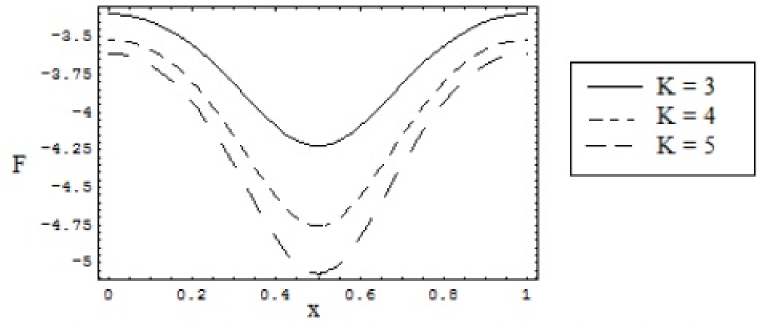

Fig. 10: Effect of **K** on F, when $\gamma = 1, \phi = 0.2, \alpha = \pi/4$, Q = 0, M=1 & G=6.

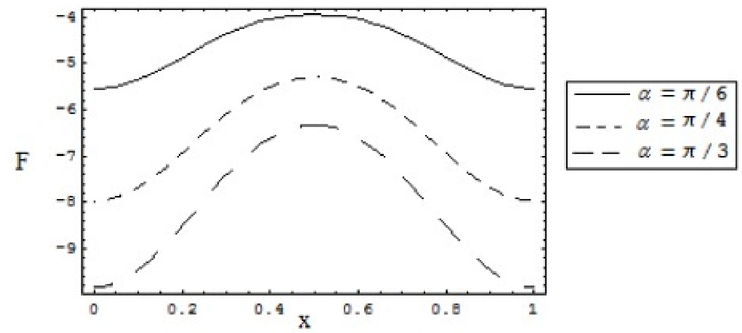

Fig. 11: Effect of α on F, when $\gamma = 5, \phi = 0.2$, G = 12, Q=0, M=1 & K = 3.

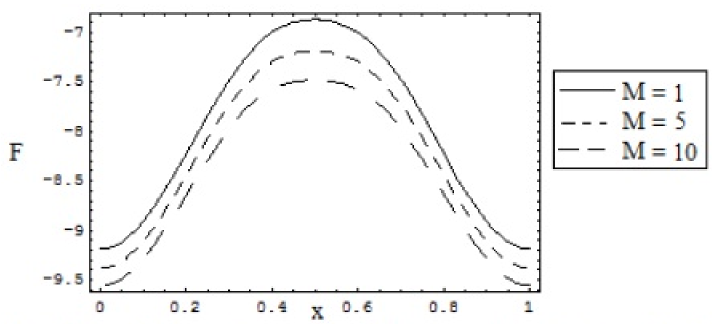

Fig. 12: Effect of M on F, when $\gamma = 8, \phi = .2, \alpha = \pi/4$, Q=0, G=12 & K=6.

4. Conclusion

In this paper we presented a theoretical approach to study the peristaltic transport of a couple stress fluid in an inclined planar channel through porous medium under the effect of magnetic field. The effect of various values of parameters on Velocity, Pressure rise and Friction force have been computed numerically and explained graphically.

We conclude the following observations:

1. The velocity u increases with increasing angle of inclination α but, decreases with increasing in couple stress parameter γ, magnetic field M and porous medium K.

2. The pressure ΔP increases with increasing in magnetic field M, porous medium K and angle of inclination α but, decreases with increasing in couple stress parameter γ.

3. The friction force F increases with decreasing in magnetic field M, porous medium K and angle of inclination α but, increases with increasing in couple stress parameter γ.

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