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ROLE OF OCTAGONAL FUZZY NUMBERS IN THREE STAGE FLOW SHOP SCHEDULING PROBLEM WITH SETUP TIME AND TRANSPORTATION TIME

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Abstract

The concept of octagonal fuzzy numbers was introduced in 2013 by one of the authors [7]. In this paper, we consider three-machine, n-job flow shop scheduling problem in which the uncertainties involved in processing time, setup time and transportation time are studied using octagonal fuzzy numbers. Our objective is to find an optimum job sequence with constraints of job-block which minimizes the total rental cost of machines. A new algorithm is proposed to solve the problem in which the operational complexities due to fuzzy environment are dealt with.

1. Introduction

A flowshop scheduling problem is one of the classical problems in production scheduling. Ever since the first results of modern scheduling theory appeared some 50 years ago, scheduling has attracted a lot of attention from both academia and industry. Johnson (1954) [1] proposed the well known Johnson's algorithm in the two stage flow shop makespan scheduling problem. Maggu and Das(1977) [2] gave an algorithm to solve job block in job scheduling. Gupta and Sharma(2011) [3] studied bicriteria in nA2 flow shop scheduling under specified rental policy in which processing time and setup time are associated with probabilities. Fuzzy numbers play an important role in handling uncertainities. Several authors have used different fuzzy numbers to solve scheduling problems. To name a few, Machon and Lee(1990) [4] used mean values to solve fuzzy processing times. Deepak Gupta et al.(2012) [5] studied two stage flowshop scheduling problem with fuzzy processing and setup times.

This paper is arranged as follows: Section 2 recalls the concept of octagonal fuzzy numbers and its operations. Section 3 describes the three stage flow shop scheduling problem. In section 4, we record an algorithm in which the operational complexities due to fuzzy environment are dealt with. In section 5, we have solved a numerical example using the proposed algorithm. Section 6 concludes the paper with a comparitive study.

2. Basic Definitions

For the sake of completeness we recall the required definitions and results from [7] where in the concept of octagonal fuzzy numbers was introduced. We consider a subclass of generalized octagonal fuzzy numbers which are linear defined as follows:

Definition 2.1 : A fuzzy number $\tilde{A}$ is said to be a linear octagonal fuzzy number or simply an octagonal fuzzy number whose membership function $\mu_{\tilde{A}}$ is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} k(\dfrac{x-a_1}{a_2-a_1}) & a_1 \le x \le a_2 \\ k & a_2 \le x \le a_3 \\ k+(1-k)(\dfrac{x-a_3}{a_4-a_3}) & a_3 \le x \le a_4 \\ \omega & a_4 \le x \le a_5 \\ k+(1-k)(\dfrac{a_6-x}{a_6-a_5}) & a_5 \le x \le a_6 \\ k & a_6 \le x \le a_7 \\ k(\dfrac{a_8-x}{a_8-a_7}) & a_7 \le x \le a_8 \\ 0 & \text{otherwise} \end{cases}$$

and represented $(a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8; k, \omega)$ where $a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, k, \omega$ are real numbers such that $a_1 \le a_2 \le a_3 \le a_4 \le a_5 \le a_6 \le a_7 \le a_8$ and $0 \le k \le \omega \le 1$.

Average High Ranking(AHR)

We use the measure of octagonal fuzzy number as AHR

$$M^{Oct}(\tilde{A}) = \frac{1}{4}[(a_1 + a_2 + a_7 + a_8)k + (a_3 + a_4 + a_5 + a_6)(1-k)] \ \ where \ \ 0 < k < 1 \quad (2.1)$$

Arithmetic Operations on Octagonal Fuzzy Numbers

If $\tilde{A} \approx (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8; k, \omega)$ and
$\tilde{B} \approx (b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8; k, \omega)$ are two octagonal fuzzy numbers, then their addition, multiplication and division respectively are defined by

$$\tilde{A} + \tilde{B} \approx (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4, a_5 + b_5, a_6 + b_6, a_7 + b_7, a_8 + b_8; k, \omega) \quad (2.2)$$

When $\tilde{A}$ and $\tilde{B}$ are two positive octagonal fuzzy numbers, then

$$\tilde{A} \otimes \tilde{B} \approx (a_1b_1, a_2b_2, a_3b_3, a_4b_4, a_5b_5, a_6b_6, a_7b_7, a_8b_8; k, \omega) \quad (2.3)$$

$$\frac{\tilde{A}}{\tilde{B}} \approx (\frac{a_1}{b_8}, \frac{a_2}{b_7}, \frac{a_3}{b_6}, \frac{a_4}{b_5}, \frac{a_5}{b_4}, \frac{a_6}{b_3}, \frac{a_7}{b_2}, \frac{a_8}{b_1}; k, \omega) \quad (2.4)$$

Ahmad Soltani et al. [8] introduced a new subtraction procedure for trapezoidal fuzzy numbers, we along lines introduce subtraction for any two octagonal fuzzy numbers. Call $\tilde{C} = \tilde{A} \ominus \tilde{B}$ where $\tilde{C} = (c_1, c_2, ..., c_8; k, \omega)$ given by

$$\begin{aligned}
c_8 &= \max(0, a_8 - b_8) \\
c_7 &= \max(0, \min(c_8, (a_7 - b_7))) \\
c_6 &= \max(0, \min(c_7, (a_6 - b_6))) \\
c_5 &= \max(0, \min(c_6, (a_5 - b_5))) \\
c_4 &= \max(0, \min(c_5, (a_4 - b_4))) \\
c_3 &= \max(0, \min(c_4, (a_3 - b_3))) \\
c_2 &= \max(0, \min(c_3, (a_2 - b_2))) \\
c_1 &= \max(0, \min(c_2, (a_1 - b_1)))
\end{aligned}$$

3. Three Stage Flow Shop Scheduling Problem with Setup Time and Transportation Time

Assumptions

1. The jobs to be processed are independent of each other

2. Pre-emption of job is not allowed
3. All the jobs and machines are available at the beginning of the processing
4. Each job is processed through each of the machine once and only once
5. A job is not available to the next machine until and unless processing on the current machine is completed
6. Machines never breakdown and are available throughout the scheduling process

Rental Situation(S)

The machines will be taken on rent as and when they are required. The first machine will be taken on rent in the beginning of the processing of the jobs, second machine will be taken on rent at a time when the first job is completed on the first machine and transported to the second machine. Third machine willl be taken on rent at a time when the first job is completed on the second machine and transported to the third machine and so on.

Problem Formulation

A three stage fuzzy flow shop problem with jobs (i=1,2,...,n) having fuzzy processing time p_{ij} and setup time q_{ij} on machines(j=1,2,3) under particular rental situation S can mathematically be formulated as,

Minimize $O_j(S_p)$ or Minimize $R_c(S_q) = \sum O_j(S_q) \times r_j$,

subject to the constraint:Rental Situation(S)

Let $T_{i,1\to2}, T_{i,2\to3}$ be the fuzzy time taken in transporting job i. Our objective is to find a sequence S_q of jobs which minimizes the rental cost of the machines(j=1,2,3).

	Machine 1	**Machine 1**		**Machine 2**	**Machine 2**		**Machine 3**	**Machine 3**
$\boldsymbol{i}$	$\boldsymbol{p_{i1}}$	$\boldsymbol{q_{i1}}$	$\boldsymbol{T_{i1\to2}}$	$\boldsymbol{p_{i2}}$	$\boldsymbol{q_{i2}}$	$\boldsymbol{T_{i2\to3}}$	$\boldsymbol{p_{i3}}$	$\boldsymbol{q_{i3}}$
1	p_{11}	q_{11}	$T_{11\to2}$	p_{12}	q_{12}	$T_{12\to3}$	p_{13}	q_{13}
2	p_{21}	q_{21}	$T_{21\to2}$	p_{22}	q_{22}	$T_{22\to3}$	p_{23}	q_{23}
3	p_{31}	q_{31}	$T_{31\to2}$	p_{32}	q_{32}	$T_{32\to3}$	p_{33}	q_{33}
-	-	-	-	-	-	-	-	-
m	p_{m1}	q_{m1}	$T_{m1\to2}$	p_{m2}	q_{m2}	$T_{m2\to3}$	p_{m3}	q_{m3}
-	-	-	-	-	-	-	-	-
n	p_{n1}	q_{n1}	$T_{n1\to2}$	p_{n2}	q_{n2}	$T_{n2\to3}$	p_{n3}	q_{n3}

4. Algorithm

Step 1: Using equation (2.1) find Average High Ranking(AHR) for the processing time, setup time and transportation time and denote them as $p'_{i1},\ \ q'_{i1},\ \ T'_{i1\to 2},\ \ p'_{i2},\ \ q'_{i2},$ $T'_{i2\to 3},\ \ p'_{i3},\ \ q'_{i3}$

Step 2: Using the formulae find

$$p''_{i1} = p'_{i1} - q'_{i3},\ \ p''_{i2} = p'_{i2} - q'_{i1},\ \ p''_{i3} = p'_{i3} - q'_{i2}$$

(or)

$$p''_{i1} = p'_{i1} - q'_{i2},\ \ p''_{i2} = p'_{i2} - q'_{i3},\ \ p''_{i3} = p'_{i3} - q'_{i1}$$

Step 3: Deploy two fictitious machines G and H with processing times as G_{i1} and H_{i2} for job i, as:
$G_{i1} = p''_{i1} + T'_{i,1\to 2} + p''_{i2} + T'_{i,2\to 3}$
$H_{i2} = p''_{i2} + T'_{i,1\to 2} + p''_{i3} + T'_{i,2\to 3}$

Step 4: For given equivalent job block $\beta(k,m)$ calculate $G_{\beta 1}$ and $H_{\beta 2}$ using the criteria of Maggu and Das [2] as:
$G_{\beta 1} = G_{k2} + G_{m2} - min(G_{m1}, H_{k2})$ and
$H_{\beta 2} = H_{k2} + H_{m2} - min(G_{m1}, H_{k2})$

Step 5: Define a new reduced problem having two fictitious machines G and H with the processing times G_{i1} and H_{i2} as mentioned in Step 3 and jobs(k,m) as job-block is replaced by single equivalent job β with processing time $G_{\beta 1}$ and $H_{\beta 2}$ as mentioned in Step 4.

Step 6: Obtain the sequence S_1 by applying Johnson's technique [1] on machines G and H.

Step 7: Obtain other sequences by putting $2^{nd}, 3^{rd}, ..., n^{th}$ jobs of the sequence S_1 in the 1^{st} position and all other jobs of S_1 in the same order. Let these sequences be $S_2, S_3, S_4, ..., S_r$.

Step 8: Prepare flow table only for those sequences $S_p(p = 1, 2, ..., r)$ which have job block $\beta(k, m)$ and find the completion time $ct_{xj}(S_p)$ of last job on machines 1,2 and 3

Step 9: Evaluate starting time $st_{yj}(S_p)$ of first job on machines 1,2 and 3 for each sequence in Step 8

Step 10: Compute Operation time $O_p(S_p)$ and Rental cost $R_c(S_q)$ of machines 1,2 and 3 for each of the sequences S_p obtained in Step 8
$O_j(S_p) = ct_{xj}(S_p) \ominus st_{yj}(S_p)$ and $R_c(S_q) = \sum O_j(S_q) \times r_j$ for p=1,2,...,r

Step 11: Find $min(R_c(S_q))$, p=1,2,...,r. Let it corresponds to $p = q$, then S_q is the optimal sequence for minimum rental cost.

5. Numerical Example

To illustrate the developed algorithm we consider the following example, where we use octagonal fuzzy numbers which are normal(i.e.,for $\omega = 1$)to denote the durations. Repeated computations for various values of k, exhibited best results only for $k < 0.5$. To cite the same we choose $k = 0.3$.

A job shop owner has one straddle milling machine, one drilling machine and one special purpose finishing machine. He has five jobs in hand each of which needs milling first, drilling next and finally finishing. The jobs(2,3) are to be processed in priority as equivalent group job. The rental costs per unit time for machines 1,2 and 3 are 10,8,5 units respectively. The processing time, set up time and transportation times are given below. Determine optimum sequence for the jobs using which will complete them in shortest possible time.

Jobs	p_{i1}	q_{i1}
1	(8,8.333,8.667,9,10,10.667,11.333,12;0.3,1)	(1,1.333,1.667,2,3,3.667,4.333,5;0.3,1)
2	(11,11.667,12.333,13,14,15.333,16.667,18;0.3,1)	(4,4.667,5.333,6,7,8,9,10;0.3,1)
3	(13,13.333,13.667,14,15,15.667,16.333,17;0.3,1)	(3,3.333,3.667,4,5,5.667,6.333,7;0.3,1)
4	(10,10.333,10.667,11,12,12.667,13.333,14;0.3,1)	(5,5.667,6.333,7,9,10,11,12;0.3,1)
5	(15,15.333,15.667,16,18,18.667,19.333,20;0.3,1)	(6,6.667,7.333,8,10,11.333,12.667,14;0.3,1)

Jobs	$T_{i1\to 2}$	p_{i2}
1	(3,3.333,3.667,4,5,5.667,6.333,7;0.3,1)	(7,7.333,7.667,8,9,9.667,10.333,11;0.3,1)
2	(4,4.333,4.667,5,6,6.667,7.333,8;0.3,1)	(9,9.667,10.333,11,13,14,15,16;0.3,1)
3	(2,2.333,2.667,3,4,4.667,5.333,6;0.3,1)	(10,10.333,10.667,11,12,12.667,13.333,14;0.3,1)
4	(3,3.333,3.667,4,5,5.667,6.333,7;0.3,1)	(9,9.333,9.667,10,11,11.667,12.333,13;0.3,1)
5	(5,6,7,8,11,12.333,13.667,15;0.3,1)	(14,14.333,14.667,15,16,16.667,17.333,18;0.3,1)

Jobs	q_{i2}	$T_{i2\to 3}$
1	(1,1.333,1.667,2,3,3.667,4.333,5;0.3,1)	(2,2.333,2.667,3,4,4.667,5.333,6;0.3,1)
2	(3,3.333,3.667,4,5,5.667,6.333,7;0.3,1)	(3,3.333,3.667,4,5,5.667,6.333,7;0.3,1)
3	(4,4.333,4.667,5,6,6.667,7.333,8;0.3,1)	(1,1.333,1.667,2,3,3.667,4.333,5;0.3,1)
4	(2,2.333,2.667,3,4,4.667,5.333,6;0.3,1)	(2,2.333,2.667,3,4,4.667,5.333,6;0.3,1)
5	(5,5.333,5.667,6,7,7.667,8.333,9;0.3,1)	(4,4.667,5.333,6,9,10.333,11.667,13;0.3,1)

Jobs	p_{i3}	q_{i3}
1	(9,9.333,9.667,10,11,11.667,12.333,13;0.3,1)	(1,1.333,1.667,2,3,3.667,4.333,5;0.3,1)
2	(10,10.333,10.667,11,12,12.667,13.333,14;0.3,1)	(5,5.667,6.333,7,9,10,11,12;0.3,1)
3	(12,12.333,12.667,13,14,14.667,15.333,16;0.3,1)	(3,3.333,3.667,4,5,5.667,6.333,7;0.3,1)
4	(5,5.667,6.333,7,9,10.333,11.667,13;0.3,1)	(4,4.667,5.333,6,7,8,9,10;0.3,1)
5	(13,13.333,13.667,14,15,15.667,16.333,17;0.3,1)	(2,2.333,2.667,3,4,4.667,5.333,6;0.3,1)

The above problem can be solved by the proposed algorithm following Steps 1 to 4

i	G_{i1}	H_{i2}
1	21.366	22.366
2	21.55	22.866
3	23.366	21.366
4	15.866	15.55
5	37.366	31.866

As per Step 5, the processing times of equivalent job block $\beta(2,3)$ using Maggu and Das criteria [2] is given by

$G_{\beta 1} = G_{22} + G_{32} - min(G_{31}, H_{32}) = 21.55 + 23.366 - 23.666 = 22.05,$

$H_{\beta 2} = H_{22} + H_{32} - min(G_{31}, H_{22}) = 22.866 + 21.366 - 23.666 = 21.366$

i	G_{i1}	H_{i2}
1	21.366	22.366
β	22.05	21.366
4	15.866	15.55
5	37.366	31.866

Using Johnson's algorithm the optimal sequence is $S_1 = 1-5-\beta-4 = 1-5-2-3-4$
By Step 8, the other optimal sequences are $S_2 = 5-1-2-3-4, S_3 = 2-1-5-3-4, S_4 = 3-1-5-2-4, S_5 = 4-1-5-2-3$
The flow tables for sequences having job block(2,3) are S_1, S_2, S_5 and the computation results are tabulated as follows:

For Sequence $\mathbf{S_1}$

Jobs	Machine 1 (FLOW IN-FLOW OUT)
1	(0,0,0,0,0,0,0,0;0.3,1)-(8,8.333,8.667,9,10,10.667,11.333,12;0.3,1)
5	(9,9.666,10.334,11,13,14.334,15.666,17;0.3,1) -(24,24.999,26.001,27,31,33.001,34.999,37;0.3,1)
2	(30,31.666,33.334,35,41,44.334,47.666,51;0.3,1)-(41,43.333,45.667,48,55,59.667,64.333,69;0.3,1)
3	(45,48,51,54,62,67.667,73.333,79;0.3,1)-(58,61.333,64.667,68,77,83.334,89.666,96;0.3,1)
4	(61,64.666,68.334,72,82,89.001,95.999,103;0.3,1)-(71,74.999,79.001,83,94,101.668,109.332,117;0.3,1)

Jobs	Machine 2 (FLOW IN-FLOW OUT)
1	(11,11.666,12.334,13,15,16.334,17.666,19;0.3,1)-(18,18.999,20.001,21,24,26.001,27.999,30;0.3,1)
5	(29,30.999,33.001,35,42,45.334,48.666,52;0.3,1)-(43,45.332,47.668,50,58,62.001,65.999,70;0.3,1)
2	(48,50.665,53.335,56,65,69.668,74.332,79;0.3,1)-(57,60.332,63.668,67,78,83.668,89.332,95;0.3,1)
3	(60,63.665,67.335,71,83,88.335,95.665,102;0.3,1)-(70,73.998,78.002,82,95,102.002,108.998,116;0.3,1)
4	(74,78.331,82.669,87,101,108.669,116.331,124;0.3,1)-(83,87.664,92.336,97,112,120.336,128.664,137;0.3,1)

Jobs	Machine 3 (FLOW IN-FLOW OUT)
1	(20,21.332,22.668,24,28,30.668,33.332,36;0.3,1)-(29,30.665,32.335,34,39,42.335,45.665,49;0.3,1)
5	(47,49.999,53.001,56,67,72.334,77.666,83;0.3,1)-(60,63.332,66.668,70,82,88.001,93.999,100;0.3,1)
2	(62,65.665,69.335,73,86,92.668,99.332,106;0.3,1)-(72,75.998,80.002,84,98,105.335,112.665,120;0.3,1)
3	(77,81.665,86.335,91,107,115.335,123.665,132;0.3,1)-(89,93.998,99.002,104,121,130.002,138.998,148;0.3,1)
4	(92,97.331,102.669,108,126,135.669,145.331,155;0.3,1)-(97,102.998,109.002,115,135,146.002,156.998,168;0.3,1)

$ct_{41}(S_1)$= (71,74.999,79.001,83,94,101.668,109.332,117;0.3,1)
$ct_{42}(S_1)$= (83,87.664,92.336,97,112,120.336,128.664,137;0.3,1)
$ct_{43}(S_1)$= (97,102.998,109.002,115,135,146.002,156.998,168;0.3,1)
$st_{11}(S_1)$=(0,0,0,0,0,0,0,0;0.3,1)
$st_{12}(S_1)$= (11,11.666,12.334,13,15,16.334,17.666,19;0.3,1)
$st_{13}(S_1)$= (20,21.332,22.668,24,28,30.668,33.332,36;0.3,1)

$O_1(S_1)$= (71,74.999,79.001,83,94,101.668,109.332,117;0.3,1)

$O_2(S_1)$= (72,75.998,80.002,84,97,104.002,110.998,118;0.3,1)

$O_3(S_1)$ = (77,81.666,86.334,91,107,115.334,123.666,132;0.3,1)

$R_c(S_1) = (O_1(S_1) \times r_1) + (O_2(S_1) \times r_2) + (O_3(S_1) \times r_3)$

= (1671,1766,1862,1957,2251,2426,2599,2774;0.3,1)with defuzzified value as 2148 units.

For Sequence $\mathbf{S_2}$

Jobs	Machine 1 (FLOW IN-FLOW OUT)
5	(0,0,0,0,0,0,0,0;0.3,1)-(15,15.333,15.667,16,18,18.667,19.333,20;0.3,1)
1	(21,22,23,24,28,30,32,34;0.3,1)-(29,30.333,31.667,33,38,40.667,43.333,46;0.3,1)
2	(30,31.666,33.334,35,41,44.334,47.666,51;0.3,1)-(41,43.333,45.667,48,55,59.667,64.333,69;0.3,1)
3	(45,48,51,54,62,67.667,73.333,79;0.3,1)-(58,61.333,64.667,68,77,83.334,89.666,96;0.3,1)
4	(61,64.666,68.334,72,82,89.001,95.999,103;0.3,1)-(71,74.999,79.001,83,94,101.668,109.332,117;0.3,1)

Jobs	Machine 2 (FLOW IN-FLOW OUT)
5	(20,21.333,22.667,24,29,31,33,35;0.3,1)-(34,35.666,37.334,39,45,47.667,50.333,53;0.3,1)
1	(39,40.999,43.001,45,52,55.334,58.666,62;0.3,1)-(46,48.332,50.668,53,61,65.001,68.999,73;0.3,1)
2	(47,49.665,52.335,55,64,68.668,73.332,78;0.3,1)-(56,59.332,62.668,66,77,82.668,88.332,94;0.3,1)
3	(60,63.666,67.334,71,81,88.001,94.999,102;0.3,1)-(70,73.999,78.001,82,93,100.668,108.332,116;0.3,1)
4	(74,78.332,82.668,87,99,107.335,115.665,124;0.3,1)-(83,87.665,92.335,97,110,119.002,127.998,137;0.3,1)

Jobs	Machine 3 (FLOW IN-FLOW OUT)
5	(38,40.333,42.667,45,54,58,62,66;0.3,1)-(51,53.666,56.334,59,69,73.667,78.333,83;0.3,1)
1	(53,55.999,59.001,62,73,78.334,83.666,89;0.3,1)-(62,65.332,68.668,72,84,90.001,95.999,102;0.3,1)
2	(63,66.665,70.335,74,87,93.668,100.332,107;0.3,1)-(73,76.998,81.002,85,99,106.335,113.665,121;0.3,1)
3	(78,82.665,87.335,92,108,116.335,124.665,133;0.3,1)-(90,94.998,100.002,105,122,131.002,139.998,149;0.3,1)
4	(93,98.331,103.669,109,127,136.669,146.331,156;0.3,1)-(98,103.998,110.002,116,136,147.002,157.998,169;0.3,1)

$ct_{41}(S_2)$= (71,74.999,79.001,83,94,101.668,109.332,117;0.3,1)

$ct_{42}(S_2)$= (83,87.665,92.335,97,110,119.002,127.998,137;0.3,1)

$ct_{43}(S_2)$= (98,103.998,110.002,116,136,147.002,157.998,169;0.3,1)

$st_{51}(S_2)$=(0,0,0,0,0,0,0,0;0.3,1)

$st_{52}(S_2)$= (20,21.333,22.667,24,29,31,33,35;0.3,1)

$st_{53}(S_2)$= (38,40.333,42.667,45,54,58,62,66;0.3,1)

$O_1(S_2)$= (71,74.999,79.001,83,94,101.668,109.332,117;0.3,1)

$O_2(S_2)$= (63,66.332,69.668,73,81,88.002,94.998,102;0.3,1)

$O_3(S_2)$= (60,63.665,67.335,71,82,89.002,95.998,103;0.3,1)

$R_c(S_2) = (O_1(S_2) \times r_1) + (O_2(S_2) \times r_2) + (O_3(S_2) \times r_3)$

= (1514,1599,1684,1769,1998,2166,2333,2501;0.3,1)with defuzzified value as 1929 units.

For Sequence S_5

Jobs	Machine 1 (FLOW IN-FLOW OUT)
4	(0,0,0,0,0,0,0,0;0.3,1)-(10,10.333,10.667,11,12,12.667,13.333,14;0.3,1)
1	(15,16,17,18,21,22.667,24.333,26;0.3,1)-(23,24.333,25.667,27,31,33.334,35.666,38;0.3,1)
5	(24,25.666,27.334,29,34,37.001,39.999,43;0.3,1)-(39,40.999,43.001,45,52,55.668,59.332,63;0.3,1)
2	(45,47.666,50.334,53,62,67.001,71.999,77;0.3,1)-(56,59.333,62.667,66,76,82.334,88.666,95;0.3,1)
3	(60,64,68,72,83,90.334,97.666,105;0.3,1)-(73,77.333,81.667,86,98,106.001,113.999,122;0.3,1)

Jobs	Machine 2 (FLOW IN-FLOW OUT)
4	(13,13.666,14.334,15,17,18.334,19.666,21;0.3,1)-(22,22.999,24.001,25,28,30.001,31.999,34;0.3,1)
1	(26,27.666,29.334,31,36,39.001,41.999,45;0.3,1)-(33,34.999,37.001,39,45,48.668,52.332,56;0.3,1)
5	(44,46.999,50.001,53,63,68.001,72.999,78;0.3,1)-(58,61.332,64.668,68,79,84.668,90.332,96;0.3,1)
2	(63,66.665,70.335,74,86,92.335,98.665,105;0.3,1)-(72,76.332,80.668,85,99,106.335,113.665,121;0.3,1)
3	(75,79.665,84.335,89,104,112.002,119.998,128;0.3,1)-(85,89.998,95.002,100,116,124.669,133.331,142;0.3,1)

Jobs	Machine 3 (FLOW IN-FLOW OUT)
4	(24,25.332,26.668,28,32,34.668,37.332,40;0.3,1)-(29,30.999,33.001,35,41,45.001,48.999,53;0.3,1)
1	(35,37.332,39.668,42,49,53.335,57.665,62;0.3,1)-(44,46.665,49.335,52,60,65.002,69.998,75;0.3,1)
5	(62,65.999,70.001,74,88,95.001,101.999,109;0.3,1)-(75,79.332,83.668,88,103,110.668,118.332,126;0.3,1)
2	(77,81.665,86.335,91,107,115.335,123.665,132;0.3,1)-(87,91.998,97.002,102,119,128.002,136.998,146;0.3,1)
3	(92,97.665,103.335,109,128,138.002,147.998,158;0.3,1)-(104,109.998,116.002,122,142,152.669,163.331,174;0.3,1)

$ct_{31}(S_5)$= (73,77.333,81.667,86,98,106.001,113.999,122;0.3,1)

$ct_{32}(S_5)$= (85,89.998,95.002,100,116,124.669,133.331,142;0.3,1)

$ct_{33}(S_5)$= (104,109.998,116.002,122,142,152.669,163.331,174;0.3,1)

$st_{41}(S_5)$=(0,0,0,0,0,0,0,0;0.3,1)

$st_{42}(S_5)$= (13,13.666,14.334,15,17,18.334,19.666,21;0.3,1)

$st_{43}(S_5)$= (24,25.332,26.668,28,32,34.668,37.332,40;0.3,1)

$O_1(S_5)$ = (73,77.333,81.667,86,98,106.001,113.999,122;0.3,1)

$O_2(S_5)$ = (72,76.332,80.668,85,99,106.335,113.665,121;0.3,1)

$O_3(S_5)$ = (80,84.666,89.334,94,110,118.001,125.999,134;0.3,1)

$R_c(S_5) = (O_1(S_1) \times r_1) + (O_2(S_1) \times r_2) + (O_3(S_1) \times r_3)$

= (1706,1807,1909,2010,2322,2501,2679,2358;0.3,1)with defuzzified value as 2209 units.

Therefore $Min\{R(S_p)\}$=(1514,1599,1684,1769,1998,2166,2333,2501;0.3,1) units and is for sequence of jobs S_2.

Hence the sequence S_2:5-1-2-3-4 is the optimal sequence with minimum rental cost (1514,1599,1684,1769,1998,2166,2333,2501;0.3,1)units.

6. Conclusion

The operation time and rental cost for three stage flow shop scheduling problem when solved using octagonal fuzzy numbers for $k < 0.5$ is much lower, when solved using trapezoidal fuzzy numbers. When continued into dodecagonal fuzzy numbers [9] with the condition that both $k_1, k_2 < 0.5$, the project cost is slightly reduced than the octagonal fuzzy numbers but the difference between them is very feeble, wherein the computation is tedious. Therefore solving the problem using octagonal fuzzy numbers is sufficient to yield best real time solution.

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