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TRANSMISSION OF INTERNAL GRAVITY WAVE TUNNELLING IN THE CONDUCTING STRATIFIED REGION ALONG WITH ROTATION

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Abstract

Transmission of internal gravity waves is investigated in the stratified regions for a conducting rotating fluid. We group the wave solution as upward and downward propagating waves using group velocity and energy approaches and test the wave solution in the different density barrier regions. The analytic solutions are derived for the weakly stratified rotating conducting waves and are analysed in the different density barrier regions for the evanescence of the waves in each region with varying density stratification. The transmission of the waves in the mixed region bounded by the discontinuities in the density profile is more compared to the region of uniform density due to the linear resonance of waves in the mixing region and shows that when the regions are weakly stratified the transmission is more otherwise waves strongly reflect from the weakly stratified region. This paper shows that the transmission of internal gravity waves is in the horizontal region along the fluid lines rather than the vertical direction and the reduction in the transmission of the waves is due to the conducting and rotating properties of the fluid along with the weakly density stratification.

Key Words : *Transmission, Evanescence, Stratified flows, Alfvén waves, Internal gravity waves, Density barrier.*

AMS Subject Classification : 76B55; 76B60; 76B70.

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1. Introduction

Waves are important solutions of the atmospheric system. The studies of the ducting of gravity waves motions in the atmosphere and oceans was first considered by Eckart [1] arising from the regions of thermal ducting(layers of increased static stability, N_0^2). It is generally accepted that convection in the troposphere generates gravity waves that propagate into the stratosphere, mesosphere and thermosphere and dynamical states of theses regions [2]. However, it is not known exactly how this occurs, what types of waves are generated. Eckart's resonences were theoretically derived by Fritts and Yuan[3]. Internal gravity waves in the upper atmosphere also play an important part in the production of certain ionospheric phenomena [4]. A full understanding of their role will depend in part on an understanding of the propagation conditions met at all levels in the atmosphere, and more specifically, of the part played by reflection and ducting [5].

The studies of gravity wave ducting in the ocean, the laboratory and the atmosphere have revealed that the formation of ducting of gravity wave activity to be important at smaller scales as well [3]. Internal gravity waves transport energy and momentum in density-stratified fluids on relatively fast time scales. These waves are responsible for a variety of processes in the mesosphere and lower thermosphere: generation of turbulence, formation of general circulation pattern of the atmosphere, deposition of net momentum, eddy conduction of heat, mixing of atmospheric constituents, fluctuation in atmospheric drag, and nonlinear interaction with tidal/planetary waves leading to variability of planetaryscale motions, momentum and energy transfer from the troposphere to the middle and upper atmosphere[22]. Perhaps most important is their role in determining the mean flow of the atmosphere.

Evanescence of wave in a certain region can also cause wave ducting. Ducting of gravity wave activity is likely to be a common occurrence in the atmosphere and the oceans and may provide a means of transporting vertically the energy and momentum associated with wave motions [3, 4]. Strong density and wind gradients or a boundary such as earths surface can trap gravity waves [5]. This phenomenon is called gravity wave ducting in which the majority of wave energy resides. The variation in the buoyancy frequency causes a natural filtering mechanism for gravity waves [8]. The ray theory[12] also predicts that the waves reflect from a level where the wave frequency matches the buoyancy frequency, N.

The solar interior also has been recognized as a potential source region for propagating gravity waves. Earlier, it has been thought that low-frequency fluid motions near the base of the convection zone can generate internal gravity waves that are capable of propagating into the radiative interior of the Sun. Interest in such waves has been renewed of late by suggestions that the angular momentum transport by them can account for the uniform rotation of the radiative interior, as inferred from helioseismic measurements [6].

Sutherland and Yewchuk [7] have derived an analytic theory for internal gravity wave tunnelling through a weakly stratified fluid. They have obtained the transmission co-efficient of internal waves crossing a weakly stratified region. This theory provides quantitative predictions of partial reflection and transmission of internal waves incident upon a weakly stratified layer.

This paper provides an analytic prediction for the transmission coefficient of internal Alfv*é*en -gravity waves crossing a region in which N^2 is reduced (weakly stratified region) for conducting and rotating fluid. The transmission co-efficient of an internal Alfv*é*en-gravity wave crossing over a barrier is computed. The effect of the rotation and magnetic field on the transmission co-efficient is studied.

2. Mathematical Formulation

We consider three dimensional motion of an electrically conducting rotating inviscid fluid with variation being in the x and z directions (i.e. horizontal and vertical directions respectively). The fluid is stratified and the stratification of the mean flow may then be described in terms of a single parameter which vary with the vertical height z, i.e. the Brunt-väisälä frequency defined by $N(z) = -\left(\frac{g}{\rho_0}\frac{d\rho_0}{dz}\right)^{\frac{1}{2}} = (g\beta)^{\frac{1}{2}}$ where $\beta = \left(\frac{1}{\rho_0}\frac{d\rho_0}{dz}\right)$. The density ρ of the fluid is considered to be uniform in the first two cases but the mixed region is bounded by discontinuous density profile. Initially the fluid is assumed to be in the state of rest. Since we consider atmospheric and astrophysical phenomena, although the flow velocities are rather small, the Reynolds number and Magnetic Reynolds number are quite large and hence an inviscid perfectly conducting flow model would appear to be a reasonable one. The rotation of the earth may not be neglected which is one of contributing factor. The perturbation of magnetic field from the horizontal uniform basic magnetic field, is considered to be very small which is of critical importance to the present analysis. It can be shown that under these assumptions the vertical disturbance velocity $\hat{\omega}$ satisfies the following linearized equation:

$$\frac{d^2\hat{\omega}}{dz^2} + \alpha^2 \frac{\left(\frac{N^2}{\omega^2 - k^2A^2} - 1\right)}{\left(1 - \frac{\Omega^2\omega^2}{\omega^4 - k^4A^4}\right)}\hat{\omega} = 0. \tag{1}$$

Here, the stratification of the mean flow is described in terms of a single parameter which may vary with z, the Brunt-väisälä frequency N, Ω

is the rotational parameter and the magnetic field is described by another parameter A, which is the Alfvéen velocity defined by $A = \left(\frac{\mu H_0^2}{\rho_0}\right)^{\frac{1}{2}}$ and $\alpha = (k^2 + l^2)^{\frac{1}{2}}$.
We assume the three-dimensional transient disturbance produced by temporary extraneous forces which is horizontally and temporally periodic in the form

$$\psi = \hat{\psi}(z)\exp[i(kz + ly - \omega t)], \tag{2}$$

where $k(> 0)$ is the horizontal wave number, $\omega(\leq N_0)$ is the wave frequency and ω/k is the phase velocity. The solution of the equation (1) is given by

$$\hat{\omega} = A_1 e^{+i\eta z} + B_1 e^{-i\eta z} \tag{3}$$

where $\eta = -\alpha\left(\frac{N^2}{\omega^2 - k^2A^2} - 1\right)^{\frac{1}{2}} / \left(1 - \frac{4\Omega^2\omega^2}{\omega^4 - k^4A^4}\right)^{\frac{1}{2}}$ are arbitrary constants. The vertical wave number for $|z| > \frac{L}{2}$, it is defined to be negative so that the incident wave and transmitted wave propagate upward. We seek to interpret these solutions as upward or downward propagating waves [14] in the following section which plays a significant role in understanding the transmission and reflection of waves at weakly stratified regions that we have considered in this paper.

2.1 Upward and Downward Propagating Waves

In a medium of which the properties vary substantially over a wavelength it is difficult to specify exactly which part of an oscillatory motion corresponds to a wave travelling in the upward direction and which one in the opposite direction there is a continuous interchange between the two. In a uniform medium, on the other hand, precise and physically important identifications may be made. We must be quite clear about the interpretation of this in a uniform medium as considered in this paper.

It is clear from governing wave equation every wave with horizontal wave number k and phase-velocity c has a vertical structure of the from (4) For the sake of definiteness we settle the branch for η by requiring that

$$c_k > 0, \quad \eta_k > 0 \tag{4}$$

$$\eta = -\alpha \frac{\left(\frac{N^2}{\omega^2 - k^2 A^2} - 1\right)^{\frac{1}{2}}}{\left(1 - \frac{4\Omega^2\omega^2}{\omega^4 - k^4 A^4}\right)^{\frac{1}{2}}}. \tag{5}$$

This implies that if

$$1 \ll \frac{N^2}{\omega^2 - k^2 A^2}, 1 \gg \frac{4\Omega^2\omega^2}{\omega^4 - k^4 A^4} \quad \text{then} \quad \eta \approx \left(\frac{-N_\alpha}{(\omega^2 - k^2 A^2)}\right)^{\frac{1}{2}}$$

if

$$1 \gg \frac{N^2}{\omega^2 - k^2 A^2}, \frac{4\Omega^2\omega^2}{\omega^4 - k^4 A^4} \gg 1 \quad \text{then} \quad \eta \approx \frac{\alpha(\omega^4 - k^4 A^4)^{\frac{1}{2}}}{2\Omega\omega}$$

$$\eta = \begin{cases} \frac{N_\alpha}{(\omega^2 - k^2 A^2)^{\frac{1}{2}}} & if \quad \omega^2 > k^2 A^2 \\ -\frac{iN_\alpha}{(\omega^2 - k^2 A^2)^{\frac{1}{2}}} & if \quad \omega^2 > k^2 A^2 \end{cases} \tag{6}$$

$$\eta = \begin{cases} \frac{\alpha\left(\omega^4 - k^4 A^4\right)^{\frac{1}{2}}}{2\Omega\omega} & if \quad \omega^2 > k^2 A^2 \\ \frac{i\alpha\left(\omega^4 - k^4 A^4\right)^{\frac{1}{2}}}{2\Omega\omega} & if \quad \omega^2 < k^2 A^2 \end{cases} \tag{7}$$

with the proper definition of branches of η and its expression under extreme situations described above it is possible to interpret which solution in (3) represents an upward or downward propagating wave. If $\omega^{>} k^2 A^2$ then η is positive, so that the phase front move upwards and if $\omega^2 < k^2 A^2$ then η is imaginary. Thus the first solution in (2) describes a wave with a upward component of velocity for $\omega^2 > k^2 A^2$. However, the influence of such a wave propagate upwards and so it is called an upward traveling wave. The complete spatial distribution of velocity associated with the first solution is $\hat{\omega} = A_1 e^{+i\eta z}$ represents a plane wave with phase front $\eta z - kct = \text{constant}$. For the range of frequency ω such that k and η are real, ω^2 must be greater than $k^2 A^2$. The vertical component of the phase velocity is $c_k = \frac{\omega}{\eta}$. The phase fronts are always perpendicular to the phase velocity. Therefore the first solution describes a wave with a downward component of phase velocity, so such a wave propagates upwards and likewise the second solution. $\hat{\omega} = B_1 e^{-i\eta z}$ be interpreted as downward traveling wave. The significance of this interpretation can be looked at in the following two ways.

(i) Group Velocity Approach : The dispersion relation which relates wave number k and the frequency ω can be written as

$$\omega = \pm \frac{\sqrt{\frac{4\eta^2\omega^2+\alpha^2}{\eta^2+\alpha^2} \pm \sqrt{\frac{4\eta^2\omega^2+\alpha^2}{\eta^2+\alpha^2} + \frac{4(k^4A^4)(\eta^2+\alpha^2)+(\alpha^2N^2k^2A^2)}{\eta^2+\alpha^2}}}}{\sqrt{2}}. \tag{8}$$

According to (6) we must take the minus sign when η and $(\omega^2 - k^2A^2)$ are positive and the plus sign when they are negative. In either case, for the first solution in (2), $\frac{\partial\omega}{\partial\eta}$ is always positive, and hence corresponds to an upward component of group velocity. Thus the first solution in (3) represents an upward-propagating wave and similarly the second solution represents a downward-propagating wave.

(ii) Energy Approach : A second view of understanding upward and downward-propagating waves comes from energy consideration. The total mean rate of working by the fluid below any level on the fluid above is pw, where p is the disturbance pressure and an over bar denotes an average over a horizontal wave length or over a period. We can show that

$$\overline{p\omega} = \frac{i\overline{\rho}(\omega^4 - k^4A^4) - (4\Omega^2\omega^2)}{\omega\alpha^2(\omega^2 + k^2A^2)} \overline{\frac{d\hat{\omega}}{dz}, \hat{\omega}}, \tag{9}$$

where $\overline{\rho}$ is the mean density, $\overline{\frac{d\hat{\omega}}{dz}, \hat{\omega}} = \left(\frac{d\hat{\omega}}{dz}, \hat{\omega}^*\right)$ [14]. Using (9) it can be shown that the total mean rate of working is given by

$$\overline{pw} = \frac{\overline{\rho}(\omega^4 - k^4A^4) - (4\Omega^2\omega^2)}{\omega\alpha^2(\omega^2 + k^2A^2)} A_q^2 \eta / \tag{10}$$

We find from (10) that the $\overline{pw}$ is positive for the solution $\hat{w} = Ae^{+i\eta z}$ and thus wave energy is flowing upwards, so it is upward propagating wave and for the second solution i.e. $\hat{\omega} = Be^{-i\eta z}$, $\overline{pw}$ is negative and wave energy is flowing downwards and hence it is downward propagating wave.

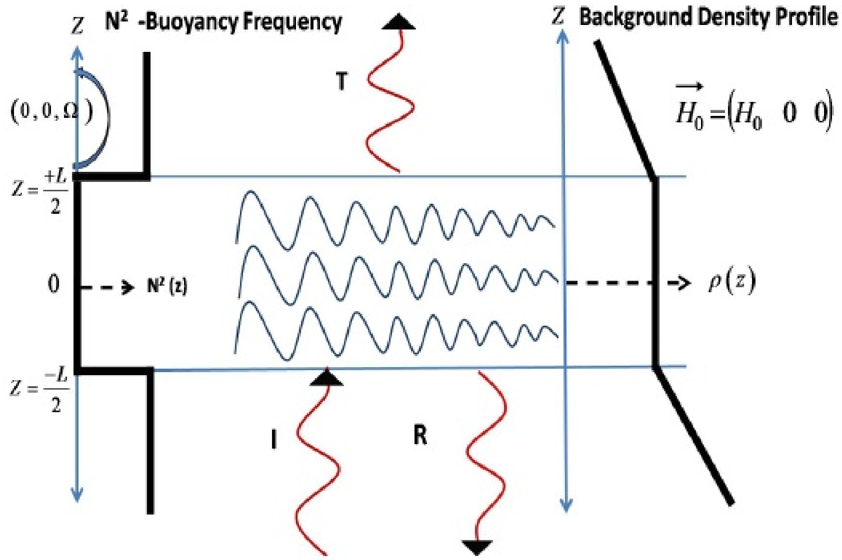

Figure 1. Schematic representation of the Physical configuration for the applied magnetic field along with Rotation and background density Profiles

Transmission across the barriers is derived in the following subsections.

3. Transmission Across N^2-barrrier 1

In this case we have a uniform density conducting fluid of finite depth L bounded on either side by a stratified conducting fluids extending to infinity. We assume

$$N^2 = \begin{cases} N_0^2 & |z| > \frac{L}{2} \\ 0 & |z| \le \frac{L}{2} \end{cases} \tag{11}$$

which is N^2-barrrier1 of depth L as shown in figure 1. With this the solution of (5) takes the form

$$\hat{\omega} = \begin{cases} A_3 e^{i\eta z} & |z| > \frac{L}{2} \\ A_2 e^{\frac{z}{\delta}} + B_2 e^{\frac{-z}{\delta}} & \frac{-L}{2} \le z \le \frac{L}{2} \\ A_1 e^{i\eta z} + B_2 e^{-i\eta z} & |z| < \frac{L}{2} \end{cases} \tag{12}$$

where $\eta = -\alpha \left(\frac{N^2}{\omega^2 - k^2 A^2} - 1\right)^{\frac{1}{2}} / \left(1 - \frac{4\Omega^2\omega^2}{\omega^4 - k^4 A^4}\right)^{\frac{1}{2}}$, $A = \sqrt{\frac{\mu H_0^2}{\rho_0}}$ and $\delta = \frac{1}{k}, k(> 0)$ is the well defined horizontal wave number and $\omega(\le N)$ is the wave frequency. As a wave packet with amplitude A_1 reaches the first interface at $z = \pm L/2$, some part of the wave is transmitted and the rest is reflected, the amplitudes being A_2 and A_1 respectively and the amplitudes being A_3 and B_2 respectively. Thereafter the waves are only transmitted in the stratified region ($|z| > \frac{L}{2}$). Since η represents the vertical wave number for $|z| > \frac{L}{2}$ it is defined to be negative so that the incident wave (with amplitude A_1)and transmitted wave(with amplitude A_3) propagate upward. Our aim here is to determine the transmission coefficient $T_m = |A_3/A_1|^2$, which represents the fraction of energy transported across N^2-barrier 1 and is given by

$$T_m = \left[1 + \frac{(1+\eta^2\delta^2)^2}{4\eta^2\delta^2}\sinh^2\left(\frac{L}{\delta}\right)\right]^{-1}. \tag{13}$$

In limit the $\Omega \to 0$ (conducting and non-rotating case)

$$T_m = \left[1 + \frac{(1+\lambda^2\delta^2)^2}{4\lambda^2\delta^2}\sinh^2\left(\frac{L}{\delta}\right)\right]^{-1}. \tag{14}$$

In limit the $\Omega \to 0$ and $A \to 0$ (non-conducting and non-rotating case)

$$T_m = \left[1 + \frac{(1+\gamma^2\delta^2)^2}{4\gamma^2\delta^2}\sinh^2\left(\frac{L}{\delta}\right)\right]^{-1}. \tag{15}$$

The transmission coefficient has been plotted for different values of frequency, N^2-barrrier width and Alfvéen velocity as plotted below.

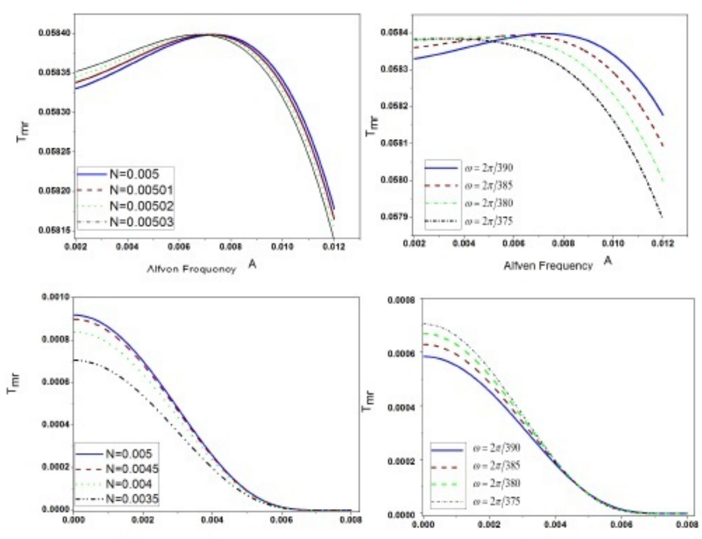

FIGURE 2. (a)Variation of transmission coefficient T_m with $(\omega^2 - k^2A^2) > N_0^2$ and $(\omega^4 - k^4A^4) > 4\Omega^2\omega^2$ with different N^2 for varying A (b)Variation of Transmission coefficient T_m with different ω for varying A (c) Variation of Transmission coefficient T_m with different N^2 for varying Ω(d) Variation of Transmission coefficient T_m with different ω for varying Ω with $\omega = 2\pi s^{-1}/390$, $N_0 = 0.0005s^{-2}$, $k = 2\pi/15km^{-1}$, L=5km

4. Transmission Across N^2-barrrier2

In the second case we assume a uniform density fluid of finite depth L is bounded on either side by a stratified fluids extending either side to infinity given by,

$$N^2 = \begin{cases} N_0^2 & |z| > \frac{L}{2} \\ N_1^2 & |z| \leq \frac{L}{2} \end{cases} \tag{16}$$

where $N_1 \leq \omega \leq N_0$. In this case the fluid in the lower is initially at rest. The fluid in the upper and lower region is uniformly stratified along with the parameter N_0^2 and middle layer with N_1^2. Solution obtained in this case is given by:

$$\hat{\omega} = \begin{cases} A_3 e^{i\eta z} & |z| > \frac{L}{2} \\ A_2 e^{\xi z} + B_2 e^{-\xi z} & \frac{-L}{2} \leq z \leq \frac{L}{2} \\ A_1 e^{i\eta z} + B_2 e^{-i\eta z} & |z| < \frac{L}{2} \end{cases} \tag{17}$$

where $\eta = -\alpha\left(\frac{N_0^2}{\omega^2 - k^2A^2} - 1\right)^{\frac{1}{2}} / \left(1 - \frac{4\Omega^2\omega^2}{\omega^4 - k^4A^4}\right)^{\frac{1}{2}}$, $\xi = \alpha\left(1 - \frac{N_1^2}{\omega^2 - k^2A^2}\right)^{\frac{1}{2}} / \left(1 - \frac{4\Omega^2\omega^2}{\omega^4 - k^4A^4}\right)^{\frac{1}{2}}$, $k = \frac{1}{\delta}\hat{\psi}(z), \frac{d\hat{\psi}}{dz}$ are continuous across the interface and hence velocity and pressure.The transmission coefficient computed for case2 is given by

$$T_{mb} = \left[1 + \frac{(\eta^2 + \xi^2)}{(4\xi^2\eta^2)}\sinh^2(\xi L)\right]^{-1}. \tag{18}$$

In the limit $N_1 \to 0$ equation (18) reduces the transmission coefficient obtained in the first case and in the limit $\Omega \to 0$ and $A \to 0$ to [7] case for non-conducting fluids respectively.

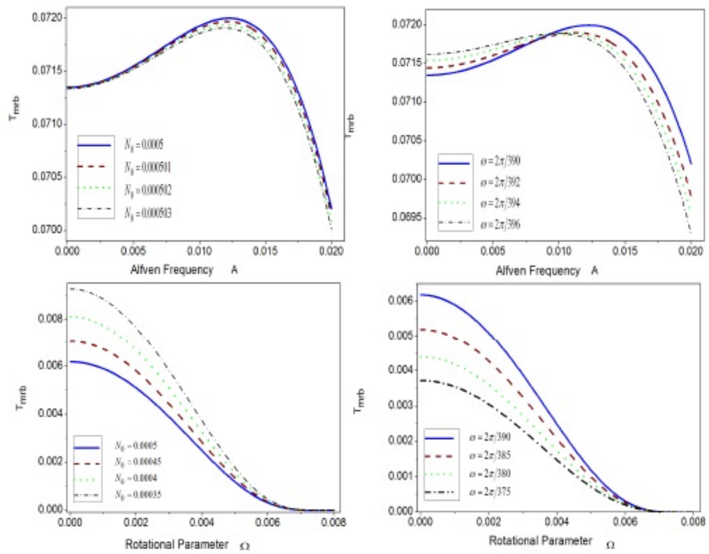

FIGURE 3. (a)Variation of transmission coefficient T_{mb} with $(\omega^2 - k^2A^2) > N_0^2$ and $(\omega^4 - k^4A^4) > 4\Omega^2\omega^2$ with different N^2 for varying A (b)Variation of Transmission coefficient T_{mb} with different ω for varying A (c) Variation of transmission coefficient T_{mb} with different N^2 for varying Ω (d) Variation of Transmission coefficient T_{mb} with different ω for varying Ω with $\omega = 2\pi s^{-1}/390$, $N_0 = 0.0005s^{-2}$, $N_1 = 0.00025s^{-2}$,$k = 2\pi km^{-1}/15$,L=5km.

5. Transmission Across Locally Mixed Region

In this case, $\rho_b(z)$ is assumed to vary continuously, even though its slope is discontinuous at at $z = \pm L/2$. More realistically, localized mixed regions within a stratified fluid are better represented by a discontinuous density profile in the form:

$$\rho_b = \begin{cases} \rho_0\left(1 - \frac{z}{H_1}\right) & |z| \le \frac{L}{2} \\ \\ \rho_0\left(1 - \frac{z}{H_0}\right) & |z| > \frac{L}{2} \end{cases} \tag{19}$$

This is called 'N^2 -barrier3' of depth L as shown in figure 1. Where $H_0 \equiv \frac{g}{N_0^2}$ and $H_1 \equiv \frac{g}{N_1^2}$ measure the strength of stratification respectively outside and within a partially mixed region of depth L. Consistent with the Boussinesq approximation we assume $H_0, H_1 \gg L, k^{-1}$. The corresponding squared buoyancy frequency is the same as that for the generalization of the N^2-barrier except for infinite spikes at $z = -L/2$ where the density changes discontinuously by $\Delta\rho_0 = \rho_0\left[\frac{(N_0^2 - N_1^2)}{g}\right]\left(\frac{L}{2}\right)$. The prescribed '$N^2$-

barrier2' in this case is as follows:

$$N^2 = \begin{cases} N_0^2 & |z| > \frac{L}{2} \\ \\ N_1^2 & |z| \leq \frac{L}{2} \end{cases} \tag{20}$$

where, $N_1 \leq \omega \leq N_0$, requiring velocity and pressure to be continuous across the interface [16] we compute the transmission coefficient in the $N_{\leq}\omega \leq N_0$ case, is given by:

$$T_{m(mix)} = \left[1 + \frac{(\eta^2+\xi^2)}{(4\xi^2\eta^2)}\sinh^2(\xi L)\Gamma^2_{rmix}\right]^{-1}. \tag{21}$$

in which, $T_{m(mix)} = \left[1 + \frac{L^2\alpha^2 N_0^2(1-\sigma^2)}{4(\omega^2-4\Omega^2)} - L\xi\coth(L\xi)\right]$. In the limit $\Omega \to 0$ in (20) reduces to non-conducting fluid results of [6]. However we have plotted the graph of $T_{m(mix)}$ against Ω in figure 4 when $\omega > 2\Omega$.

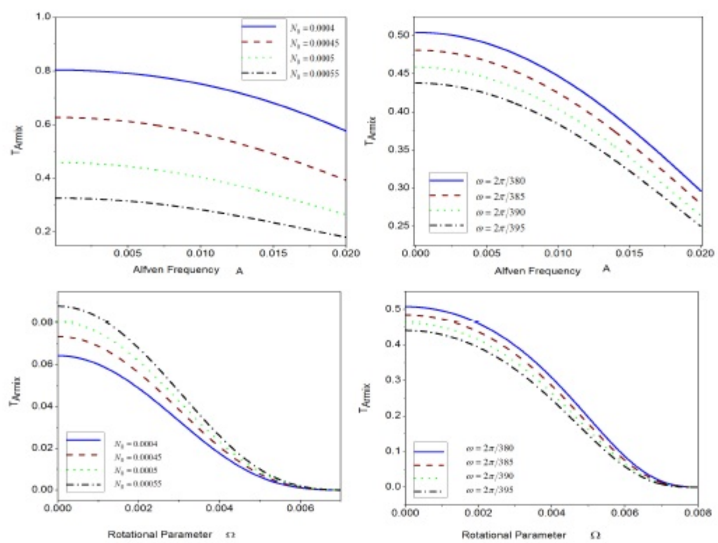

FIGURE 4. (a)Variation of transmission coefficient $T_{m(mix)}$ with $(\omega^2 - k^2A^2) > N_0^2$ and $(\omega^4 - k^4A^4) > 4\Omega^2\omega^2$ with different N^2 for varying A (b)Variation of Transmission coefficient $T_{m(mix)}$ with different ω for varying A (c) Variation of transmission coefficient $T_{m(mix)}$ with different N^2 for varying Ω (d) Variation of Transmission coefficient $T_{m(mix)}$ with different ω for varying Ω with $\omega = 2\pi s^{-1}/390$, $N_0 = 0.0005s^{-2}$, $N_1 = 0.00025s^{-2}$,$k = 2\pi km^{-1}/15$, L=5km

The contour plots for the variation of the transmission coefficient for a conducting rotating fluid in the N^2-barrier1, N^2-barrier2, N^2-barrier3 is given in figure 5.

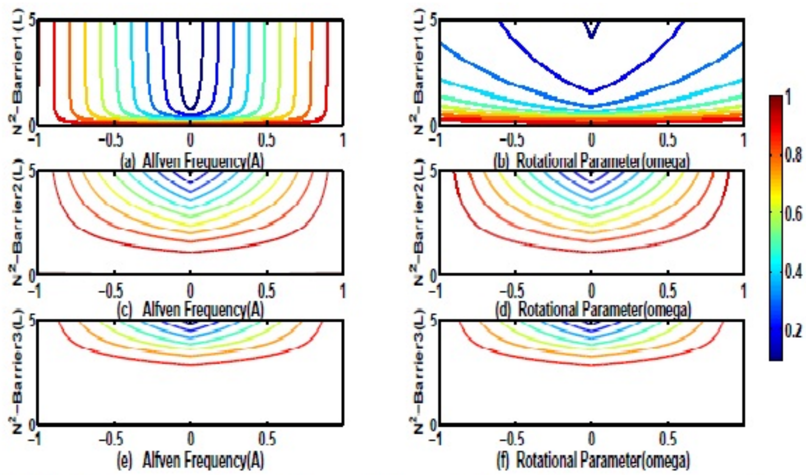

FIGURE 5. The contour plots of the Transmission co-efficient for barrier1,barrier2,barrier3.

6. Results and Conclusions

Transmission of internal gravity waves is analysed in the stratified regions for a conducting rotating fluid. We group the wave solution as upward and downward propagating waves using group velocity and energy approach. The wave equation is derived and the wave solution analysed for transmission in the all the three regions. We have obtained the transmission coefficient for the effect of rotation and magnetic field for different density variations. The analytical solutions has been obtained considering the vertical wave numbers (in the region $|z| \leq L/2$) and (in the region $|z| > L/2$) defined as real or imaginary. Derived transmission coefficients are plotted in these regions shows a decrease in the transmission of the internal gravity waves revealing the evanescence in the N2 reduced regions.

In the N^2-barrrier1 evanescence of the internal gravity waves is more horizontal as shown in figure 2 where transmission is slowly decreasing with increasing Alfvén velocity both in case of varying frequency, ω and buoyancy frequency, N and in figure 3 buoyancy frequency, N is weakly stratified and transmission in this case is also reduced and hence evanescence occur in N^2-barrrier2 where density is assumed to vary continuously whereas in N^2-barrrier3 localised mixed regions with stratification transmission is more and reduced in the barrier region with exactly going to the existing limits for the case of non-conducting and non-rotating fluids. The contour plots of the transmission of waves using N2-barrrier lengths (L), rotational parameter (Ω), Alfvén velocity (A) of the internal gravity waves also reveal that existence of evanescence in the transmission along the different density barriers. In figure5 we see that the transmission is high in the middle region and slowly reduced in the above and below regions which reveals the more transmission along the horizontal region. Hence in the N^2-barrrier1 we observe that the transmission is decreased and waves are along the horzontal direction signifies the trapping of the upward propagating internal gravity waves in the stratified region and in case of N^2-barrrier3 which is mixed N^2 region the transmission is more comparatively to the other two cases.

We find that, the effect of rotational and magnetic fields is to make the wave to propagate along the fluid lines rather than allow it to propagate upwards which is depicted in graphs. This is because the gravity waves propagate along the fluid lines rather than allow it to propagate upwards due to the effect of rotation and magnetic field. The above results conclude that rotation and magnetic field signifies the evanescence in the barrier region.

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