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ON LONGEST PATH POLYNOMIAL (LP POLYNOMIAL) IN GRAPHS

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Abstract

Let G be a simple graph with no isolated vertices. Let $l_p(G,k)$ be the collections of longest path sets of G with length k and let $l(G,k) = |l_p(G,k)|$. Then, the Longest Path polynomial $L_p(G,x)$ is given by $L_p(G,x) = \sum\limits_{k=\gamma_l(G)} l(G,k)x^k$, where $\gamma_1(G)$ is the minimum cardinality of the length of the longest path between two vertices. In this paper, we discussed the LP polynomial of the graphs such as path, cycle, complete graph, triangular snake graph, complete bipartite graph, wheel graph.

1. Introduction

Unless mentioned or otherwise, a graph in this paper shall mean a simple finite graph and without isolated vertices. We denote the vertex set and edge set of the graph G by $V(G)$ and $E(G)$ respectively. For standard terminology and notations we follow [2]. A walk of length k is a finite sequence $v_0e_1\ v_1e_2\ v_2 \cdots e_k\ v_k$ whose terms are alternatively

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Key Words : *LP polynomial (Longest Path polynomial), Path, Cycle, Triangular snake, Wheel.*

vertices and edges such that the ends of an edge e_i are v_{i-1} and v_i for $1 \leq i \leq k$. A walk in which all the vertices are distinct is called a path. The number of edges in the sequence is called the length of the path.

Definition 1.1 : Let G be a simple graph with no isolated vertices. Let $l_p(G, k_)$ be the collections of longest path sets of G with length k and let $l(G, k) = |l_p(G, k)|$. Then, the Longest Path polynomial $L_p(G, x)$ is given by $L_p(G, x) = \sum\limits_{k=\gamma_l(G)} l(G(k)x^k$, where $\gamma_l(G)$ is the minimum cardinality of the length of the longest path between two vertices.

Note 1.2 :

1. Minimum cardinality of the length of the longest path between two vertices is notated as $\gamma_l(G)$.

   (In other words,$\gamma_l(G)$ is known to be longest path number of G).

2. Number of longest path sets of G with length k is notated as $l(G, k)$.

Theorem 1.3 : The LP polynomial for cycle C_n is given by

$$L_p(C_n, x) = \begin{cases} \sum\limits_{k=\lfloor \frac{n}{2} \rfloor+1}^{n-1} nx^k & \text{if } n \text{ is odd} \\ \\ \sum\limits_{k=\frac{n}{2}+1}^{n-1} nx^k + \frac{n}{2}x^{\frac{n}{2}} & \text{if } n \text{ is even} \end{cases}$$

for $n \geq 3$.

Proof : Let C_n be the cycle with n vertices and n edges.

The minimum cardinality of the length of the longest path between two vertices of $C_n, \gamma_l(G)$ for even

$$\gamma_l(G) = \frac{n}{2}$$

and for odd

$$\gamma_l(G) = \lfloor \frac{n}{2} \rfloor + 1.$$

Number of longest path sets of C_n with length k, $l(C_n, k)$ is

$$l(C_n, k) = n; \text{ if } \lfloor \frac{n}{2} \rfloor + 1 \leq k \leq n - 1 \text{ for } n \text{ is odd}$$

$$l(C_n, k) = \begin{cases} \frac{n}{2} & \text{if } k = \frac{n}{2} \\ \\ n & \text{if } \frac{n}{2} \leq k \leq n-1 \end{cases} \quad \text{for } n \text{ is even}$$

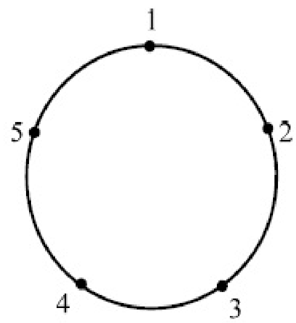
The LP polynomial of C_n is $L_p(C_n, x) = \sum\limits_{k=\gamma_l(G)} l(G,k)x^k$.

$$L_p(C_n, x) = \begin{cases} \sum\limits_{k=\lfloor \frac{n}{2} \rfloor +1}^{n-1} nx^k & \text{if } n \text{ is odd} \\ \\ \sum\limits_{k=\frac{n}{2}+1}^{n-1} nx^k + \frac{n}{2}x^{\frac{n}{2}} & \text{if } n \text{ is even} \end{cases}$$

This is true for all $n \geq 3$.

Hence proved.

Illustration 1.4 :



C₅

Longest path between two vertices are given below.

From vertex 1 to other vertices :

$l(1,2) = \{15432\}$ (length 4)
$l(1,3) = \{1543\}$ (length 3)
$l(1,4) = \{1234\}$ (length 3)
$l(1,5) = \{12345\}$ (length 4)

From vertex 2 to other vertices :

$l(2,3) = \{21543\}$ (length 4)
$l(2,4) = \{2154\}$ (length 3)
$l(2,5) = \{2345\}$ (length 3)

From vertex 3 to other vertices :

$l(3,4) = \{32154\}$(length 4)

$l(3,5) = \{3215\}$ (length 3)

From vertex 4 to other vertices :

$l(4,5) = \{43215\}$ (length 4).

Minimum cardinality of the length of the longest path between two vertices,

$$\gamma_l(G) = \lfloor \frac{n}{2} \rfloor + 1 = \lfloor \frac{5}{2} \rfloor + 1 = 3.$$

Longest path of length 1 and 2 :

There is no longest path of length 1 and 2 $l(G,1) = l(G,2) = 0$.

Longest path of length 3 : $l(G,3)$.

$$\begin{aligned} l_p(G,3) &= \{l(1,3), l(1,4), l(2,4), l(2,5), l(3,5)\} \\ l(G,3) &= |l_p(G,3) = 5. \end{aligned}$$

Longest path of length 4 : $l(G,4)$.

$$\begin{aligned} l_p(G,4) &= \{l(1,2), l(1,5), l(2,3), l(3,4), l(4,5)\} \\ l(G,4) &= |l_p(G,4)| = 5. \end{aligned}$$

That is, for odd vertices, k varies as $\lfloor \frac{n}{2} \rfloor + 1 \leq k \leq n-1$.

The LP polynomial of C_n is $L_p(G,x) \sum\limits_{k=\lfloor \frac{n}{2} \rfloor + 1}^{n-1} nx^k$.

For $n = 5$,

$$\begin{aligned} L_p(C_5, x) &= \sum_{k=\lfloor \frac{n}{2} \rfloor + 1}^{5-1} 5x^k \\ &= \sum_{k=3}^{4} 5x^k \\ &= 5x^3 + 5x^4. \end{aligned}$$

Note 1.5 : The LP polynomial of cycle $C_n, n \geq 3$ for some nodes is given below :

1. $L_p(C_3, x) = 3x^2$
2. $L_p(C_4, x) = 4x^3 + 2x^2$
3. $L_p(C_5, x) = 5x^4 + 5x^3$
4. $L_p(C_6, x) = 6x^5 + 6x^4 + 3x^3$
5. $L_p(C_7, x) = 7x^7 + 7x^5 + 7x^4$.

Theorem 1.6 : The LP polynomial of path P_n is given by $L_p(P_n, x) = \sum\limits_{k=1}^{n-1} (n-k)x^k$ for $n \geq 2$.

Proof : Let P_n be the path with n vertices and $n-1$ edges.

The minimum cardinality of the length of longest path between two vertices k of path P_n, $\gamma_l(G) = 1$.
The number of longest path sets of P_n with length k is

$$l(P_n, k) = n - k; \quad \text{if} \quad 1 \leq k \leq n-1 \;\; \text{for} \;\; n \geq 2.$$

The LP polynomial for P_n is

$$\begin{aligned} L(P_n, x) &= \sum_{k=\gamma_l(G)}^{n} l(P_n, k)x^k \\ &= (n-1)x + (n-2)x^2 + (n-3)x^3 + \cdots + (n-(n-1))x^{n-1} \\ &= \sum_{k=1}^{n-1}(n-k)x^k. \end{aligned}$$

That is $L_p(P_n, x) = \sum_{k=1}^{n-1}(n-k)x^k$.
This is true for all $n \geq 2$.
Hence proved.
Illustration 1.7 :

P_4

Consider the above graph G with vertices $V = \{1, 2, 3, 4\}$ and edges $E = \{12, 23, 34\}$.
Longest path between two vertices are given below.

From vertex 1 to other vertices :

$l(1,2) = \{12\}$ (length 1)
$l(1,3) = \{123\}$ (length 2)
$l(1,4) = \{1234\}$ (length 3)

From vertex 2 to other vertices :

$l(2,3) = \{23\}$ (length 1)
$l(2,4) = \{234\}$ (length 2)

From vertex 3 to other vertices :

$l(3,4) = \{34\}$(length 1)

Longest path of length 1 $l(G,1)$:

$l_p(G,1) = \{l(1,2), l(2,3), l(3,4)\}$
$l(G,1) = |l_p(G,1)| = 3.$

Longest path of length 2 : $l(G, 2)$.

$$\begin{aligned} l_p(G, 2) &= \{l(1, 3), l(2, 4), \} \\ l(G, 2) &= |l_p(G, 2)| = 2. \end{aligned}$$

Longest path of length 3 : $l(G, 3)$.

$$\begin{aligned} l_p(G, 3) &= \{l(1, 4)\} \\ l(G, 3) &= |l_p(G, 3)| = 1. \end{aligned}$$

There is no path of length more than 3.
that is, $l(G, k) = 0$ for $k \geq 4$.
The LP polynomial of C_n is

$$\begin{aligned} L_p(G, x) &= \sum_k l(G, k)x^k \\ &= l(G, 1)x + l(G, 2)x^2 + l(G, 3)x^3 \\ &= 3x + 2x^2 + x^3. \end{aligned}$$

Note 1.8 : The LP polynomial of path graph $P_n, n \geq 2$ for some nodes is given below :

1. $L_p(P_2, x) = x$
2. $L_p(P_3, x) = x^2 + 2x$
3. $L_p(P_4, x) = x^3 + 2x^2 + 3x$
4. $L_p(P_5, x) = x^4 + 2x^3 + 3x^3 + 4x$
5. $L_p(P_6, x) = x^5 + 2x^4 + 3x^3 + 4x^2 + 5x$.

Theorem 1.9 : The LP polynomial of the complete graph K_n is given by

$$L_p(K_n, x) = nC_2 x^{n-1} \quad \text{for} \quad n \geq 3.$$

Proof : Let K_n be the complete graph with n vertices and $\frac{n(n-1)}{2}$ edges.
The minimum cardinality of length of longest path between two vertices of K_n, is $n-1$ (that is, $\gamma_l(G) = n - 1$).
In K_n, there is no other longest path number greater than $n - 1$.

The number of longest path sets of K_n with length k is,

$$l(K_n, k_= nC_2; \quad \text{for} \;\; k = n-1 \;\; \text{and}$$

$$l(K_n, k) = 0; \quad \text{for} \;\; k > n-1.$$

The LP polynomial of K_n is

$$\begin{aligned} L_p(K_n, x) &= \sum_{k=\gamma_l(G)} l(K_n, k)x^k \\ &= \sum_{k=n-1} l(K_n, k)x^k \\ &= nC_2 x^{n-1}. \end{aligned}$$

That is, $L_p(K_n, x) = nC_2 x^{n-1}$.
This is true for all $n \geq 3$.
Hence proved.

Note 1.10 : The LP polynomial of complete graph $K_n, n \geq 3$ for some nodes is given below :

1. $L_p(K_3, x) = 3x^2$
2. $L_p(K_4, x) = 6x^3$
3. $L_p(K_5, x) = 10x^4$
4. $L_p(K_6, x) = 15x^5$.

Theorem 1.11 : The LP polynomial of the complete bipartite graph $K_{n,n}$ is given by

$$L_p(K_{n,n}, x) = \sum_{k=n+1}^{n+2} n(k-n+1)x^k \;\; \text{for‘} \; n \geq 2.$$

Proof : Let $K_{n,n}$ be the complete bipartite graph with n vertices and n^2 edges.
The minimum cardinality of the length of longest path between two vertices of $K_{n,n}$,$\gamma_l(G) = n+1$.
The number of longest path sets of $K_{n,n}$ with length k,

$$l(K_{n,n}, k) = n(k-n+1); n+1 < k < n+2 \;\; \text{for} \;\; n > 2.$$

The LP polynomial of $K_{n,n}$ is

$$\begin{aligned} L_p(K_{n,n},x) &= \sum_{k=\gamma_l(G)} l(K_{n,n},k)x^k \\ &= n[(n+1)-n+1]x^{n+1} + n[(n+2)-n+1]x^{n+2} \\ &= \sum_{k=n+1}^{n+2} n(k-n+1)x^k. \end{aligned}$$

That is

$$L_p(K_{n,n},x) = \sum_{k=n+1}^{n+2} n(k-n+1)x^k.$$

In other words

$$L_p(K_{n,n},x) = n^2x^{2n-1} + n(n-1)x^{2n-2}.$$

This is true for all $n \geq 2$.

Hence proved.

Note 1.12 : The LP polynomial of complete bipartite graph $K_{n,n}, n \geq 2$ is given below :

1. $L_p(K_{2,2},x) = 4x^3 + 2x^2$
2. $L_p(K_{3,3},x) = 9x^5 + 6x^4$
3. $L_p(K_{4,4},x) = 16x^7 + 12x^6$
4. $L_p(K_{5,5},x) = 25x^9 + 20x^8$.

Definition 1.13 : The Dutch windmill graph (friendship graph) is the graph obtained by taking n copies of the cycle graph C_3 with a vertex in common.

Theorem 1.14 : The LP polynomial of the Friendship graph F_n is given by

$$L_p(F_n,x) = 3nx^2 + 2n(n-1)x^4 \quad \text{for} \quad n \geq 1.$$

Proof : Let F_n be the graph obtained from the one point union of n copies of C_3.

The graph F_n has $2n+1$ vertices and $3n$ edges.

The minimum cardinality of length of the longest path between two vertices of F_n, $\gamma_l(G) = 2$.

Therefore in F_n, the longest path numbers are 2 and 4.

The number of longest path sets of F_n with length k are

$$l(F_n, 2) = 3n \quad \text{and} \quad l(F_n, 4) = 2n(n-1).$$

The LP polynomial of F_n is

$$\begin{aligned} L_p(F_n, x) &= \sum_{k=\gamma_l(G)} l(F_n, k)x^k \\ &= \sum_{k=2,4} l(F_n, k)x^k \\ &= l(F_n, 2)x^2 + l(F_n, 4)x^4 \\ &= 3nx^2 + 2n(n-1)x^3. \end{aligned}$$

That is, $L_p(F_n, x) = 3nx^2 + 2n(n-1)x^4$.

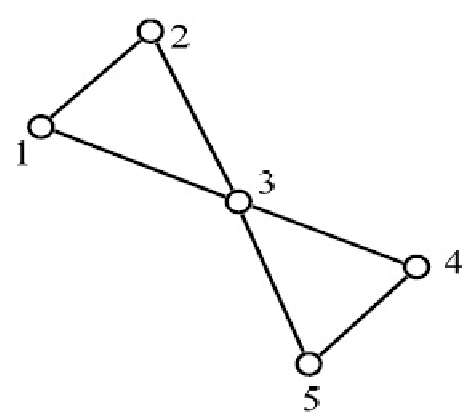
This is true for all $n \geq 1$.

Hence proved.

Note 1.15 : The LP polynomial of Friendship graph $F_n, n \geq 1$ is given below:

1. $L_p(F_1, x) = 3x^2$
2. $L_p(F_2, x) = 4x^4 + 6x^2$
3. $L_p(F_3, x) = 12x^4 + 9x^2$
4. $L_p(F_4, x) = 24x^4 + 12x^2$.

Illustration 1.16 :



F_2

Theorem 1.17 : The LP polynomial of the complete bipartite graph $K_{n,n}$ is given by

$$L_p(K_{n,n}, x) = \sum_{k=n+1}^{n+2} n(k-n+1)x^k \quad \text{for} \quad n \geq 2.$$

Proof : Let $K_{n,n}$ be the complete bipartite graph with n vertices and n^2 edges.
The minimum cardinality of length of longest path between two vertices of $K_{n,n}$, $\gamma_l(G) = n+1$.
The number of longest path sets of $K_{n,n}$ with length k

$$l(K_{n,n}, k) = n(k-n+1); \; n+1 \leq k \leq n+2 \quad \text{for} \quad n \geq 2.$$

The LP polynomial for $K_{n,n}$ is

$$\begin{aligned} L_p(K_{n,n}, x) &= \sum_{k=\gamma_l(G)} l(K_{n,n}, k)x^k \\ &= n[(n+1)-n-1]x^{n+1} + n[(n+2)-n+1]x^{n+2} \\ &= \sum_{k=n+1}^{n+2} n(k-n+1)x^k. \end{aligned}$$

That is, $L_p(K_{n,n}, x) = \sum_{k=n+1}^{n+2} n(k-n+1)x^k$.
In other words, $L_p(K_{n,n}, x) = n^2x^{2n-1} + n(n-1)x^{2n-2}$.
This is true for all $n \geq 2$.
Hence proved.
Note 1.18 : The LP polynomial of complete bipartite graph $K_{n,n}, n \geq 2$ is given below:

1. $L_p(K_{2,2}, x) = 4x^3 + 2x^2$
2. $L_p(K_{3,3}, x) = 9x^5 + 6x^4$
3. $L_p(K_{4,4}, x) = 16x^7 + 12x^6$
4. $L_p(K_{5,5}, x) = 25x^9 + 20x^8$.

Theorem 1.19 : The LP polynomial of Wheel graph W_n is given by $L_p(W_n, x) = nC_2x^{n-1}$ for $n \geq 4$.
Proof : Let W_n be a wheel graph has n vertices and $2(n-1)$ edges.
The minimum cardinality of length of the longest path between two vertices of W_n, $\gamma_l(G) = n-1$.

There is no other longest path number greater than $n-1$.

The number of longest path sets of W_n with length k,

$$l(W_n, k) = nC_2; \quad \text{for} \quad k = n-1$$

$$l(W_n, k) = 0; \quad \text{for} \quad k > n-1.$$

The LP polynomial of W_n is

$$L_p(W_n, x) = \sum_{k=\gamma_l(G)} l(W_n, k)x^k$$

$$= 7\sum_{k=n-1} l(W_n, , k)x^k$$

$$= nC_2 x^{n-1}.$$

That is, $L_p(W_n, x) = nC_2 x^{n-1}$.

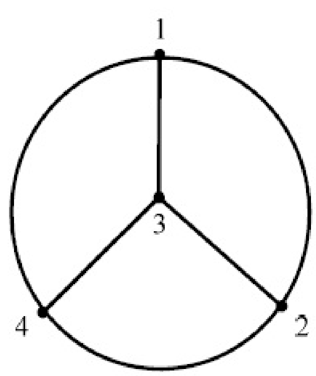
This is true for all $n \geq 4$.

Hence proved.

Note 1.20 : The LP polynomial of Wheel graph $W_n, n \geq 4$ for some nodes is given below:

1. $L_p(W_n, x) = 6x^3$
2. $L_p(W_5, x) = 10x^4$
3. $L_p(W_6, x) = 15x^5$
4. $L_p(W_7, x) = 21x^6$.

Illustration 1.21 :



$\mathbf{W_4}$

Theorem 1.22 : The LP polynomial of Triangular snake graph TS_n is given by

$$L_p(TS_n, x) = \sum_{k=4,k \text{ is even}}^{2n} 4(n+1-\frac{k}{2})x^k + 3nx^2 \quad \text{for} \quad n \geq 2.$$

Proof : Let TS_n be a triangular snake graph whose block-cut point graph is a path. Equivalently, it is obtained from a path $P = v_1, v_2, \cdots, v_{n+1}$ by joining v_i and v_{i+1} to a new vertex $u_1, u_2, u_3, \cdots, u_n$.
The graph TS_n has $2n+1$ vertices and $3n$ edges, where n is the number of blocks in the triangular snake.
Let $V(TS_n) = \{u_i/1 \leq i \leq n\} \ \cup \ \{v_i/1 \leq i \leq n-1\}$ and

$$E(TS_n) = \{u_iu_{i+1}/1 \leq i \leq n\} \cup \{u_iv_i/1 \leq i \leq n\} \cup \{v_iu_{i+1}/1 \leq i \leq n\}.$$

The minimum cardinality of length of the longest path between two vertices of TS_n, $\gamma_l(G) = 2$.
The number of longest path sets of TS_n with length k,

$$l(TS_n, k) = 3n; \quad \text{for} \quad k = 2$$

$$l(TS_n, k) = 4(n+1-\frac{k}{2}); \quad \text{for} \quad 4 \leq k \leq 2n, \quad k \quad \text{is even}/$$

The LP polynomial of TS_n is

$$\begin{aligned} L_p(TS_n, x) &= \sum_{k=\gamma_l(G)} l(TS_n, k)x^k \\ &= \sum_{k=4,k \ \text{is even}}^{2n} 4(n+1-\frac{k}{2})x^k + 3nx^2. \end{aligned}$$

That is

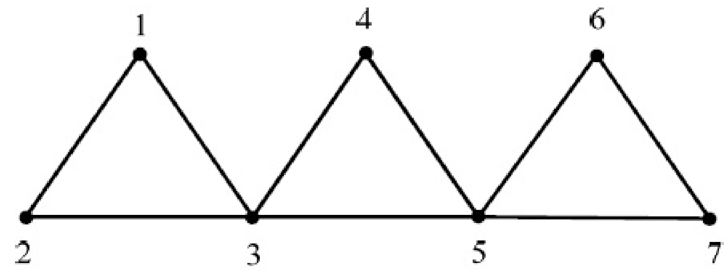
$$L_p(TS_n, x) = \sum_{k=4,k \ \text{is even}}^{2n} 4(n+1-\frac{k}{2})x^k + 3nx^2.$$

This is true for all $n \geq 2$.
Hence proved.
Note 1.23* : The LP polynomial of $TS_n, n \geq 2$ is given below:

1. $L_p(TS_2, x) = 4x^4 + 6x^2$

2. $L_p(TS_3, x) = 4x^6 + 8x^4 + 9x^2$
3. $L_p(TS_4, x) = 4x^8 + 8x^6 + 12x^4 + 12x^2$
4. $L_p(TS_5, x) = 4x^{10} + 8x^8 + 12x^6 + 16x^4 + 15x^2$.

Illustration 1.24 :



TS$_3$

Theorem 1.25 : The LP polynomial of graph $P_n \times C_3$ given by $L_p(P_n \times C_3, x) = 3nC_2x^{2n-1}$ for $n \geq 2$.

Proof : Let $P_n \times C_3$ be the graph obtained by joining 3 copies of P_n with n copies of C_3.

Let the vertices of $P_n \times C_3$ be $\{a_{ij}/1 \leq i \leq 3, 1 \leq j \leq n\}$ and the edges of $P_n \times C_3$ be $\{a_{ij}a_{ij+1}/1 \leq i \leq 3, 1 \leq j \leq n-1\} \cup \{a_{1i}a_{2i}/1 \leq i \leq n\} \cup \{a_{1i}a_{3i}/1 \leq i \leq n\} \cup \{a_{2i}a_{3i}/1 \leq i \leq n\}$.

The graph $P_n \times C_3$ has $3n$ vertices and $6n-3$ edges.

In $P_n \times C_3, \gamma_l(G) = 3n-1$.

The number of longest path sets of $P_n \times C_3$ with length k,

$$l(P_n \times C_3, k) = 3nC_2; \quad \text{for} \quad k = 3n-1$$

$$l(P_n \times C_3, k) = 9; \quad \text{for} \quad k > 3n-1.$$

The LP polynomial of $P_n \times C_3$ is

$$\begin{aligned} L_p(P_n \times C_3, x) &= \sum_{k-\gamma_l(G)} l(P_n \times C_3, k)x^k \\ &= 3nC_2x^{3n-1}. \end{aligned}$$

That is, $L_p(P_n \times C_3, x) = 3nC_2x^{3n-1}$.

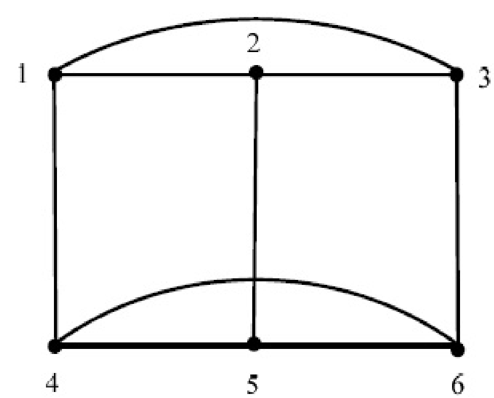
This is true for all $n \geq 2$.

Hence proved.

Note 1.26 : The LP polynomial of $P_n \times C_3, n \geq 2$ is given below:

1. $L_p(P_2 \times C_3, x) = 15x^5$
2. $L_p(P3 \times C_3, x) = 36x^8$
3. $L_p(P_4 \times C_3, x) = 66x^{11}$
4. $L_p(P_5 \times C_3, x) = 105x^{14}$.

Illustration 1.27 :



P₂ x C₃

Theorem 1.28 : The LP polynomial of the graph $C_n \times C_n$ is $L_p(C_n \times C_n, x) = (n^2 - 1)C_2x^{n^2-1}$ for $n \geq 4$.

Proof : Let $C_n \times C_n$ be the graph obtained by joining n copies of C_n with n copies of C_n.

The graph $C_n \times C_n$ has $2n^2$ edges and n^2 vertices.

Let the vertices of $C_n \times C_n$ be $\{a_{i1}, a_{i2}, a_{i3}, \cdots, a_{in}/1 \leq i \leq n\}$.

Here each row and each column is respectively adjacent and $\{a_{1i}, a_{ni}/1 \leq i \leq n\}$ are adjacent and $\{a_{i1}, a_{in}/1 \leq i \leq n\}$ are adjacent.

The minimum cardinality of length of the longest path between two vertices of $C_n \times C_n$, $\gamma_l(G) = n^2 - 1$.

There is no other longest path greater than $n^2 - 1$.

The number of longest path of length k,

$$l(C_n \times C_n, k) = (n^2 - 1)C_2; \quad \text{for} \;\; k = n^2 - 1$$

$$l(C_n \times C_n, k) = 0; \quad \text{for} \;\; k > n^2 - 1.$$

The LP polynomial of $C_n \times C_n$ is

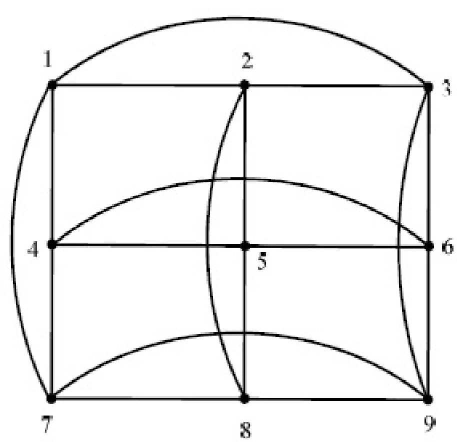
$$\begin{aligned} L_p(C_n \times C_n, x) &= \sum_{k=\gamma_l(G)} l(C_n \times C_n, k)x^k \\ &= \sum_{k=n^2-1} l(C_n \times C_n, k)x^k \\ &= (n^2 - 1)C_2 x^{n^2-1}. \end{aligned}$$

That is, $L_p(C_n \times C_n, x) = (n^2 - 1)C_2 x^{n^2-1}$.

This is true for all $n \geq 3$.

Hence proved.

Illustration 1.29 :



C₃ x C₃

Note 1.30 : The LP polynomial of $C_n \times C_n, n \geq 3$ is given below:

1. $L_p(C_3 \times C_3, x) = 36x^8$

2. $L_p(C_4 \times C_4, x) = 120x^{15}$

3. $L_p(C_5 \times C_5, x) = 3000x^{24}$

4. $L_p(C_6 \times C_6, x) = 630x^{35}$

Theorem 1.31 : The LP polynomial of the graph A_n is $L_p(A_n, x) = 2nC_2x^{2n-1}$ for $n \geq 3$.

Proof : Let A_n be an antiprism graph which corresponds to the skeleton of an antiprism. A semiregular polyhedron (antiprism) constructed with $2n$-gons and $2n$ triangles. The graph A_n has $2n$ vertices and $4n$ edges.

The minimum cardinality of length of the longest path between two vertices of A_n, $\gamma_l(G) = 2n - 1$.

The number of longest path of length k,

$$l(A_n, k) = 2nC_2 \ ; \quad \text{for} \quad k = 2n - 1$$

$$l(A_n, k) = \text{for} \quad k > 2n - 1.$$

The LP polynomial of A_n is

$$\begin{aligned} L_p(A_n, x) &= \sum_{k=\gamma_l(G)} l(A_n, k)x^k \\ &= \sum_{k=2n-1} l(A_n, k)x^k \\ &= 2nC_2x^{2n-1}. \end{aligned}$$

That is, $L_p(A_n, k)x^k = 2nC_2x^{2n-1}$.

This is true for all $n \geq 3$.

Hence proved.

Note 1.32 : The LP polynomial of $A_n, n \geq 3$ is given below:

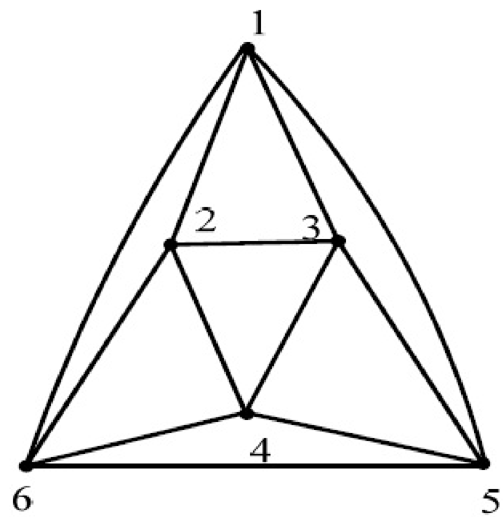
1. $L_p(A_3, x) = 15x^5$

2. $L_p(A_4, x) = 28x^7$

3. $L_p(A_5, x) = 45x^9$

4. $L_p(A_6, x) = 66x^{11}$.

Illustration 1.33 :



$\mathbf{A_3}$

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