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BOUNDARY SET ON SOFT BIMINIMAL SPACES

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Abstract

The aim of this paper is to introduce the concept of some fundamental properties of boundary set on soft biminimal spaces.

1. Introduction

In 2000, V. Popa and T. Noiri [14] introduced the concepts of minimal structure (briefly m-structure). They also introduced the concepts of m_X-open set and m_X-closed set and characterize those sets using m_X-closure and m_X-interior operators respectively. J.C. Kelly [7] defined the study of bitopological spaces in 1963. In 2010, C. Boonpok [2] introduced the concept of biminimal structure space and studied $m_X^1 m_X^2$-open sets and $m_X^1 m_X^2$-closed sets in biminimal structure spaces. Russian researcher Molodtsov [5], initaited the concept of soft sets as a new mathematical tool to deal with uncertainties

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Key Words : *Soft minimal, Soft biminimal space, Boundary.*

while modeling problems in engineering physics, computer science, economics, social sciences and medical sciences in 1999. In 2015, R. Gowri and S. Vembu [11] introduced Soft minimal and soft biminimal spaces. In this paper, we introduce the concept of boundary set on soft biminimal spaces and some of their simple properties.

2. Preliminaries

Definition 2.1 [11] : Let X be an initial universe set, E be the set of parameters and $A \subseteq E$. Let F_A be a nonempty soft set over X and $\tilde{P}(F_A)$ is the soft power set of F_A. A subfamily $\tilde{m}$ of $\tilde{P}(F_A)$ is called a soft minimal set over X if $F_\emptyset \in \tilde{m}$ and $F_A \in \tilde{m}$.
$(F_A, \tilde{m})$ or $(X, \tilde{m}, E)$ is called a soft minimal space over X. Each member of $\tilde{m}$ is said to be $\tilde{m}$ -soft open set and the complement of an $\tilde{m}$-soft open set is said to be $\tilde{m}$-soft closed set over X.

Definition 2.2 [11] : Let X be an initial universe set and E be the set of parameters. Let $(X, \tilde{m_1}, E)$ and $(X, \tilde{m_2}, E)$ be the two different soft minimals over X. Then $(X, \tilde{m_1}, \tilde{m_2}, E)$ or $(F_A, \tilde{m_1}, \tilde{m_2})$ is called a soft biminimal spaces.

Definition 2.3 [11] : A soft subset F_B of a soft biminimal space $(F_A, \tilde{m}_1, \tilde{m}_2)$ is called $\tilde{m}_1\tilde{m}_2$-soft closed if $\tilde{m}cl_1(\tilde{m}cl_2(F_B)) = F_B$. The complement of $\tilde{m}_1\tilde{m}_2$-soft closed set is called $\tilde{m}_1\tilde{m}_2$-soft open.

Proposition 2.4 [11] : Let $(F_A, \tilde{m}_1, \tilde{m}_2)$ be a soft biminimal space over X. Then F_B is a $\tilde{m}_1\tilde{m}_2$-soft open soft subsets of $(F_A, \tilde{m}_1, \tilde{m}_2)$ if and only if $F_B = \tilde{m}Int_1(\tilde{m}Int_2(F_B))$.

Definition 2.5 [5] : Let U be an initial universe and E be a set of parameters. Let $P(U)$ denote the power set of U and A be a nonempty subset of E. A pair (F, A) is called a soft set over U, where F is a mapping given by $F : A \rightarrow P(U)$.
In other words, a soft set over U is a parametrized family of subsets of the universe U. For $\epsilon \in A$. $F(\epsilon)$ may be considered as the set of ϵ - approximate elements of the soft set (F, A). Clearly, a soft set is not a set.

Example 2.6 [5] : Let $U = \{u_1, u_2\}$, $E = \{x_1, x_2, x_3\}$, $A = \{x_1, x_2\} \subseteq E$ and
$F_A = \{(x_1, \{u_1, u_2\}), (x_2, \{u_1, u_2\})\}$. Then

$$F_{A_1} = \{(x_1, \{u_1\})\},$$
$$F_{A_2} = \{(x_1, \{u_2\})\},$$
$$F_{A_3} = \{(x_1, \{u_1, u_2\})\},$$
$$F_{A_4} = \{(x_2, \{u_1\})\},$$

$F_{A_5} = \{(x_2, \{u_2\})\}$,
$F_{A_6} = \{(x_2, \{u_1, u_2\})\}$,
$F_{A_7} = \{(x_1, \{u_1\}), (x_2, \{u_1\})\}$,
$F_{A_8} = \{(x_1, \{u_1\}), (x_2, \{u_2\})\}$,
$F_{A_9} = \{(x_1, \{u_1\}), (x_2, \{u_1, u_2\})\}$,
$F_{A_{10}} = \{(x_1, \{u_2\}), (x_2, \{u_1\})\}$,
$F_{A_{11}} = \{(x_1, \{u_2\}), (x_2, \{u_2\})\}$,
$F_{A_{12}} = \{(x_1, \{u_2\}), (x_2, \{u_1, u_2\})\}$,
$F_{A_{13}} = \{(x_1, \{u_1, u_2\}), (x_2, \{u_1\})\}$,
$F_{A_{14}} = \{(x_1, \{u_1, u_2\}), (x_2, \{u_2\})\}$,
$F_{A_{15}} = F_A$,
$F_{A_{16}} = F_\emptyset$.

are all soft subsets of F_A . so $|\tilde{P}(F_A)| = 2^4 = 16$.

3. Boundary Set On Soft Biminimal Spaces

In this section, we introduce the concept and study some fundamental properties of boundary set on soft biminimal spaces.

Definition 3.1 : Let $(F_A, \tilde{m}_1, \tilde{m}_2)$ be a soft biminimal space (SBMS), F_B be a soft subset of F_A and $x \in F_A$. Then x is called $\tilde{m}_1\tilde{m}_2$-boundary point of F_B if $x \in \tilde{m}_iCl(\tilde{m}_jCl(F_B)) \cap \tilde{m}_iCl(\tilde{m}_jCl(F_A - F_B))$. We denote the set of all $\tilde{m}_1\tilde{m}_2$-boundary point of F_B by $\tilde{m}Bdr_{ij}(F_B)$ where $i, j = 1, 2$, and $i \neq j$. From definition we have $\tilde{m}Bdr_{ij}(F_B) = \tilde{m}_iCl(\tilde{m}_jCl(F_B)) \cap \tilde{m}_iCl(\tilde{m}_jCl(F_A - F_B))$.

Example 3.2 : Let $X = \{u_1, u_2\}$, $E = \{x_1, x_2, x_3\}$, $A = \{x_1, x_2\} \subseteq E$ and
$F_A = \{(x_1, \{u_1, u_2\}), (x_2, \{u_1, u_2\})\}$. Then

$F_{A_1} = \{(x_1, \{u_1\})\}$,
$F_{A_2} = \{(x_1, \{u_2\})\}$,
$F_{A_3} = \{(x_1, \{u_1, u_2\})\}$,
$F_{A_4} = \{(x_2, \{u_1\})\}$,
$F_{A_5} = \{(x_2, \{u_2\})\}$,
$F_{A_6} = \{(x_2, \{u_1, u_2\})\}$,
$F_{A_7} = \{(x_1, \{u_1\}), (x_2, \{u_1\})\}$,
$F_{A_8} = \{(x_1, \{u_1\}), (x_2, \{u_2\})\}$,

$$
\begin{aligned}
&F_{A_9} = \{(x_1,\{u_1\}),(x_2,\{u_1,u_2\})\},\\
&F_{A_{10}} = \{(x_1,\{u_2\}),(x_2,\{u_1\})\},\\
&F_{A_{11}} = \{(x_1,\{u_2\}),(x_2,\{u_2\})\},\\
&F_{A_{12}} = \{(x_1,\{u_2\}),(x_2,\{u_1,u_2\})\},\\
&F_{A_{13}} = \{(x_1,\{u_1,u_2\}),(x_2,\{u_1\})\},\\
&F_{A_{14}} = \{(x_1,\{u_1,u_2\}),(x_2,\{u_2\})\},\\
&F_{A_{15}} = F_A,\\
&F_{A_{16}} = F_\emptyset \text{ are all soft subsets of } F_A.
\end{aligned}
$$

$\tilde{m}_1 = \{F_\emptyset, F_A, F_{A_8}, F_{A_{10}}\}$ and $\tilde{m}_2 = \{F_\emptyset, F_A, F_{A_1}, F_{A_{12}}\}$.

Hence, $\tilde{m}Bdr_{12}(\{(x_1,\{u_1\})\}) = \{(x_1,\{u_1\}),(x_2,\{u_2\})\}$, $\tilde{m}Bdr_{21}(\{(x_1,\{u_1\})\}) = F_A$ and $\tilde{m}Bdr_{12}(\{(x_1,\{u_2\}),(x_2,\{u_1\})\}) = F_A = \tilde{m}Bdr_{21}(\{(x_1,\{u_2\}),(x_2,\{u_1\})\})$

Theorem 3.3 : Let $(F_A,\tilde{m}_1,\tilde{m}_2)$ be a soft biminimal space. Let F_B be a soft subset of F_A. Then $\tilde{m}Bdr_{ij}(F_B) = \tilde{m}Bdr_{ij}(F_A - F_B)$ where $i,j = 1,2$, and $i \neq j$

Proof :
$$
\begin{aligned}
\tilde{m}Bdr_{ij}(F_B) &= \tilde{m}_iCl(\tilde{m}_jCl(F_B)) \cap \tilde{m}_iCl(\tilde{m}_jCl(F_A - F_B))\\
&= \tilde{m}_iCl(\tilde{m}_jCl(F_A - (F_A - F_B)) \cap \tilde{m}_iCl(\tilde{m}_jCl(F_A - F_B)))\\
&= \tilde{m}Bdr_{ij}(F_A - F_B).
\end{aligned}
$$
□

Theorem 3.4 : Let $(F_A,\tilde{m}_1,\tilde{m}_2)$ be a soft biminimal space and F_B, F_C be a soft subset of F_A. Then for any $i,j = 1,2$, and $i \neq j$, Then the following are true.

i) $\tilde{m}Bdr_{ij}(F_B) = \tilde{m}_iCl(\tilde{m}_jCl(F_B)) \setminus \tilde{m}_iInt(\tilde{m}_jInt(F_B))$

ii) $\tilde{m}Bdr_{ij}(F_B) \cap \tilde{m}_iInt(\tilde{m}_jInt(F_B)) = F_\emptyset$

iii)$\tilde{m}Bdr_{ij}(F_B) \cap \tilde{m}_iInt(\tilde{m}_jInt(F_A - F_B)) = F_\emptyset$

iv) $\tilde{m}_iCl(\tilde{m}_jCl(F_B)) = \tilde{m}Bdr_{ij}(F_B) \cup \tilde{m}_iInt(\tilde{m}_jInt(F_B))$

v) $F_A = \tilde{m}_iInt(\tilde{m}_jInt(F_B)) \cup \tilde{m}Bdr_{ij}(F_B) \cup \tilde{m}_iInt(\tilde{m}_jInt(F_A - F_B))$ is a pairwise disjoint union

vi) $\tilde{m}_iCl(\tilde{m}_jCl(F_B)) = \tilde{m}Bdr_{ij}(F_B) \cup F_B$.

Proof :

(i)
$$
\begin{aligned}
\tilde{m}Bdr_{ij}(F_B) &= \tilde{m}_iCl(\tilde{m}_jCl(F_B)) \cap \tilde{m}_iCl(\tilde{m}_jCl(F_A - F_B))\\
&= \tilde{m}_iCl(\tilde{m}_jCl(F_B)) \cap (F_A - \tilde{m}_iInt(\tilde{m}_jInt(F_B)))\\
&= \tilde{m}_iCl(\tilde{m}_jCl(F_B)) \setminus \tilde{m}_iInt(\tilde{m}_jInt(F_B)).
\end{aligned}
$$

(ii) $\tilde{m}Bdr_{ij}(F_B) \cap \tilde{m}_iInt(\tilde{m}_jInt(F_B))$
$$
\begin{aligned}
&= [\tilde{m}_iCl(\tilde{m}_jCl(F_B)) \setminus \tilde{m}_iInt(\tilde{m}_jInt(F_B))] \cap \tilde{m}_iInt(\tilde{m}_jInt(F_B))\\
&= F_\emptyset
\end{aligned}
$$

iii) $\tilde{m}Bdr_{ij}(F_B) \cap \tilde{m}_iInt(\tilde{m}_jInt(F_A - F_B))$
$= \tilde{m}Bdr_{ij}(F_A - F_B)) \cap \tilde{m}_iInt(\tilde{m}_jInt(F_A - F_B))$
$= F_\emptyset$

iv) $\tilde{m}_iCl(\tilde{m}_jCl(F_B))$
$= [\tilde{m}_iCl(\tilde{m}_jCl(F_B)) \setminus \tilde{m}_iInt(\tilde{m}_jInt(F_B))] \cup \tilde{m}_iInt(\tilde{m}_jInt(F_B))$
$= \tilde{m}Bdr_{ij}(F_B) \cup \tilde{m}_iInt(\tilde{m}_jInt(F_B))$

v) $\tilde{m}_iInt(\tilde{m}_jInt(F_B)) \cup \tilde{m}Bdr_{ij}(F_B) \cup \tilde{m}_iInt(\tilde{m}_jInt(F_A - F_B))$
$= \tilde{m}_iCl(\tilde{m}_jCl(F_B)) \cup [F_A - \tilde{m}_iCl(\tilde{m}_jCl(F_B))]$
$= F_A$.

By (ii) and (iii) $\tilde{m}Bdr_{ij}(F_B) \cap \tilde{m}_iInt(\tilde{m}_jInt(F_B)) = F_\emptyset$ and
$\tilde{m}Bdr_{ij}(F_B) \cap \tilde{m}_iInt(\tilde{m}_jInt(F_A - F_B)) = F_\emptyset$
Now, $\tilde{m}_iInt(\tilde{m}_jInt(F_B)) \cap \tilde{m}_iInt(\tilde{m}_jInt(F_A - F_B)) \subseteq F_B \cap (F_A - F_B)$
$= F_\emptyset$

Therefore $F_A = \tilde{m}_iInt(\tilde{m}_jInt(F_B)) \cup \tilde{m}Bdr_{ij}(F_B) \cup \tilde{m}_iInt(\tilde{m}_jInt(F_A - F_B))$ is a pairwise disjoint union.

vi) $\tilde{m}Bdr_{ij}(F_B) \cup F_B = [\tilde{m}_iCl(\tilde{m}_jCl(F_B)) \cap \tilde{m}_iCl(\tilde{m}_jCl(F_A - F_B))] \cup F_B$
$= [\tilde{m}_iCl(\tilde{m}_jCl(F_B)) \cup F_B] \cap [\tilde{m}_iCl(\tilde{m}_jCl(F_A - F_B)) \cup F_B]$
$= \tilde{m}_iCl(\tilde{m}_jCl(F_B)) \cap ([F_A - \tilde{m}_iInt(\tilde{m}_jInt(F_B))] \cup F_B)$
$= \tilde{m}_iCl(\tilde{m}_jCl(F_B)) \cap F_A$
$= \tilde{m}_iCl(\tilde{m}_jCl(F_B))$. □

Theorem 3.5 : Let $(F_A, \tilde{m}_1, \tilde{m}_2)$ be a SBMS and F_B be a soft subset of F_A. Then for any $i, j = 1, 2$, and $i \neq j$. Then
i) F_B is $\tilde{m}_i\tilde{m}_j$- soft closed if and only if $\tilde{m}Bdr_{ij}(F_B) \tilde{\subseteq} F_B$.
ii) F_B is $\tilde{m}_i\tilde{m}_j$- soft open if and only if $\tilde{m}Bdr_{ij}(F_B) \tilde{\subseteq} (F_A - F_B)$.

Proof :
i) Assume that F_B is $\tilde{m}_i\tilde{m}_j$- soft closed,

Thus $\tilde{m}_iCl(\tilde{m}_jCl(F_B)) = F_B$.

Let $x \in \tilde{m}Bdr_{ij}(F_B)$.

Then, $x \in \tilde{m}_iCl(\tilde{m}_jCl(F_B)) \cap \tilde{m}_iCl(\tilde{m}_jCl(F_A - F_B))$

$\Longrightarrow x \in F_B \cap (\tilde{m}_iCl(\tilde{m}_jCl(F_A - F_B))$

$\Longrightarrow x \in F_B$

Therefore, $\tilde{m}Bdr_{ij}(F_B)\tilde{\subseteq}F_B$.

Conversely, Let $\tilde{m}Bdr_{ij}(F_B)\tilde{\subseteq}F_B$

Then $\tilde{m}Bdr_{ij}(F_B) \cap (F_A - F_B) = F_\emptyset$

Now, $(\tilde{m}_iCl(\tilde{m}_jCl(F_B)) \cap (F_A - F_B)$

$$= (\tilde{m}_iCl(\tilde{m}_jCl(F_B)) \cap [\tilde{m}_iCl(\tilde{m}_jCl(F_A - F_B)) \cap (F_A - F_B)]$$
$$= [\tilde{m}_iCl(\tilde{m}_jCl(F_B)) \cap \tilde{m}_iCl(\tilde{m}_jCl(F_A - F_B)] \cap (F_A - F_B)$$
$$= \tilde{m}Bdr_{ij}(F_B) \cap (F_A - F_B)$$
$$= F_\emptyset$$

Therefore $\tilde{m}_iCl(\tilde{m}_jCl(F_B))\tilde{\subseteq}F_B$.

But $F_B\tilde{\subseteq}\tilde{m}_iCl(\tilde{m}_jCl(F_B))$.

Therefore $F_B = \tilde{m}_iCl(\tilde{m}_jCl(F_B))$.

Hence F_B is $\tilde{m}_i\tilde{m}_j$- soft closed.

ii) Assume that F_B is $\tilde{m}_i\tilde{m}_j$- soft open.

Thus $\tilde{m}_iInt(\tilde{m}_jInt(F_B)) = F_B$.

Now, $\tilde{m}Bdr_{ij}(F_B) \cap F_B = [\tilde{m}_iCl(\tilde{m}_jCl(F_B)) \setminus \tilde{m}_iInt(\tilde{m}_jInt(F_B))] \cap F_B$

$$= [\tilde{m}_iCl(\tilde{m}_jCl(F_B)) \setminus F_B] \cap F_B$$
$$= F_\emptyset$$

Hence, $\tilde{m}Bdr_{ij}(F_B)\tilde{\subseteq}(F_A - F_B)$.

Conversely, Let $\tilde{m}Bdr_{ij}(F_B)\tilde{\subseteq}(F_A - F_B)$.

Thus $\tilde{m}Bdr_{ij}(F_B) \cap F_B = F_\emptyset$.

That implies, $[\tilde{m}_iCl(\tilde{m}_jCl(F_B)) \setminus \tilde{m}_iInt(\tilde{m}_jInt(F_B))] \cap F_B = F_\emptyset$.

Since $F_B\tilde{\subseteq}\tilde{m}_iCl(\tilde{m}_jCl(F_B))$, we have $(F_B \setminus \tilde{m}_iInt(\tilde{m}_jInt(F_B))) \cap F_B = F_\emptyset$.

But $\tilde{m}_iInt(\tilde{m}_jInt(F_B))\tilde{\subseteq}F_B$.

Hence, $F_B = \tilde{m}_iInt(\tilde{m}_jInt(F_B))$.

F_B is $\tilde{m}_i\tilde{m}_j$- soft open. □

Theorem 3.6 : Let $(F_A, \tilde{m}_1, \tilde{m}_2)$ be a SBMS and F_B be a soft subsets of F_A. Then

$\tilde{m}Bdr_{ij}(F_B) = F_\emptyset$ if and only if F_B is $\tilde{m}_i\tilde{m}_j$- soft closed and $\tilde{m}_i\tilde{m}_j$- soft open where $i, j = 1, 2$, and $i \neq j$

Proof : Let $\tilde{m}Bdr_{ij}(F_B) = F_\emptyset$.

Thus by Theorem 3.5, we have F_B is $\tilde{m}_i\tilde{m}_j$- soft closed and $\tilde{m}_i\tilde{m}_j$- soft open.

Conversely, let F_B is $\tilde{m}_i\tilde{m}_j$- soft closed and $\tilde{m}_i\tilde{m}_j$- soft open .

Then $\tilde{m}_iCl(\tilde{m}_jCl(F_B)) = F_B$ and

$\tilde{m}_iInt(\tilde{m}_jInt(F_A - F_B)) = F_A - F_B$

$$\begin{aligned}\tilde{m}Bdr_{ij}(F_B) &= \tilde{m}_iCl(\tilde{m}_jCl(F_B)) \cap \tilde{m}_iCl(\tilde{m}_jCl(F_A - F_B)).\\ &= F_B \cap (F_A - F_B)\\ &= F_\emptyset.\end{aligned}$$

□

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