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PAIRWISE SEMI BICOMPACT AND PAIRWISE SEMI LINDELOFF BISPACES

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Abstract

The notion of pairwise semi-bicompact and pairwise semi-Lindeloff spaces was introduced by S. Balasubramanian [2] in bitopological spaces. Here we have studied the idea of pairwise semi-bicompact and pairwise semi-Lindeloff spaces in a more general structure of a bispace and investigate how far several results as valid in a bitopological space are affected in a bispace.

1. Introduction

The notion of a bitopological space was introduced by J. C. Kelly [9] in 1963. Later several ideas like compactness, connectedness, separation axioms etc. were studied in a bitopological space. Subsequently the idea of semi compactness has been studied in a bitopological space in [2]. The notion of a σ space or simply a space was introduced by

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A. D. Alexandroff [1] generalizing the idea of a topological space where only countable union of open sets were taken to be open. In 1995, Das and Lahiri [11] studied the notion of semi open sets in a σ space. They also generalized the notion of a bitopological space to a bispace using the idea of σ space. Banerjee and Saha [3] studied the idea of pairwise semi open sets in a more general structure of a bispace.
In this paper we have studied the idea of semi bicompactness and semi Lindeloffness in a bispace where by semi open cover of X, we mean a collection of semi open sets in a bispace (X, τ_1, τ_2) which cover X. This idea of semi bicompactness is not the same used in [4].

2. Pairwise Semi Bicompact

Definition 2.1 [1] : A set X is called an Alexandroff space or σ space or simply a space if in it is chosen a system of subsets F satisfying the following axioms:

(1) The intersection of a countable number of sets from F is a set in F.

(2) The union of a finite number of sets from F is a set in F.

(3) The void set φ is a set in F.

(4) The whole set X is a set in F.

Sets of F are called closed sets. Their complementary sets are called open sets. It is clear that instead of closed sets in the definition of the space, one may put open sets with subject to the conditions of countable summability, finite intersectibility and the condition that X and void set φ should be open. The collection of all such open sets will sometimes be denoted by τ and the space by (X, τ). Note that a topological space is a space but in general, τ is not a topology as can be easily seen by taking $X = R$ and τ as the collection of all F_σ sets in R.
Definition 2.2 [1] : To every set M of a space (X, τ) we correlate its closure $\overline{M}$, the intersection of all closed sets containing M. The closure of a set M may also be denoted by $\tau cl(M)$ or simply clM when there is no confusion about τ.
Generally the closure of a set in a space is not a closed set. By the axioms, it easily follows that

(1) $\overline{M \cup N} = \overline{M} \cup \overline{N}$;

(2) $M \subset \overline{M}$;

(3) $\overline{M} = \overline{\overline{M}}$;

(4) $\overline{\varphi} = \varphi$.

(5) $\overline{A} = A \cup A'$ where A' denotes the set of all limit point of A.

Definition 2.3 [11] : The interior of a set M in a space (X, τ) is defined as the union of all open sets contained in M and is denoted by $\tau - int(M)$ or $int(M)$ when there is no confusion about τ.

Definition 2.4 [9] : A set X on which are defined two arbitrary topologies P, Q is called a bitopological space and is denoted by (X, P, Q).

Definition 2.5 [10] : Let X be a nonempty set. If τ_1 and τ_2 be two collections of subsets of X such that (X, τ_1) and (X, τ_2) are two spaces, then X is called a bispace and is denoted by (X, τ_1, τ_2).

Definition 2.6 [3] : Let (X, τ_1, τ_2) be a bispace. We say that a subset A of X is τ_i semi open with respect to τ_j (in short τ_i, s. o. w. r. to τ_j) or (τ_i, τ_j) semi open or simply (i, j) semi open if and only if there exists a τ_i open set 0 such that $0 \subset A \subset \tau_j c\ l(0)$, $i, j = 1, 2, i \neq j$.

A set $A \subset (X, \tau_1, \tau_2)$ is called (i, j) semi closed if $X - A$ is (i, j) semi open. Intersection of all (i, j) semi closed set containing a given set $A \subset (X, \tau_1, \tau_2)$ is called (i, j) semi closure of A.

Definition 2.7 (cf. [2]) : Let (X, τ_1, τ_2) be a bispace. Then (X, τ_1, τ_2) is said to be (i, j) semi bicompact if every (i, j) semi open cover of X has a finite subcover. (X, τ_1, τ_2) is said to be locally (i, j) semi bicompact if each $x \in X$ has (i, j) semi open neighbourhood whose (i, j) semi closure is (i, j) semi bicompact.

Theorem 2.1 : Every (i, j) semi bicompact is locally (i, j) semi bicompact.

Proof : Let (X, τ_1, τ_2) be (i, j) semi bicompact. The whole set X is τ_i open, so τ_i semi open set with respect to τ_j (τ_i s.o,.w.r.to τ_j). Since φ is τ_i open, it is (i, j) semi open and so $X - \varphi$ i.e. X is (i, j) semi closed. So (i, j) semi closure of X is X. So each point $x \in X$ has a neighbourhood viz. X whose (i, j) semi closure is (i, j) semi bicompact. Hence X is locally (i, j) semi bicompact.

Example 2.1 : Example of a semi bicompact space.

Let $X = [0, 2]$, $\tau_1 = \{\varphi, X, G_i\}$, $\tau_2 = \{\varphi, X, F_i\}$ where G_i's and F_i's are the countable subsets of irrational numbers from [0,1] and [1,2]. Then (X, τ_1, τ_2) is a bispace but not a bitopological space. Then (τ_1, τ_2) semi open sets are as follows:

(i) Any G_i is τ_1 s. o. w. r. t. τ_2 i.e. (τ_1, τ_2) semi open.

(ii) Any subset of [0,1] containing at least one irrational number from [0,1] is (τ_1, τ_2) semi open.

(iii) $[0, 1] \cup P_i$ where P_i is any subset of rationals in [1, 2] is (τ_1, τ_2) semi open.

(iv) X is (τ_1, τ_2) semi open.

So any (τ_1, τ_2) semi open cover of X must contains the member X. So (X, τ_1, τ_2) is (τ_1, τ_2) semi bicompact.

Example 2.2 : Let $X = \mathbb{R}$ and let α be a fixed real number. Let $\tau_1 = \{X, \varphi, G_i, F_i\}$ where G_i's are countable subsets of $\mathbb{R} - \{\alpha\}$ and F_i's are the cofinite subsets of $\mathbb{R}$ containing α and $\tau_2 = \{$the collection of all F_σ sets$\}$, then (X, τ_1, τ_2) is a bispace which is not a bitopological space. Then τ_2 closure of any set is that set itself. So τ_1 open sets are the only τ_1 s.o. sets w.r.to τ_2. Because for any set $A, A = \tau_2 clA$ and so $G_i = \tau_2 clG_i$ for all $G_i \in \tau_1$. So $G_i \subset G_i \subset \tau_2 clG_i$ for all $G_i \in \tau_1$. Therefore, any (τ_1, τ_2) semi open cover (τ_1 open cover) of X has a finite subcover. So $(X, \tau_1, , \tau_2)$ is (i, j) semi bicompact.

Theorem 2.2 : Let $X, \tau_1, , \tau_2)$ be a bispace and $Y \subset X$. So $(Y, \tau_{1/Y}, \tau_{2/Y})$ is a subspace. If U is τ_1 s.o. set w.r. to τ_2 then $U \cap Y$ is $\tau_{1/Y}$ s.o.w.r. to $\tau_{2/Y}$ if Y is τ_2 open.

Proof : Since U is τ_1 s.o. sets w.r. to τ_2, there exists a τ_1 open set G such that $G \subset U \subset \tau_2 clG$. So $G \cap Y \subset U \cap Y \subset \tau_2 clG \cap Y \subset \tau_2 cl(G \cap Y)$, if Y is τ_2 open. So $U \cap Y$ is $\tau_{1/Y}$ s.o.w.r. to $\tau_{2/Y}$. Similarly $U \cap Y$ is $\tau_{2/Y}$ s.o. w. r. to $\tau_{1/Y}$ if Y is τ_1 open.

Theorem 2.3 : Let $A \subset X$. Then if a subset U of A is $(\tau_{i/A}, \tau_{j/A})$ semi open, then $U = A \cap G$ where G is (i, j) semi open, conversely if A is τ_j open and G be any (i, j) semi open then $U = A \cap G$ is $(\tau_{i/A}, \tau_{j/A})$ semi open.

Proof : Let U be a $(\tau_{i/A}, \tau_{j/A})$ semi open set. So there exists $\tau_{i/A}$ open set O such that $0 \subset U \subset \tau_{j/A} cl(0)$. Since O is $\tau_{i/A}$ open, there exists τ_i open set V such that $0 = A \cap V$. So $A \cap V \subset U \subset \tau_{j/A} cl(A \cap V)$. Again, $\tau_{j/A} cl(A \cap V) = \tau_j cl(A \cap V) \cap A \subset \tau_j cl(A \cap V) \subset \tau_j clV$. Therefore, $A \cap V \subset U \subset \tau_j clV$. Let $G = U \cup V$, then $U = A \cap G$, because $A \cap G = A \cap (U \cup V) = (A \cap U) \cup (A \cap V) = U \cup (A \cap V) = U$, since $A \cap V \subset U$.

Again since $A \cap V \subset U \subset \tau_j clV$, it follows that $(A \cap V) \cup V \subset U \cup V \subset (\tau_j clV) \cup V$ i.e., $V \subset G \subset \tau_j clV$. Therefore, G is (i, j) semi open.

Conversely, let G be (i, j) semi open. So there exists τ_i open set O such that $0 \subset G \subset \tau_j cl0$ which implies that $0 \cap A \subset G \cap A \subset \tau_j cl0 \cap A \subset \tau_j cl(0 \cap A)$, since A is τ_j open. Therefore, $U = A \cap G$ is $(\tau_{i/A}, \tau_{j/A})$ semi open.

Theorem 2.4 : Let (X, τ_1, τ_2) be a bispace and $A \subset X$. If A is (τ_1, τ_2) semi bicompact then $(A, \tau_{1A}, \tau_{2/A})$ is $(\tau_{1/A}, \tau_{1/A})$ semi bicompact. Converse part holds if A is τ_2 open.

Proof : First suppose that A is (τ_1, τ_2) semi bicompact. Let $\{U_i : i \in I)$ be $(\tau_{1/A}, \tau_{2/A})$ semi open over for A. Then $U_i = V_i \cap A$ where V_i is (τ_1, τ_2) semi open set, by Theorem 2.3. So $\{V_i : i \in I\}$ is a (τ_1, τ_2) semi open cover for A. Hence there exists a finite subcover say $\{V_1, V_2, \cdots, V_n\}$ for A i.e. $A \subset (V_1 \cup V_2 \cup \cdots \cup V_n)$ which implies that $(V_1 \cup V_2 \cup \cdots \cup V_n) \cap A = A$ i.e. $(V_1 \cap A) \cup (V_2 \cap A) \cdots (V_n \cap A) = A$. Therefore, $U_1 \cup U_2 \cup \cdots \cup U_n = A$ and so $\{U_1, U_2, \cdots, U_n\}$ is a finite subcover for A. Hence $(A, \tau_{1/A}, \tau_{2/A})$ is $(\tau_{1/A}, \tau_{2/A})$ semi bicompact.

Converely let A be $(\tau_{1/A}, \tau_{2/A})$ semi bicompact. Let $\{U_i : i \in I\}$ be any (τ_1, τ_2) semi open cover for A. So $A \subset \cup\{U_i : i \in I\}$ which implies that $\cup \{U_i : i \in I\} \cap A = A$ i.e. $\cup \{(U_i \cap A) : i \in I\} = A$. So $\{U_i \cap A\}$ is a $(\tau_{1/A}, \tau_{2/A})$ semi open cover for A, since A is τ_2 open. Hence there exists a finite subcover say $\{(U_1 \cap A), (U_2 \cap A), \cdots, (U_n \cap A)\}$ for A i.e. $(U_1 \cap A) \cup (U_2 \cap A) \cup \cdots \cup (U_n \cap A) = A$ and so $A \subset U_1 \cup U_2 \cup \cdots \cup U_n$. Hence A is (τ_1, τ_2) semi bicompact. Similarly if A is $(\tau_{2/A}, \tau_{1/A})$ semi bicompact, then A is (τ_2, τ_1) semi bicompact.

Theorem 2.5 : If A is τ_i closed subspace of a (i, j) semi bicompact space X. Then $(A, \tau_{1/A}, \tau_{2/A})$ is $(\tau_{i/A}, \tau_{j/A})$ semi bicompact. Moreover A is (i, j) semi bicompact if A is τ_j open.

Proof : Let $A \subset (X, \tau_1, \tau_2)$ be τ_i closed and let (X, τ_1, τ_2) be (i, j) semi bicomact. Let $\{U_i : i \in I\}$ be an $(\tau_{i/A}, \tau_{j/A})$ semi open cover for A in $(A, \tau_{1/A}, \tau_{2/A})$. Then $U_i = G_i \cap A$ where G_i is (i, j) semi open in X. So $\{G_i : i \in I\} \cup (X - A)$ is (i, j) semi open cover for X, since $X - A$ is τ_i open and hence (i, j) semi open. Since (X, τ_1, τ_2) is (i, j) semi bicompact, there exists finite number of sets $\{G_1, G_2, \cdots, G_k\}$ such that $G_1 \cup G_2 \cup \cdots \cup G_k \cup (X - A) = X$. So $\{U_1, U_2, \cdots, U_k\}$ form a finite subcover for A. So $(A, (\tau_{1/A}, \tau_{2/A})$ is $(\tau_{i/A}, \tau_{j/A})$ semi bicompact. Moreover A is (i, j) semi bicompact if A is τ_j open.

Theorem 2.6 : Let (X, τ_1, τ_2) be (i, j) semi bicompact. Then (X, τ_i) is compact, $i, j = 1, 2;\ i \neq j$.

Proof : Let $\{G_i : i \in I\}$ be any τ_i open cover for X. Since every τ_i open is (i, j) semi open, so $\{G_i : i \in I\}$ is a (i, j) semi open cover for X and so has finite subcover. So (X, τ_i) is compact.

Corollary 2.1 : If (X, τ_1, τ_2) is (i, j) semi bicompact then (X, τ_i) is locally compact.

Theorem 2.7 : Let (X, τ_1, τ_2) be a bispace. Then $(X, \tau_1, \tau_2)$is semi bicompact if and only if every class of (i, j) semi closed sets having finite intersection property (F.I.P.), the intersection of the entire collection is non empty.

Proof : Let (X, τ_1, τ_2) be (i, j) semi bicompact. Let $\{F_i : i \in I\}$ be a collection of (i, j) semi closed sets having F.I.P. If possible, let $\bigcap\limits_{i \in I} F_i = \varphi$ then $(\bigcap\limits_{i \in I} F_i)^c = X$. i.e., $\cup(X - F_i) = X$. So $\{(X - F_i) : i \in I\}$ is collection of semi open cover for X. So there exists a finite subcover say $\{X - F_1, X - F_2, \cdots, X - F_n\}$ such that $(X - F_1) \cup (X - F_2) \cup \cdots \cup (X - F_n) = X$ which imples that $\bigcap\limits_{i=1}^{n} F_i = \varphi$, a contradiction. So $\bigcap\limits_{i \in I} F_i \neq \varphi$.

Conversely let $\{G_i : i \in I\}$ be any (i, j) semi open cover for X. Then $\bigcup\limits_{i \in I} G_i = X$ which implies that $\bigcap\limits_{i \in I}(X - G_i) = \varphi$. So by the condition the collection $\{X - G_i : i \in I\}$ of (i, j) semi closed sets does not have F.I.P. So there exists a finite subcollection such that $\bigcup\limits_{i=1}^{n}(X - G_i = \emptyset$ which implies that $c \bigcup\limits_{i=1}^{n} G_i = X$. So (X, τ_1, τ_2) is (i, j) semi bicompact.

Theorem 2.8 : The bispace (X, τ_1, τ_2) is (i, j) semi bicompact if and only if every class of (i, j) semi closed sets with empty intersection has a finite subclass with empty intersection.

Proof : Let (X, τ_1, τ_2) be (i, j) semi bicompact and let $\{F_i\}_{i \in I}$ be a collection of (i, j) semi closed sets with empty intersection i.e., $\bigcap\limits_{i \in I} F_i = \varphi$ and so $\bigcup\limits_{i \in I}(X - F_i) = X$. Therefore, $\{(X - F_i)\}_{i \in I}$ is a (i, j) semi open cover for X. Since (X, τ_1, τ_2) is (i, j) semi bicompact, there exists a finite subcover such that $\bigcup\limits_{i=1}^{n}(X - F_i) = X$. This implies that $\bigcap\limits_{i=1}^{n} F_i = \varphi$.

Conversely let every class of (i, j) semi closed sets with empty intersection has a finite subclass with empty intersection. Let $\{G_i\}_{i \in I}$ be a semi open cover for X. Then $\bigcup\limits_{i \in I} G_i = X$ and so, $\bigcap\limits_{i \in I}(X - G_i) = \varphi$. By the condition, there exists a finite sub collection such that $\bigcap_{i=1}^{n}(X - G_i) = \varphi$ i.e. $\bigcup_{i=1}^{n} G_i = X$. So (X, τ_1, τ_2) is (i, j) semi bicompact.

Corollary 2.2 : If (X, τ_1, τ_2) be (i, j) semi bicompact then, (i) for every class of τ_i closed sets having FIP the intersection of entire collection is non empty and (ii) every class of τ_1 closed with empty intersection has a finite subclass with empty intersection.

Remark 2.1 : The converse part may not be true which is shown in the following example.

Example 2.3 : Let $X = [0, \infty), \tau_1 = \{X, \varphi, Q, Q \cup F_1\}$ where $X - F_i$ are the finite subsets of irrational numbers in X and Q is the set of all rational number in X. Then (X, τ_1) is a topological space and also compact. Let $\{G_i\}$ be an τ_1 open cover for X. Let $G_i \neq X$ for each i. So there exists a set G_1 in the collection such that $G_1 = Q \cup F_1$ [since Q can not cover whole X]. So $X - F_1$ is a finite set, say $\{x_1, x_2, \cdots, x_k\}$. So there exist finite numbers of sets containing the points $x_1, x_2, \cdots, x_k$. Thus these finite number of sets together with G_1 forms a finite subcover. So (X, τ_1) is compact. So every family of τ_1 closed set possessing F.I.P, the intersection of entire collection is non empty. Let $\tau_2 = \{X, \varphi, Q_i\}$ where Q_i's are countable subsets of rational numbers in X. Then $\tau_2 cl\ Q = X$. Let P be the set of all irrational numbers in X. Let $A_n = Q \cup ([0, n) \cap P), n = 1, 2, 3, \cdots$. So $Q \subset A_n \subset X = \tau_2 cl\ Q$. This implies each A_n is τ_1 s. o. w. r. to τ_2. Also $\bigcup_{n=1}^{\infty} A_n = X$. So $\{A_n\}$ is semi-open cover for X which has no finite subcover.

Definition 2.8 [10] : A cover B of a bispace (X, τ_1, τ_2) is said to be pairwise open if $B \in \tau_1 \cup \tau_2$ and B contains at least one member from each of τ_1 and τ_2.

Definition 2.9 : A cover B of a bispace (X, τ_1, τ_2) is said to be pairwise semi open if each member of B is either (τ_1, τ_2) semi open or (τ_2, τ_1) semi open and B contains at least one (τ_1, τ_2) semi open set and at least one (τ_2, τ_1) semi open set.

Definition 2.10 [10] : (X, τ_1, τ_2) is said to be pairwise bicompact if every pairwise open cover of it has a finite subcover.

Definition 2.11 : (X, τ_1, τ_2) is said to be pairwise semi bicompact if every pairwise semi open cover of ithas a finite subcover.

Definition 2.12 [cf. 5] : Let (X, τ_1, τ_2) and (Y, σ_1, σ_2) be two bispaces. Then a mapping $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is called $\tau_1\sigma_1$ (res. $\tau_2\sigma_2$) semi continuous if and only if for each $0 \in \sigma_1(res.\ \sigma_2), f^{-1}(0)$ is $\tau_1(res.\ \tau_2)$ semi open with respect to $\tau_2(res.\ \tau_1)$. We say $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is pairwise semi continuous if it is both $\tau_1\sigma_1$ and

$\tau_2\sigma_2$ semi continuous.

Definition 2.13 [cf.5] : Let (X, τ_1, τ_2) and (Y, σ_1, σ_2) be two bispaces. Then a mapping $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is called $\tau_1\sigma_1(res.\ \tau_2\sigma_2)$ semi open if and only if for each $0 \in \tau_1(Res.\tau_2), f(0)$ is $\sigma_2(res.\ \sigma_1)$ semi open with respect to $\sigma_2(eres.\ \sigma_1)$.

We say $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is pairwise semi open if it is both $\tau_1\sigma_1$ and $\tau_2\sigma_2$ semi open.

Theorem 2.9 : Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be a pairwise semi continuous onto mapping. If (X, τ_1, τ_2) is pairwise semi bicompact then Y is pairwise bicompact.

Proof : Let $\{G_i\}_{i\in I}$ be a pairwise open cover for Y. Then $\{f^{-1}(G_i)\}_{i\in I}$ is pairwise semi open cover for X. Therefore, there exists a finite subcover of X such that $\bigcup\limits_{i=1}^{n} f^{-1}(G_i) = X$. This implies that $\bigcup\limits_{i=1}^{n} G_i = f(X) = Y$. So Y is pairwise bicompact.

Theorem 2.10 : Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be pairwise continuous and pairwise open. If Y is (i, j) semi bicompact then X is $(i, j))$ semi bicompact $i, j = 1, 2, i \neq j$.

Proof : Let $\{G_\alpha\}_{\alpha\in I}$ be (i, j) semi open cover for X. So for each G_α, there exists $U_\alpha \in \tau_i$ such that $U_\alpha \subset G_\alpha \subset \tau_j clU_\alpha$. This implies that $f(U_\alpha) \subset f(G_\alpha) \subset f(\tau_j clU_\alpha) \subset \sigma_j clf(U_\alpha)$, since $f : (X, \tau_j) \rightarrow (Y, \sigma_j)$ is continuous. Thus $f(G_\alpha)$ is (i, j) semi open in Y which implies that $f(\bigcup\limits_{\alpha\in I} G_\alpha) = f(X) = Y$ i.e., $\bigcup\limits_{\alpha\in I} f(G_\alpha) = Y$. So $\{f(G_\alpha)\}_{\alpha\in I}$ is (i, j) semi open cover for Y. Since Y is (i, j) semi bicompact, there exists a finite subcover $\{f(G_1), f(G_2), \cdots, f(G_k)\}$ such that $\bigcup\limits_{\alpha=1}^{k} f(G_\alpha) = Y$ i.e. $\bigcup\limits_{\alpha=1}^{k} G_\alpha = f^{-1}(Y) = X$. Hence X is (i, j) semi bicompact.

3. Pairwise Semi Lindeloff

Definition 3.1 (cf. [2]) : Let (X, τ_1, τ_2) be a bispace. Then X is said to be (i, j) semi Lindeloff if each (i, j) semi open cover of X has a countable subcover. Clearly every (i, j) semi bicompact space is $(i, j))$ semi Lindeloff but converse may not be true as shown in the following example.

Example 3.2 : Let $X = R$ and let α be a fixed real number. Let $\tau_1 = \{X, \varphi, G_i, F_i\}$ where G_i's are the countable subsets of $R - \{\alpha\}$ and F_i's are the co-countable subsets of R containing α and τ_2 be the collection of all F_σ sets, then (X, τ_1, τ_2) is a bispace which is not a bitopological space. Then as in the example 2.2, τ_1 open sets are the only τ_1 s.o. sets w. r . to τ_2. Then any (τ_1, τ_2) semi open cover (i.e., τ_1 open cover) has countable subcover. So (X, τ_1, τ_2) is (τ_1, τ_2) semi Lindeloff. Again, let $A = \{x_1, x_2, \cdots\}$ be countable subsets of R such that $x_i \neq \alpha$. Let $\mathcal{B} = \{\{x_i\} : x_i \in A\}$ then $\mathcal{B}$ is collection of τ_1 open sets and $R - A$ is a τ_1 open set. Then $\mathcal{B}$ together with $R - A$ is an (τ_1, τ_2) semi open cover (infact, τ_1 open cover) of X which has no finite subcover. So X is not (τ_1, τ_2) semi bicompact.

Theorem 3.1 : A τ_i is closed subspace of a (i, j) semi Lindeloff bispace is (i, j) semi Lindeloff.

The proof is straightforward and so is omitted.

Theorem 3.2 : Let (X, τ_1, τ_2) be (i, j) semi Lindeloff bispace. Then (X, τ_i) is Lindeloff space, $i, j = 1, 2; i \neq j$.

The proof is straightforward and so is omitted.

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