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TOTAL PENALTY COST IN THE FUZZY SCHEDULE ON PARALLEL MACHINES

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Abstract

The main objective of this paper is minimizing the total penalty cost in the fuzzy schedule of the jobs on the parallel machines. This cost is composed of the total earliness and the total tardiness cost.

1. Introduction

The study of earliness and tardiness penalties in scheduling models is a relatively recent area of inquiry. For many years scheduling research focused on single performance measures. Most of the literature deals with regular measure such as mean flow time mean lateness, percentage of jobs tardy, mean tardiness etc. in deterministic time but the environment in modern society is neither fixed nor probabilistic. So, here we are

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Key Words : *Parallel Machines, fuzzy scheduling, Total penalty cost, Total earliness cost, Total tardiness cost.*

considering fuzzy environment i.e. the processing time of each job is in indeterminist environment.

The concept of penalizing both earliness and tardiness has spawned a new and rapidly developing line of research in the scheduling field. Because the use of both earliness and tardiness penalties in fuzzy environment give rise to a non-regular performance measure, it has led to new methodological issues in the design of solution procedures, [1].

In this paper an algorithm is developed here to minimize the total penalty cost due to earliness or lateness of sequence jobs in fuzzy environment on the parallel machines by using Average High Ranking (AHR) for the processing time when the processing time in the triangular fuzzy number and the Graded Mean Integration Representation when the processing time in the trapezoidal fuzzy number.

Also, we justified this algorithm by numerical examples.

Finally we can take advantage of this research and then apply it in laboratories and production plants that machines run parallel so as to reduce the time and increase production.

Parallel Machines

Parallel machine scheduling is a kind of important multi machine scheduling in which every machine has same work function and every job can be processed by any of the available machines, that is mean the parallel machine scheduling consider several available identical machines to execute a set of jobs $N = \{1, 2, \cdots, n\}$. It is a widely studied optimization problem, [2].

The following figure illustrate four parallel machines with n jobs, [3].

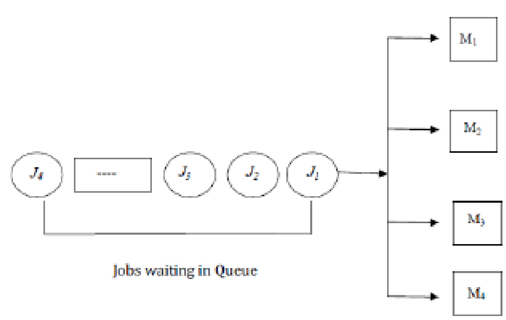

Figure (1) illustrate four parallel machines with n jobs

3. Fuzzy Processing Time

The processing time of a job can vary in many ways, may be due to environmental factor or due to the different work places. We find that when a contractor takes the work from a department, he calculates total expenditure at the time of allotment. But due to many factors like non available of labor, weather not favorable, or sometimes abnormal conditions, cost may vary. Hence due to these reasons work can be completed late and creates due date problem i.e. order cant be delivered on time, on the other hand if the work completes before the due time it arises the inventory problem, [1].
In this paper processing time of a job considered in the triangular fuzzy number and trapezoidal fuzzy number.

4. Triangular Fuzzy Number

Triangular fuzzy number is a fuzzy number represented with three situations as $\tilde{A} = (a_1, a_2, a_3)$ where a_1 and a_3 denote the lower and upper limits of support of a fuzzy set $\tilde{A}$. The membership value of the x denoted by $\mu_{\tilde{A}}(x), x \in R^+$ can be calculated according to the following formula

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & x \leq a_1 \\ \frac{x-a_1}{a_2-a_1} & a_1 < x < a_2 \\ \frac{a_3-x}{a_3-a_2} & a_2 < x < a_3 \\ 0 & x \geq a_3 \end{cases}$$

The following figure represents a triangular fuzzy number, [4]

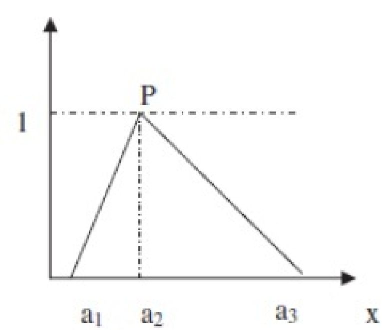

Figure (2) Triangular Fuzzy Number

5. Average High Ranking (AHR)

The processing time of the jobs are calculated by using Yager's Average High Ranking formula (AHR) when the processing time in the three situations (a, b, c) where a in favorable (High) condition, b Normal (Medium) condition and c in worse (Bad) condition, [5]

$$(AHR) = \frac{3b + c - a}{3}.$$

6. Trapezoidal Fuzzy Number

Trapezoidal fuzzy number is a fuzzy number represented with four situations as $\tilde{A} = (a_1, a_2, a_3, a_4)$. The membership value of the x denoted by $\mu_{\tilde{A}}(x), x \in R^+$ can be calculated according to the following formula

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & x < a_1, x > a_4 \\ \frac{x-a_1}{a_2-a_1} & a_1 \leq x \leq a_2 \\ 1 & a_2 \leq x \leq a_3 \\ \frac{a_4-x}{a_4-a_3} & a_3 \leq x \leq a_4 \end{cases}$$

The following figure represents a trapezoidal fuzzy number, [6]

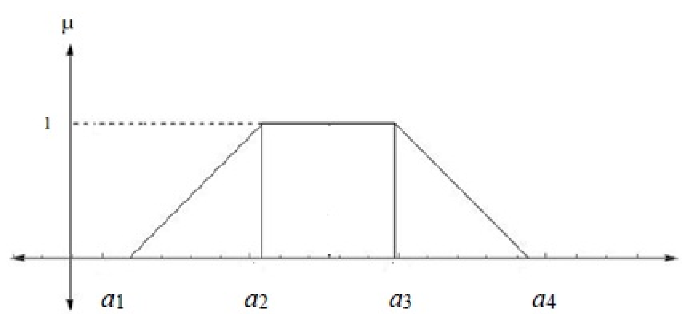

Figure (3) Trapezoidal Fuzzy Number

Graded Mean Integration Representation (GMIR)

The graded mean integration representation method based on the integral value of graded mean h level of generalized fuzzy number for fuzzifying generalized fuzzy member. The generalized fuzzy number is defined as follows:

Suppose A is a generalized fuzzy number. It is described as any fuzzy subset of the real line R, whose membership function μA satisfies the following conditions.

- $\mu_{\tilde{A}}(x)$ is a continuous mapping from R to $[0,1]$,
- $\mu_{\tilde{A}}(x) = 0, -\infty < x \leq\cong a_1$,
- $\mu_{\tilde{A}}(x) = L(x)$ is strictly increasing on $[a_1, a_2]$,
- $\mu_{\tilde{A}}(x) = w_A, a_2 \leq x \leq a_3$,
- $\mu_{\tilde{A}}(x) = R(x)$ is strictly decreasing on $[a_3, a_4]$,
- $\mu_{\tilde{A}}(x) = 0, a_4 \leq x < \infty$

where $0 < w_A \leq 1$ and a_1, a_2, a_3 and a_4 are real numbers.

Generalized fuzzy numbers are denoted as $A = (a_1, a_2, a_3, a_4; w_A)_{LR}$. The graded mean h-level value of generalized fuzzy number $A = (a_1, a_2, a_3, a_4; w_A)_{LR}$ is given by $\frac{h}{2}\{L^{-1}(h) + R^{-1}(h)\}$. Then the graded Mean Integration Representation of $P(\tilde{A})$ with grade $\mu_{\tilde{A}}(x)$, where

$$P(\tilde{A}) = \frac{\int_0^{w_A} \frac{h}{2}\{L^{-1}(h) + R^{-1}(h)\}dh}{\int_0^{w_A} h\ dh}$$

where $0 < h \leq w_A$ and $0 < w_A \leq 1$.

Let A be a trapezoidal fuzzy number and denoted as $A = (a_1, a_2, a_3, a_4)$. Then we can get the graded mean Integration Representation of A by the formula, [7]

$$(GMIR) = P(\tilde{A}) = \frac{\int_0^1 \frac{h}{2}[(a_1 + a_4) + h(a_2 - a_1 - a_4 + a_3)]dh}{\int_0^{w_A} h\ dh} = \frac{a_1 + 2a_2 + 2a_3 + a_4}{6}.$$

Assumption and Notation

- AHR — Average high ranking of the processing time (a, b, c).
- GMIR — Graded mean integration representation of the processing time (a, b, c, d)
- P_i — Processing time
- d_i — Due date for the job i.
- c_i — Completion time of job i.
- T_i — Max. $\{0, c_i - d_i\}$.
- E_i — Max. $\{0, d_i - c_i\}$.
- sl_i — Slack time of job i.
- e_i — penalty per unit time for the earliness of job i.
- l_i — penalty per unit time for the tardiness of job i.

An important special case in the family of E/T problems involves minimizing the sum of absolute deviations of job completion time form a DDD having processing time in fuzzy environment. In particular, the objective function can be written as

$$f(s) = \Sigma|c_i, d_i| = \Sigma(E_i + T_i).$$

When we write the objective function in this form, it is clear that earliness and tardiness are penalized at the rate e_i and l_i for all the jobs.

Algorithm

Step 1 : Find average high ranking (AHR) of the fuzzy processing time of all the jobs in the triangular fuzzy number (a_1, a_2, a_3) by the formula $[3a_2 + a_3 - a_1]/3$, we denoted it by A_i. Find graded mean integration representation (GMIR) of the fuzzy processing time of all the jobs in the trapezoidal fuzzy number (a_1, a_2, a_3, a_4) by the formula $[a_1 + 2a_2 + 2a_3 + a_4]/6$, we denoted it by G_i.

Step 2 : Find the slack time of all the jobs $sl_i = |A_i - d_i|$ if the processing time in the triangular fuzzy number and $sl_i = |G_i - d_|$ if the processing time in the trapezoidal fuzzy number.

Step 3 : Arrange the jobs in increasing order of their slack time. If two jobs have the same slack time then considers the jobs of lowest processing time at the earlier position.

Step 4 : Using the sequence obtained in step 3 to find the total penalty of all the jobs using earliness (e_i) and lateness (l_i) penalty cost to find.

2. Numerical Examples

Example (1) : Consider 7-jobs with processing time in fuzzy environment with distinct due date on four parallel machines. Penalty cost (e_i) for earliness and (l_i) lateness is given in the following table.

Table (1) Data for example (1)

Job	P_i	d_i	e_i	l_i
1	(8,9,10)	12	2	3
2	(15,16,17)	18	2	3
3	(8,9,10)	17	2	3
4	(5,6,7)	10	2	3
5	(9,10,11)	14	2	3
6	(9,10,11)	8	2	3
7	(10,11,12)	13	2	3

Solution : By applying the steps (1) and (2) of the algorithm we get the following data.

Table (2) Data for steps (1) and (2) for example (1)

Job	P_i	A_i	d_i	sl_i	e_i	l_i
1	(8,9,10)	29/3	9	2/3	2	3
2	(15,16,17)	50/3	16	2/3	2	3
3	(8,9,10)	29/3	8	5/3	2	3
4	(5,6,7)	20/3	4	8/3	2	3
5	(9,10,11)	32/3	10	2/3	1	3
6	(9,10,11)	32/3	9	5/3	2	3
7	(10,11,12)	35/3	10	5/3	2	3

From step (3) of the algorithm the **optimal sequence** is **1-5-2-3-6-7-4.**

The following table shows the total flow time of the system and the total optimized penalty cost due to earliness/tardiness of the jobs.

Table 3 : Total optimized penalty cost due to earliness/tardiness of the jobs for example (1)

Job	M_1	M_2	M_3	M_4	d_i	sl_i	Cost
1	0-29/3				9	2/3	(2/3)(2) = 4/3
5		0-32/3			10	2/3	(2/3)(3) = 2
2			0-50/3		16	2/3	(2/3)(3) = 2
3				9-29/3	8	5/3	(5/3)(3) = 5
6	29/3 -61/3				9	(34/3)	(34/3)(3) =34
7				29/3-64/3	10	34/3	(34/3)(3) = 34
4		32/3-52/3			4	40/3	(40/3)(3) = 40

Total penalty cost = 118.33.

Example 2 : Consider 5-jobs with processing time in fuzzy environment with distinct due date on three parallel machines. Penalty cost (e_i) for earliness and (l_i) lateness is given in the following table.

Table 4 : Data for Example 2

Job	P_i	d_i	e_i	l_i
1	(8,9,11,13)	10	2	3
2	(8,9,10,11)	9	2	3
3	(6,8,9,10)	8	2	3
4	(9,10,12,14)	11	2	3
5	(10,12,13,14)	12	2	3

Solution :

By applying the steps (1) and (2) of the algorithm we get the following data.

Table 5 : Data for steps (1) and (2) for Example 2

Job	P_I	G_i	d_i	sl_i	e_i	l_i
1	(8,9,11,13)	61/6	10	1/6	2	3
2	(8,9,10,11)	57/6	9	1/2	2	3
3	(6, 8, 9,10)	50/6	8	1/3	2	3
4	(9,10,12,14)	67/6	11	1/6	2	3
5	(10,12,13,14)	74/6	12	1/3	2	3

From step (3) of the algorithm the **optimal sequence is 1-4-3-5-2.**

The following table shows the total flow time of the system and the total optimized penalty cost due to earliness/tardiness of the jobs.

Table 6 : total optimized penalty cost due to earliness/tardiness of the jobs for Example 2

Job	M_1	M_2	M_3	d_i	sl_i	Cost
1	0-61/6			10	1/6	$(1/6)(2) = 1/3$
4		0-676		11	1/6	$(1/6)(3) = 1/2$
3			0-50/6	8	1/3	$(1/3)(3) = 1$
5			60/6-124/6	12	52/6	$(52/6)(3) = 26$
2	61/6-118/6			9	64/6	$(64/6)(3) = 32$

Total penalty cost $= 59.83$.

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