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A COMPARITIVE STUDY ON P-FUZZY BIRINGS USING BIRINGS

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Abstract

We give definition for P-Fuzzy algebra on the Algebra A, Fuzzy rings, P-fuzzy rings and P-fuzzy sub birings. Then we study the connection of P-fuzzy subalgebras with fuzzy Bi-rings. Some Properties of P-Fuzzy subrings are stated and proved.

Introduction

Fuzzy sets originated in a seminar paper by Lotfi A. Zadeh in 1965. A fuzzy set A is defined as a map from A to the real unit interval $I = [0, 1]$. It has grown by leaps and bounds and innumerable numbers have appeared in various journals. Applications of fuzzy sets and fuzzy logic were introduced by Mamdani in 1975.

A group is an algebraic system with one binary operation. Generalizing Z along with two operations $+$ and $\cdot$ using the properties distributive and associative under $\cdot$, the concept of Ring arises.

Key Words : *P-fuzzy subalgebras, P-fuzzy rings and P-fuzzy subrings.*

The definitions given to the concept of P-Fuzzy set, P-fuzzy subalgebra on the algebra A and fuzzy rings also involve different operations. Now we unify and generalize these definitions using P-fuzzy Birings. The theorems proved also highly generalize the existing ones.

1. Preliminaries

Definition 1.1 : If $(P, \leq)$ is a partially ordered set and X is a nonempty set, then any mapping $A \to P$ is a P-fuzzy subset of A or a P-fuzzy set on A, denoted by P^A.

Definition 1.2 : Let A be a non-empty set $*$

(i) $(P, *)$ be a monoid, where 1 is the unity for .

(ii) $(P, \leq)$ be a Partially ordered set aith 1 as the greatest element.

(iii) $*$ be isotone in both variables.

P always denotes such a structure.

Definition 1.3 : A P-fuzzy set $\mu \in P^A$ is called a P-fuzzy algebra or fuzzy subalgebra on the algebra A, if

- For any $n - ary(n \geq 1)$ operation $f \in F$
  $\mu(f(x_1, \cdots, x_n) \geq \mu(x_1) * \cdots * \mu(x_n)$ for all $x_1, \cdots, x_n \in A$.
- For any constant (nullary operation) C $\mu(c) \geq \mu(x)$ for all $x \in A$.

(**Note** : If A is a group then nullary operation is consequence of $n - ary$ operation).

Definition 1.4 : Let G be a group. A fuzzy subset μ of a group G is called a fuzzy subgroup of the group G if

- $\mu(xy) \geq \min(\mu(x), \mu(y))$ for every $x, y \in G$ and
- $\mu(x^{-1}) = \mu(x)$ for every $x \in G$.

Definition 1.5 : A fuzzy subset μ of a ring R is called a Fuzzy subring of R if for all $x, y \in R$. then

- $\mu(x - y) \geq \min(\mu(x), \mu(y))$
- $\mu(xy) \geq \min(\mu(x), \mu(y))$ for all $x, y \in R$.

Proposition 1.6 : Let μ be a fuzzy subring of $(R, +, .)$ then

- μ is a fuzzy subgroup of $(R, +)$
- μ is a fuzzy subgroupoid of $(R, \cdot)$.

2. P-Fuzzy Subbi-Ring

Definition 2.1 : A P-fuzzy subset $\mu \in P^A$ is called a P-Fuzzy subring of R of the algebra A if for all $x, y \in R$ then

- $\mu(x - y) \geq (\mu(x), \mu(y))$
- $\mu(xy) \geq \min(\mu(x), \mu(y))$ for all $x, y \in R$.

Definition 2.2 : A non-empty set $(R, +, .)$ with two binary operations '$+'$ and '$\cdot'$ is called P-Fuzzy biring if $R = R_1 \cup R_2$ where R_1 and R_2 are proper subsets of R and

- $(R_1, +, .)$ is a P-Fuzzy ring
- $(R_2, +, .)$ is a P - Fuzzy ring.

Note : A P-Fuzzy biring R is called finite if R contains only finite number of elements. If R has infinite number of elements then R is of infinite order.

Definition 2.3 : A P-Fuzzy biring $R = R_1 \cup R_2$ is called P-Fuzzy commutative biring if both R_1 and R_2 are commutative rings. Even if one of R_1 or R_2 is not a commutative biring then the biring is a non-commutative biring. Also the biring R has a monounit if a unit exists which is common to both R_1 and R_2. If R_1 and R_2 are rings which has separate unit then the biring $R = R_1 \cup R_2$ is a biring with unit.

Definition 2.4 : Let $R = (R_1 \cup R_2, +, .)$ be a biring. The map $\mu : R \to [0, 1]$ is called P-Fuzzy subbiring of the ring R if there exists two fuzzy subsets μ_1 (of R_1) and $-\mu_2$ (of R_2) such that

- $(\mu_1, +, .)$ is a P-Fuzzy Subring of $(R_1, +, .)$
- $(\mu_2, +, .)$ is a P-Fuzzy Subrinng of $(R_2, +, .)$
- $\mu = \mu_1 \cup \mu_2$.

3. Propertie of P-Fuzzy Sub bi-ring

Theorem 3.1 : Every t-level subset of a P-Fuzzy sub-birinng μ of a biring $R = R_1 \cup R_2$ need not in general be a sub-biring of the biring R.

Proof : By definition $R = (R_1 \cup R_2, +, .)$ be a biring and $\mu = \mu_1 \cup \mu_2$ be a P-Fuzzy sub bi-ring of the biring R. Then the bilevel subset of the P-Fuzzy sub-biring μ of the biring R is $G^t\mu = G_1^t\mu_1 \cup G_2^t\mu_2$ for every $t \in \{0, \min\{\mu_1(0), \mu_2(0)\}\}$.
If $t \in \{0, \min\{\mu_1(0), \mu_2(0)\}\}$ then the bilevel subset be a sub-biring.
If $t \in \{0, \min\{\mu_1(0), m_2(0)\}\}$ then the bilevel subset need not in general be a sub-biring of the biring R.

Theorem 3.2 (Characterization Theorem) : Let R be a biring where $R = R_1 \cup R_2$ and $\mu = \mu_1 \cup \mu_2$ be a P-Fuzzy sub-biring of the biring R. A non empty subset $S = S_1 \cup S_2$ of R is a P-Fuzzy subring of R if and only if $R_1 \cup S = S_1$ and $R_2 \cup S = S_2$ are P-Sub biring of R_1 and R_2.

Proof : Let R be a biring where $R = R_1 \cup R_2$ and $\mu = \mu_1 \cup \mu_2$ be a P-Fuzzy sub-biring of the biring R.
Consider a non empty subset $S = S_1 \cup S_2$ of R is a P-Fuzzy subring of R. It is clear that R and S are P-Fuzzy sub-biring, also S_1 and S_2 are sub biring.

$\Rightarrow R_1 \cup S = S_1$ is a P-Fuzzy subring, since S is a P-Fuzzy subring,

$\Rightarrow R_2 \cup S = S_2$ is a P-Fuzzy subring, since S? is a P-Fuzzy subring.

Conversly, $R_1 \cup S = S_1$ and $R_2 \cup S = S_2$ are P-Fuzzy subring.

- S_1 and S_2 is a P-Fuzzy subring, since S is a P-Fuzzy subring.
- $S = S_1 \cup S_2$ of R is a P-Fuzzy subring of R.

Theorem 3.3 : Every P-Fuzzy sub-biring of a ring R is a P-Fuzzy subring of the ring R and not conversely.

Proof : Using the definition of P-Fuzzy subbiring, Let $R = (R_1 \cup R_2, +, .)$ be a biring. The map $\mu : R \to [0, 1]$ is called P- Fuzzy subbiring of the ring R if there exists two fuzzy subsets μ_1 (of R_1) and μ_2 (of R_2) such that

- $(\mu_1, +, .)$ is a P-Fuzzy Subring of $(R_1, +, .)$
- $(\mu_2, +, .)$ is a P-Fuzzy Subring of $(R_2, +, .)$
- $\mu = \mu_1 \cup \mu_2$.

Every P-fuzzy sub-biring of a ring R is a P-fuzzy subring of R.
Conversely, if μ is a P-fuzzy subring of R and there does not exist two P-fuzzy subring of R.

Theorem 3.4 : If $\mu = \mu_1 \cup \mu_2$ is any P-Fuzzy sub-biring of the biring $R = R_1 \cup R_2$ and if $\mu(x) < \mu(y)$ for some $x, y \in R$ then $\mu(x-y) = \mu(x) = \mu(y-x)$.

Proof : By the definition of P-Fuzzy subbiring, let $R = (R_1 \cup R_2, +, .)$ be a biring. The map $\mu : R \to [0,1]$ is called P-Fuzzy subbiring of the ring R if there exists two fuzzy subsets μ_1 (of R_1) and μ_2 (of R_2) such that

- $(\mu_1, +, .)$ is a P-Fuzzy Subring of $(R_1, +, .)$
- $(\mu_2, +, .)$ is a P-Fuzzy Subring of $(_R2, +, .)$
- $\mu = \mu_1 \cup \mu_2$.

Since under the operation addition there exists the inverse function subtraction $\to f = \{+, -\}$.
Since $x.y \in A$

- $\mu(x-y) \geq \min(\mu(x), \mu(y)) \geq \mu(x)$ (since $\mu(x) < \mu(y)$) (1)

  Similarly

- $\mu(y-x) \geq \min(\mu(x), \mu(y)) \geq \mu(x)$ (since $\mu(x) < \mu(y)$) (2)

Also under the operation

$\mu(xy) \geq \min(\mu(x), \mu(y)) \geq \mu(x)$ (since $\mu(x) < \mu(y)$). (3)

From result (1), (2) and (3)

$\Rightarrow$ If $\mu(x) < \mu(y)$ then $\mu(x-y) = \mu(x) = \mu(y-x)$.

Threorem 3.5 : The intersection of two P-fuzzy subbirings of a ring R is a P-fuzzy subbirings of R.

Theorem 3.6 : The union of the two P-fuzzy subirings of a ring R need not be a ring R.

Proof : Let A' and B' be two P-fuzzy subbirings of a ring R. Also one is not contained in the other and A' and B' are non-empty sets.
Consider $A' \cup B' = C'$.

Now, let e be the identity element of A' under the operation $+$ and satisfy the property of abelian under the same operation. Similarly, e' be the identity element of B' under the operation $+$ and satisfy the property of abelian under the same operation.
By definition of ring, A' and B' are associative under . and A' and B' satisfies distributive law.
If $e = e'$ then $A' \cup B'$ is again a P-fuzzy ring, otherwise contradicts.
Theorem 3.7 : Let R be a P-fuzzy finite biring and S be a P-fuzzy subring of R then the order of S divides order of R.

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