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SOME SEQUENCE SPACES AND MATRIX TRANSFORMATIONS WITH VEDIC RELATIONS

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Abstract

The most general linear operator transforming one sequence space into another sequence space is actually given by an infinite matrix. The purpose of the paper is to establish some results in sequence spaces with matrix transformation as vedic relation.

1. Introduction

The idea of the sequence spaces was motivated through the classical results of summability theory which were first introduced by Cesaro, Borel, Norlund, Riesz and others. The first attempt to study summability, the most general linear operator transforming one sequence space into another sequence space is actually given by an infinite matrix. Therefore, the matrix transformations as methods in abstract sense were introduced by

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the celebrated mathematician Toeplitz in 1911, be used to characterize matrix transformations and gave the necessary and sufficient conditions for an infinite matrix to be regular. Regular in the sense that it preserves the limit of the convergent sequences [1]. We find some classical sequences in Mishra et.al. [5], Ray [6] and Baral [7].

Definition 1.1 : A *sequence* is a function with domain set as the set of natural numbers and range set as the set of real numbers.

Definition 1.2 : A *sequence space* is a function space whose elements are functions from the set of natural numbers to the field K of real or complex numbers.

Definition 1.3 : By ω, we denote the space of all complex valued sequences. Any vector subspace of ω is called a sequence space. We write 1_∞, c_0 and c for the Banach spaces of all bounded, null sequence and convergent respectively. The sequence spaces are typically equipped with a norm, or at least the structure of topological vector spaces.

Definition 1.4 : A *norm* on a linear space X is a function which assigns a non-negative real number $\|x\|$ to each x in X with the following properties: for each x, y in X and for $k \in K$,

(i) $\|x\| = 0$ iff $x = 0$, (ii) $\|kx\| = |k|\|x\|$ and (iii) $\|x + y\| \leq \|x\| + \|y\|$.

Definition 1.5 : *Banach space* is a normed linear space which is complete in the matrix defined by its norms. This means that every Cauchy sequence is convergent in the Banach space. Many of the best known function spaces are the Banach spaces. Thus, a Banach Space $(X, \|\cdot\|)$ is a complete normed space.

Definition 1.6 : A *paranorm* 'g' defined on a linear space X is a function $g : X \to R$ having the following usual properties:

(i) $g(\theta) = 0$, where θ is the 0 element in X,

(ii) $g(x) = g(-x)$, for all x,

(iii) $g(x + y) \leq g(x) + g(y)$ for all x, y,

(iv) The scalar multiplication is continuous, and (v) $g(x) = 0 \Rightarrow x = \theta$.

Definition 1.7 : . A paranormed space is a linear space X together with a paranormal g defined over the real field. A total paranorm is a paranorm such that $g(x) = 0 \to x = 0$. Every paranormed space (total paranormed space) is a semi metric linear space.

Conversely, any semi-metric (metric) linear space can be made into a paranormed (total paranormed) space. So, the total paranormed space and the semi-metric linear spaces are essentially the same.

Example 1.8 : R^n is a normed linear space with the norm;
(i) $\|x_n\| = \left[\sum_{i=1}^{n} |x_i|^n\right]^{1/n}$ and (ii) $\|x_\infty\| = \max_{0\le x\le 1} |x_i|$.

Example 1.9 : The space $c[a,b]$ is a normed linear space with the norm $\|f\| = \sup_{x\in(a,b)} |f(x)|$ where $c[a,b]$ is a set of continuous functions on $[a,b]$. Also, $1_\infty, c_0$ and c are the normal linear space with the norm $\|x\| = \sup_k |x_n|$ but not with norm $\|x\| = \lim_{n\to\infty} |x_n|$.

Definition 1.8 : A *seminorm* is a finite non-negative function p on a vector space E (over the field of real or complex number) satisfying the following: $p(\lambda x) = |\lambda| p(x)$ and $p(x+y) \le p(x) + p(y)$, for all $x, y \in E$ and scalar λ. Every semi-norm is a paranormed (total paranormed) but not conversely. Every normed linear spaces may be regarded as a metric space together with metric $d(x,y)$, that is distance between x and y is $\|x - y\| = d(x,y)$.

We consider the following spaces :

$$l_\infty(p) = \{x = (x_k) : \sup_k |x_k|^{p_k} < \infty\}, \quad c_0(p) = \{x = (x_k) : |x_k|^{p_k} \to 0` \ (k \to \infty)\}$$

and $c(p) = \{x = (x_k) : |x_k - l|^{p_k} \to 0 \text{ for some } l \in C\}$.

Then, the space $c_0(p)$ is metric linear space paranormed by $\|x\| = g(x) = \sup_k |x_k|^{\frac{pk}{M}}$ and the spaces $l_\infty(p)$and $c(p)$ are paranormed by $g(x) = \sup_k |x_k|^{\frac{pk}{M}}$ if $\inf p_k > 0$ [3].

We have the following properties related to paranormed spaces :

(i) $Sl_\infty(p)$, $Sc(p)$ and $Sc_0(p)$ are the paranormed spaces with the paranorm $g(x) = \sup_{p_k} |\Delta x_k|^{\frac{pk}{M}}$ where $M = \max(1, \sup p_k)$, iff $(0 < \inf p_k < \sup p_k < \infty)$ and

(ii) $p = \{p_k\}$ is a bounded sequence, then $Sc_0(p)$ is a paranormed spaces with the paranorm $g(x) = \sup_{p_k} |\Delta x_k|^{\frac{pk}{M}}$ [4].

Now, we introduce new sequence space $\{X(p,\lambda)\}_t = \{x = (x_k) : (t_k x_k) \in X(p,\lambda)\}$ where $X(p,\lambda) = \{x = (x_k) : \lambda x \in X\}$ where $\lambda = \begin{bmatrix} 1 & 0 & \cdots \\ 2 & 1 & \cdots \\ \cdots & \cdots & \cdots \end{bmatrix}$ and $X = \{1_\infty, c_0$ and $c\}$.

Those spaces are paranormed by $g^*(x) = g(tx)$ where g is paranorm in $X(p, \lambda)$.
If X is a sequence space, then we define dual space of sequence space X as

$$X^\beta = \{a = (a_k) : \sum_{k=1}^{\infty} a_k x_k \text{ is convergent for each } x \in X\}.$$

If X and Y are two sequence spaces and $A = (a_{nk}), (n, k = 1, 2, \cdots, \infty)$ be an infinite matrix of complex numbers, then we write $A_x = (A_n(x))$ if and only if $A_n(x) = \sum\limits_{k=1}^{\infty} a_{nk} x_k$ converges to each $n \in N$. If $x = (x_k) \in X \Rightarrow A_x \in Y$, then A defines a *matrix transformations* from X into Y. By $A \in (X, Y)$, we mean the class of matrices A such that $A : X \to Y$, where $(A_x)_n = l_\infty(p, \lambda)_t, \sum\limits_{k=1}^{\infty} a_{nk} x_k, n \in \mathbb{N}$.

2. Main Results

Theorem 2.1 : Let $p_k > 0$ for every $k \in \mathbb{N}$, then $l_\infty^\beta(p, \lambda)_t = \overline{M_\infty(p, \lambda)_t}$, where $\overline{M_\infty(p, \lambda)_t} = \bigcap\limits_{N=2}^{\infty} \{a = (a_k) : \sum\limits_{k=1}^{\infty} |\Delta^2 a_k| N^{\frac{1}{p_k}} < \infty\}$and $\Delta^2 a_k = \Delta a_k - \Delta a_{k+1}$.

Proof: Let $a \in \overline{M_\infty(p, \lambda)_t}$ and $x \in l_\infty(p, \lambda)_t$ and we choose an integer $N > \max(1, \sup\limits_k |u_k|^{p_k})$, then we have $\left|\sum\limits_{k=1}^{m} a_k x_k\right| = \left|\sum\limits_{k=1}^{m} (\Delta a_k - \Delta a_{k+1}) v_k\right|$, where

$$\begin{aligned} v_k &= \sum_{i=1}^{k} (k - i + 1) t_i x_i \\ &= \left|\sum_{k=1}^{m} \Delta^2 a_k v_k\right| \\ &\leq \sum_{k=1}^{\infty} |\Delta^2 a_k||v_k| \\ &\leq \sum_{k=1}^{\infty} |\Delta^2 a_k| N^{\frac{1}{p_k}} < \infty, \end{aligned}$$

thus, we have

$$\overline{M_\infty(p, \lambda)_t} \subseteq l_\infty^\beta(p, \lambda)_t.$$

Theorem 2.2 : Let $p_k > 0$ for every $k \in \mathbb{N}$, then $(A \in l_\infty(p, \lambda), l_\infty)$ if and only if $\sup\limits_n \sum\limits_{k=1}^{\infty} |\Delta^2 a_{nk}| N^{\frac{1}{p_k}} < \infty$ for every integer $N > 1$.

Proof : Let the condition holds, then we have $\sup\limits_n \sum\limits_{k=1}^{\infty} |\Delta^2 a_{nk}| N^{\frac{1}{p_k}} < \infty$. We take $tx \in l_\infty(p, \lambda)$ then $\lambda\ tx \in l_\infty(p)_t$, and hence we get $\sup\limits_k |\lambda\ tx|^{p_k} < \infty$. So there exists an integer $N \geq 1$ such that $|\lambda tx| \leq N^{\frac{1}{p_k}}$ then we have $\left|\sum_{k=1}^{\infty} a_{nk}x_k\right| = \left|\sum_{k=1}^{\infty} \Delta^2 a_{nk}v_k\right|$ where $v_k = \sum_{i=1}^{k}(k-i+1)x_i \leq \sum_{k=1}^{\infty} |\Delta^2 a_k||v_k| \leq \sup_n \sum_{k=1}^{\infty} |\Delta^2 a_{nk}|N^{\frac{1}{p_k}} < \infty$. Hence, it follows that $\sum_{k=1}^{\infty} a_{nk}x_k$ converges for each $n \in \mathbb{N}$ and $Ax \in l_\infty$.

On the other hand, let $A \in (l_\infty(p,\lambda), l_\infty)$. As a contrary, let us assume that there exists an integer such that $\sup_n \sum_{k=1}^{\infty} |\Delta^2 a_{nk}|N^{\frac{1}{p_k}} < \infty$. Then, the matrix $(\Delta^2 a_{nk}) \notin (l_\infty(p,\lambda) l_\infty)$ and so there exists $ay = (y_k) \in l_\infty$ with $\sup_k |y_k| = 1$ such that $\sum_k \Delta^2 a_{nk}y_k \neq 0(1)$. Although, if we define the sequence $\mu = \{\mu_k\}$ by

$$\begin{aligned} \mu_k &= y_{k-2} - 2y_{k-1} + y_k \quad \text{with} \quad y_j = 0 \quad \text{for} \quad j \leq 0, \\ &= t_{k-2}y_{k-2} - 2t_{k-1}y_{k-1} + t_k y_k, \quad \text{putting} \quad k = 1, 2, \cdots \\ &= t_1 y_1 + t_2(y_2 - 2y_1) + \cdots \end{aligned}$$

then $\mu = l_\infty(p,\lambda)_t$ and therefore we get $\sum_{k=1}^{\infty} a_{nk}\mu_k = \sum_{k=1}^{\infty} |\Delta^2 a_{nk}y_k$. It follows that the sequence $\{A_n(\mu)\} \notin l_\infty$ which is contradiction to our assumption. Hence, we have $\sup_n \sum_{k=1}^{\infty} |\Delta^2 a_{nk}|N^{\frac{1}{p_k}} < \infty$.

This completes the proof of the theorem.

4. Vedic Relations

Vedic Mathematics is an ancient system of mathematics which provides multidimensional thinking ability to human brain. It is based on 16 basic sutras and 13 up- sutras [8]:

The First Sutra in sanskrit :Ek**ā**dhikena P**ū**rvena.

The First Sutra in English : Ek**ā**dhikena P**ū**rvena.

"By one more than the previous one" like 1, 1+1, 2+1,3+1, 4+1, 5+1, 6+1, 7+1, 8+1 that is, 1, 2, 3, 4, 5, 6, 7, 8, 9 (Sequence).

The vedic matrix is a nine by nine square array of numbers formed by taking a multiplication table and replacing each number by digit sum :

42 becomes 6, 56 becomes 11 which becomes 2 and so on.

So the first row consists of 1, 2, 3, 4, 5, 6, 7, 8, 9 and

the second row 2, 4, 6, 8, 1, 3, 5, 7, 9 and so on sequences.

If we add the first and the last numbers in each row or column, we get the following sequence 10, 11, 12, 13, 14, 15, 16, 17, 18 [6].
For example, in the second row we see that 8+1 = 9; 2+7 = 9; 6+3 = 9 ; 4+5 = 9. These pairs of numbers can be written as ordered pairs: (1, 8), (2, 7), (3, 6) and (4, 5) which form the matrix have some relations [8] :
(1, 8) : $1+8 = 9$; $18 = 9 \times 2$ or $81 = 9 \times 9$,
(2, 7) : $2+7 = 9$; $27 = 9 \times 3$ or $72 = 9 \times 8$,
(3, 6) : $3+6 = 9$; $36 = 9 \times 4$ or $63 = 9 \times 7$ and
(4, 5) : $4+5 = 9$; $45 = 9 \times 5$ or $54 = 9 \times 6$.

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