International J. of Math. Sci. & Engg. Appls. (IJMSEA)
ISSN 0973-9424, Vol. 11 No. II (August, 2017), pp. 9-18

TOTAL ACCURATE EDGE DOMINATION NUMBER IN GRAPHS

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Abstract

An accurate edge dominating set D of a graph $G = (V, E)$ is an total accurate edge dominating set, if $< D >$ has no isolated edges. The total accurate edge domination number $\gamma_{tae}(G)$ is the minimum cardinality of an total accurate edge dominating set. In this paper, the exact value on $\gamma_{tae}(G)$ were obtained in terms of edges, maximum degree and diameter of G. We also investigate the total accurate edge domination number for cartesian product, corona, join and strong product graphs.

Key Words : *Edge dominating set, Accurate edge dominating set, Total Accurate edge Dominating set.*

AMS Subject Classification : 05C,05C05, 05C70.

1. Introduction

The graph G be a finite, simple, non-trivial, undirected and connected (p, q) graph with vertex set $V(G)$ and edge set $E(G)$. As usual P_p, C_p, W_p and K_p are respectively the path, cycle, wheel and complete graph.

In general, the maximum degree of a vertex v is denoted by $\Delta(G)$. The greatest distance between any two vertices of a connected graph G is called the diameter of G and is denoted by $diam(G)$. For any real number x, $\lceil x \rceil$ denotes the smallest integer not less than x and $\lfloor x \rfloor$ denotes the greatest integer not greater than x. In general $< X >$ to denote the subgraph induced by the set of vertices X.

A set $F \subseteq E(G)$ is said to be an edge dominating set if every edge in $< E(G) - F >$ is adjacent to some edges in F. The Edge domination number of G is the cardinality of smallest edge dominating set of G and is denoted by $\gamma'(G)$. This concept was introduced by Mitchell and Hedetniemi [8].

A edge dominating set F of a graph $G = (V, E)$ is an accurate edge dominating set, if $< E - F >$ has no edge dominating set of cardinality $|F|$. The accurate edge domination number $\gamma_{ae}(G)$ is the minimum cardinality of an accurate edge dominating set [12].

Let G_1 and G_2 be the graphs of order p_1 and p_2 respectively. The corona of two graphs G_1 and G_2 is the graph $G_1 \circ G_2$ obtained by taking one copy of G_1 and p_1 copies of G_2 and then joining the i^{th} vertex of G_1 to every vertex of the i^{th} copy of G_2.

The join of two graphs G_1 and G_2 is the graph $G_1 + G_2$ with the vertex set $V(G_1 + G_2) = V(G_1) \cup V(G_2)$ and the edge set $E(G_1 + G_2) = E(G_1) \cup E(G_2) \cup E_{G_1}^{G_2}$, where $E_{G_1}^{G_2} = \{uv/u \in V(G_1), v \in V(G_2)\}$.

Strong product of two graphs G_1 and G_2 is a graph $G = G_1 \boxtimes G_2$ is defined as the graph on the vertex set $V(G_1) \times V(G_2)$ with vertices $u = (u_1, u_2)$ and $v = (v_1, v_2)$ connected by an edge if and only if either $u_1 = v_1$ and $u_2v_2 \in E(G_2)$ or $u_2 = v_2$ and $u_1v_1 \in E(G_1)$ or $u_1v_1 \in E(G_1)$ and $u_2v_2 \in E(G_2)$.

The cartesian product of the graphs G and H, written as $G \times H$, is the graph with vertex set $V(G) \times V(H)$, two vertices (u_1, u_2) and (v_1, v_2) being adjacent in $G \times H$ if and only if either $u_1 = v_1$ and $u_2v_2 \in E(H)$, or $u_2 = v_2$ and $u_1v_1 \in E(G)$.

In this paper we follow the notations of [4].

2. Preliminary Notes

We need the following results to prove further results.

Theorem 2.1 [14] : For any cycle C_p,

$$\gamma_t'(C_p) = \begin{cases} \frac{p}{2} & where\ p = 4n,\ n \geq 1 \\ \lceil \frac{p}{2} \rceil & where\ p = 4n+1\ and\ p = 4n+3,\ n \geq 1 \\ \frac{p}{2}+1 & where\ p = 4n+2,\ n \geq 1 \end{cases}$$

In the next section, we discuss Total accurate edge domination number of a graph.

3. Total Accurate Edge Domination Number of a Graph

We define new parameter total accurate edge domination number of a graph.

An accurate edge dominating set D of a graph $G = (V, E)$ is an total accurate edge dominating set, if $< D >$ has no isolated edges. The total accurate edge domination number $\gamma_{tae}(G)$ is the minimum cardinality of an total accurate edge dominating set.

From the figure 3.1, the total accurate edge dominating set is $A = \{4, 5, 11, 13, 15, 17\}$. Therefore $\gamma_{tae}(G) = |A| = 6$.

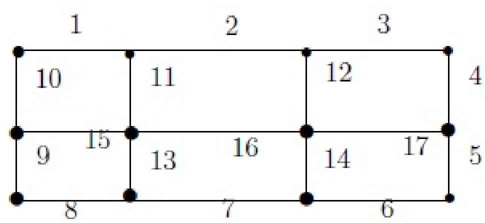

figure 3.1

4. Main Results

Theorem 4.1 : For any complete graph K_p with $p \geq 5$, $\gamma_{tae}(K_p) = \Delta(G) - diam(G)$.

Proof : Let $G = K_p$ be the complete graph with p vertices and q edges. Let $V(G) = \{v_1, v_2, ...v_p\}$ be the vertex set of G. Let $v \in V(G)$ be the vertex of maximum degree and $\Delta(v) = p - 1$. Choose any two vertices $v_i, v_j \in V(G)$ for $1 \leq i \leq p$, $1 \leq j \leq p$ then $d(v_i, v_j) = 1$ that is $diam(G) = 1$. Let $D = \{e_1, e_2, ...e_l / 1 \leq l \leq q\}$ be the minimum accurate edge dominating set of G. It is state forward that the induced subgraph $< D >$ has no isolated edges. Clearly, D itself form the minimum total accurate edge dominating set of G. Thus,

$$\begin{aligned} |D| &= |V(G)| - 2, \\ |D| &= p - 2, \\ |D| &= (p-1) - 1, \\ \gamma_{tae}(K_p) &= \Delta(G) - diam(G). \end{aligned}$$

Hence the proof.

Theorem 4.2 : For any cycle C_p, $\gamma_{tae}(C_p) = \lfloor \frac{q}{2} \rfloor + 1$.

Proof : Let $E = \{e_1, e_2, ...e_q\}$ be the edge set of C_p. Let $D = \{e_1, e_2, ...e_{\lfloor \frac{q}{2} \rfloor + 1}\}$ be the minimum accurate edge dominating set of C_p. It is obvious that the induced subgraph $< D >$ has no isolated edges. So that D itself forms the minimum total accurate edge dominating set of C_p. By Theorem 2.1,

$$\begin{aligned} |D| &\leq |E(C_p)|, \\ |D| &\leq q, \\ |D| &= \lfloor \frac{q}{2} \rfloor + 1, \\ \gamma_{tae}(C_p) &= \lfloor \frac{q}{2} \rfloor + 1. \end{aligned}$$

Hence the proof.

Theorem 4.3 : For any wheel W_p, $\gamma_{tae}(W_p) = \lceil \frac{p+1}{2} \rceil$.

Theorem 4.4 : Let P_{p_1} and P_{p_2} be the two paths of order p_1 and p_2 respectively with $p_1 < p_2$ and $H = P_{p_1} + P_{p_2}$ be the join of P_{p_1}, P_{p_2}. Then $\gamma_{tae}(H) = \lceil \frac{p_2}{2} \rceil + (p_1 - 1)$.

Proof : Let $G_1 = P_{p_1}$ and $G_2 = P_{p_2}$ be two paths labeled in order as $v_1e_1v_2e_2...e_{p_1-1}v_{p_1}$ and $v_1^{'}e_1^{'}v_2^{'}e_2^{'}...e_{p_2-1}^{'}v_{p_2}^{'}$ respectively with $p_1 < p_2$. Let $D = \{e_1^{''}, e_2^{''}, ...e_l^{''}/1 \leq l \leq q\}$ be the accurate edge dominating set of H. Suppose $D \subseteq E(G_1) \cup E(G_2)$ then the induced subgraph $< D >$ has isolated edges, which is a contradiction. So that $D \subseteq E(G_1) \cup E(G_2) \cup E_{G_1}^{G_2}$ forms the minimum total accurate edge dominating set of H. Thus,

$$\begin{aligned} |D| &\leq |V(G_2)| + |V(G_1)|, \\ |D| &= \lceil \frac{p_2}{2} \rceil + (p_1 - 1), \\ \gamma_{tae}(H) &= \lceil \frac{p_2}{2} \rceil + (p_1 - 1). \end{aligned}$$

Hence the proof.

Theorem 4.5 : Let P_{p_1} be any path of length grater than one and C_3 be a cycle, $\gamma_{tae}(P_{p_1}+C_3)=\gamma^{'}(P_{p_1}+C_3)+1$.

Proof : Let $H=P_{p_1}+C_3$ be the join graph. Consider the minimum edge dominating set $D=\{e_1,e_2,...e_{\lceil\frac{p_1}{2}\rceil+1}\}$ and $|D|=\gamma^{'}(P_{p_1}+C_3)+1$. But the set D is not accurate edge dominating set of H except P_3+C_3. Consider an edge $x\in E(G)-D$ such that $A=D\cup\{x\}$ forms the minimum accurate edge dominating set of H and the induced subgraph $<A>$ has no isolated edges. So that D is total accurate edge dominating set of G with minimum cardinality. Suppose $G=P_3+C_3$, the set D itself forms minimum accurate edge dominating set of H. But D is not total accurate edge dominating set of H. For any edge $y\in E(H)-D$ and $A=D\cup\{y\}$ forms the total accurate edge dominating set of G with minimum cardinality. Thus,

$$\begin{aligned}
|A| &\leq |V(P_{p_1})|,\\
|A| &= (\lceil\frac{p_1}{2}\rceil+1)+1,\\
|A| &= |D|+1,\\
\gamma_{tae}(P_{p_1}+C_3) &= \gamma^{'}(P_{p_1}+C_3)+1.
\end{aligned}$$

Hence the Proof.

Theorem 4.6 : Let H be the cartesian product of P_{p_1} and C_3. If $n\geq 1$ is an positive integer and $p_1\geq 3$ then,

$$\gamma_{tae}(P_{p_1}\times C_3)=\begin{cases}\frac{3p_1+1}{2} & if\ p_1=2n+1\\ \frac{3p_1}{2} & if\ p_1=2n+2\end{cases}$$

Proof : Let $G_1=P_{p_1}$ be any graph of order $p_1\geq 3$ and $G_2=C_3$ be a cycle. The cartesian product $H=G_1\times G_2$ is the graph with p vertices and q edges. Let $D=\{e_1,e_2,...e_{p_1+1}\}$ be the accurate edge dominating set of H with minimum cardinality. But the induced subgraph $<D>$ has isolated edges, which is not total accurate edge dominating set of H. Consider $F\subseteq E(H)-D$ and $A=D\cup F$ forms the minimum total accurate edge dominating set of H.

We have the following cases

Case 1 : Suppose $p_1=2n+1$.

Let $\{v_1,v_2,...v_{2n+1}\}$ be the vertices of P_{p_1} and $\{u_1,u_2,u_3\}$ be the vertices of C_3. Let

$V(H) = \{v_1^{'}, v_2^{'}, ...v_{3(2n+1)}^{'}\}$ be the vertex set of H and $D = \{e_1, e_2, ...e_{2n+2}\}$ be the minimum γ_{ae}-set of H. But the induced subgraph $< D >$ has isolated edges, which is a contradiction. So we consider $\{e_i/1 \leq i \leq q\} \subseteq E(H) - D$ such that $A = D \cup \{e_i/1 \leq i \leq q\}$ for forms the minimum total accurate edge dominating set of H. Therefore,

$$\begin{aligned} |A| &= |D| + n, \\ |A| &= (2n+2) + n = 3n + 2, \\ |A| &= \frac{3p_1 + 1}{2}, \\ \gamma_{tae}(P_{p_1} \times C_3) &= \frac{3p_1 + 1}{2}. \end{aligned}$$

Case 2 : Suppose $p_1 = 2n + 2$.
Let $\{v_1, v_2, ...v_{2n+2}\}$ be the vertices of P_{p_1} and $\{u_1, u_2, u_3\}$ be the vertices of C_3. Let $V(H) = \{v_1^{'}, v_2^{'}, ...v_{3(2n+2)}^{'}\}$ be the vertex set of H and $D = \{e_1, e_2, ...e_{2n+3}\}$ be the minimum γ_{ae}-set of H. But the induced subgraph $< D >$ has isolated edges, which is a contradiction. So we consider $\{e_i/1 \leq i \leq q\} \subseteq E(H) - D$ such that $A = D \cup \{e_i/1 \leq i \leq q\}$ forms the minimum total accurate edge dominating set of H. Therefore,

$$\begin{aligned} |A| &= |D| + n, \\ |A| &= (2n+3) + n = 3n + 3, \\ |A| &= \frac{3p_1}{2}, \\ \gamma_{tae}(P_{p_1} \times C_3) &= \frac{3p_1}{2}. \end{aligned}$$

Hence the proof.

Theorem 4.7 : Let P_3 be a path and P_{p_1} be any path with $p_1 \geq 4$, $\gamma_{tae}(P_3 \times P_{p_1}) \leq \lceil \frac{q}{6} \rceil + diam(P_{p_1})$.

Proof : Let $G_1 = P_3$ and $G_2 = P_{p_1}$ be two path labeled in order as $v_1 e_1 v_2 e_2 v_3$ and $u_1 e_1^{'} u_2 e_2^{'} .... e_{p_1 - 1}^{'} u_{p_1}$ respectively. The cartesian product $H = G_1 \times G_2$ is the graph with p vertices and q edges. The diameter, $diam(P_{p_1}) = p_1 - 1 = \lceil \frac{q}{6} \rceil$. Let $D = \{e_1^{''}, e_2^{''}, ...e_{p_1}^{''}\}$ be the accurate edge dominating set of H with minimum cardinality. But the induced subgraph $< D >$ has isolated edges, which is a contradiction. Consider $F \subseteq E(H) - D$ and $A = D \cup F$ forms the minimum total accurate edge dominating set of H.

Thus,

$$\begin{aligned} |A| &\leq |E(H)| + |D| - 1, \\ |A| &\leq \lceil \frac{q}{6} \rceil + p_1 - 1, \\ \gamma_{tae}(P_3 \times P_{p_1}) &\leq \lceil \frac{q}{6} \rceil + diam(P_{p_1}). \end{aligned}$$

Hence the proof.

Theorem 4.8 : Let H be the strong product of $P_3 \boxtimes P_{p_1}$. If $n \geq 1$ is an positive integer and $p_1 \geq 4$ then,

$$\gamma_{tae}(P_3 \boxtimes P_{p_1}) = \begin{cases} \frac{5p_1}{2} - 4 & if\ p_1 = 2n+2 \\ \frac{5p_1 - 7}{2} & if\ p_1 = 2n+3 \end{cases}$$

Proof : Let P_3 be a path and P_{p_1} be any path of order $p_1 \geq 4$ and $H = P_3 \boxtimes P_{p_1}$ be the strong product graph of P_3 and P_{p_1}.

We have the following cases

Case 1 : Suppose $p_1 = 2n + 2$.

Let $D = \{e_1, e_2, ...e_{\frac{3p_1}{2}-2}\}$ be the minimum γ_{ae}-set of H. But the induced subgraph $< D >$ has isolated edges. Consider $F = \{e_1, e_2, ...e_{p_1-2}\} \subseteq E(H) - D$ such that $A = D \cup F$ forms the minimum total accurate edge dominating set of H. Therefore,

$$\begin{aligned} |A| &= |D| + |F|, \\ |A| &= (\frac{3p_1}{2} - 2) + (p_1 - 2), \\ \gamma_{tae}(P_3 \boxtimes P_{p_1}) &= \frac{5p_1}{2} - 4. \end{aligned}$$

Case 2 : Suppose $p_1 = 2n + 3$.

Let $D = \{e_1, e_2, ...e_{\lceil \frac{3p_1}{2} \rceil - 1}\}$ be the minimum γ_{ae}-set of H. But the induced subgraph $< D >$ has isolated edges. Consider $F = \{e_1, e_2, ...e_{p_1-3}\} \subseteq E(H) - D$ such that $A = D \cup F$ forms the minimum total accurate edge dominating set of H. Therefore,

$$\begin{aligned} |A| &= |D| + |F|, \\ \gamma_{tae}(P_3 \boxtimes P_{p_1}) &= \frac{5p_1 - 7}{2}. \end{aligned}$$

Hence the proof.

Theorem 4.9 : Let P_{p_1} be any path with $p_1 \geq 2$, $\gamma_{tae}(P_{p_1} \boxtimes P_{p_1}) \geq \gamma^{'}(P_{p_1} \boxtimes P_{p_1}) + \lfloor \frac{p_1}{2} \rfloor$.

Proof : Let $H = P_{p_1} \boxtimes P_{p_1}$ be the strong product with p vertices and q edges. Let $D = \{e_1, e_2, ...e_l/1 \leq l \leq q\}$ be the minimum edge dominating set of H and $|D| = \gamma^{'}(P_{p_1} \boxtimes P_{p_1})$. Choose an edge $x \in E(H) - D$ and let $D_1 = D \cup \{x\}$ be the minimum γ_{ae}-set of H. But the induced subgraph $< D_1 >$ does not form a total accurate edge dominating set of H except P_2 and P_3. For $p_1 = 2$ *and* 3, the set $A = D_1$ forms the minimum total accurate edge dominating set of H. For $p_1 > 3$, the set D_1 is not a total accurate edge dominating set of H. Consider $F \subseteq E(H) - D_1$ and $A = D_1 \cup F$ forms the total accurate edge dominating set of H with minimum cardinality. Thus,

$$\begin{aligned} |A| &\geq |D| + |V(P_{p_1})|, \\ \gamma_{tae}(P_{p_1} \boxtimes P_{p_1}) &\geq \gamma^{'}(P_{p_1} \boxtimes P_{p_1}) + \lfloor \frac{p_1}{2} \rfloor \end{aligned}$$

Hence the Proof.

Theorem 4.10 : Let any path P_{p_1}, $p_1 \geq 1$ and a cycle C_3. Then $\gamma_{tae}(C_3 \circ P_{p_1}) \geq \gamma_{ae}(C_3 \circ P_{p_1})$ for P_2, P_3, P_4 and equality holds for remaining paths.

Proof : Let $G_1 = C_3$ and $G_2 = P_{p_1}$ be any two graphs of order three and $p_1 \geq 1$. Let $H = C_3 \circ P_{p_1}$ be the corona graph with p vertices and q edges. Let $D = \{e_l/1 \leq l \leq q\}$ be the minimum edge dominating set of H. In particular, $|D| = p_1$ for the graph $C_3 \circ P_{3p_1}$ and $C_3 \circ P_{3p_1 - 1}$. Also $|D| = p_1 + 1$ for the graphs $C_3 \circ P_{3p_1 - 2}$. The set D is the minimum accurate edge dominating set for only P_3 and P_4.

Case 1 : Suppose $G_2 = P_{p_1}$ except $p_1 = 2, 3\, and\, 4$.

Consider $F \subseteq E(H) - D$ and $D_1 = D \cup F$ forms the minimum accurate edge dominating set of H and the set $A = D_1$ itself forms the total accurate edge dominating set of G with minimum cardinality. Clearly, $|A| = |D_1|$. Therefore, $\gamma_{tae}(C_3 \circ P_{p_1}) = \gamma_{ae}(C_3 \circ P_{p_1})$.

Case 2 : Suppose $G_2 = P_2\, and\, P_4$.

Consider $x \in E(H) - D_1$ and $A = D_1 \cup \{x\}$ forms the minimum total accurate edge dominating set of H. Clearly, $|A| \geq |D_1|$. Therefore $\gamma_{tae}(C_3 \circ P_{p_1}) \geq \gamma_{ae}(C_3 \circ P_{p_1})$.

Case 3 : Suppose $G_2 = P_3$.

For any two edges $x, y \in E(H) - D_1$ and $A = D_1 \cup \{x, y\}$ forms the minimum total accurate edge dominating set of H. Clearly, $|A| \geq |D_1|$. Therefore $\gamma_{tae}(C_3 \circ P_{p_1}) \geq \gamma_{ae}(C_3 \circ P_{p_1})$.

Hence the proof.

Theorem 4.11 : Let P_2 and P_{p_1} be two paths with $p_1 \geq 3$, $\gamma_{tae}(P_2 \circ P_{p_1}) \leq \lfloor \frac{\Delta(P_2 \circ P_{p_1})}{2} \rfloor +$

$(p_1 - 1)$.

Proof : Let $H = P_2 \circ P_{p_1}$ be the corona graph such that $|V(H)| = p$, $|E(H)| = q$ and $\Delta(H) = p_1$. Let $D = \{e_1, e_2, ...e_t/1 \leq t \leq q\}$ be the accurate edge dominating set of H with minimum cardinality and the induced subgraph $< D >$ has no isolated edges. Clearly, $A = D$ is the minimum total accurate edge dominating set of H. Otherwise, consider $F \subseteq E(H) - D$ and $A = D \cup F$ forms the minimum total accurate edge dominating set of H. Thus,

$$\begin{aligned} |A| &\leq |\frac{V(P_{p_1})}{2}| + |V(P_{p_1}) - 1|, \\ |A| &\leq \lfloor\frac{p_1}{2}\rfloor + (p_1 - 1), \\ \gamma_{tae}(P_2 \circ P_{p_1}) &\leq \lfloor\frac{\Delta(P_2 \circ P_{p_1})}{2}\rfloor + (p_1 - 1) \end{aligned}$$

Hence the Proof.

5. Conclusion

In this paper we discussed the total accurate edge number of graphs and some operation on graphs.

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