International J. of Math. Sci. & Engg. Appls. (IJMSEA)
ISSN 0973-9424, Vol. 12 No. I (April, 2018), pp. 79-88

DEGREE EQUITABLE CO-ISOLATED LOCATING DOMINATION IN GRAPHS

N. MEENAL[1] AND S. MUTHAMMAI[2]
[1] Assistant Professor, PG & Research Department of Mathematics,
J. J. College of Arts & Science (Autonomous),
Pudukkottai . 622 422, Tamil Nadu, India
[2] Principal, Alagappa Government Arts College,
Karaikudi . 630 003, Tamil Nadu, India

Abstract

Let $G(V,E)$ be a connected graph. A dominating set $S \subseteq V$ is called a co-isolated locating dominating set, if for any two vertices v, $w \in V-S$, $N(v) \cap S \neq N(w) \cap S$ and there exists atleast one isolated vertex in $\langle V-S \rangle$. A dominating set $S \subseteq V$ is called a degree equitable dominating set, if for every $u \in V-S$, there exists a vertex $v \in S$ such that $uv \in E(G)$ and $|deg(u) - deg(v)| \leq 1$, where $deg(u)$ is the degree of u in G and $deg(v)$ is the degree of v in G. A dominating set $S \subseteq V$ is called a degree equitable co-isolated locating dominating set if it is both degree equitable dominating set and co-isolated locating dominating set. The minimum cardinality of a degree equitable co-isolated locating dominating set is called the degree equitable co-isolated locating domination number and is denoted by γ^e_{clid}. This paper aims at the study of this new parameter for connected graph G.

1. Introduction

Let $G = (V,E)$ be a simple graph of order p. For any $v \in V(G)$, the neighborhood

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Key Words : *Degree equitable dominating set, Co-isolated locating dominating set and Degree equitable co-isolated locating dominating set.*

AMS Subject Classification : 05C69.

$N_G(v)$ (or simply $N(v)$) of v is the set of all vertices adjacent to v in G. A non-empty set $S \subseteq V(G)$ of a graph G is a dominating set, if every vertex in $V(G) - S$ is adjacent to atleast one vertex in S.

A new type of domination known as Degree Equitable Domination introduced by Swaminathan [9]. A dominating set $S \subseteq V$ is called a **degree equitable dominating set**, if for every $u \in V - S$, there exists a vertex $v \in S$ such that $uv \in E(G)$ and $|deg(u) - deg(v)| \leq 1$, where $deg(u)$ is the degree of u in G and $deg(v)$ is the degree of v in G. If S is a degree equitable dominating set, then any super set of S is also a degree equitable dominating set. The **degree equitable domination number** γ_d^e is the minimum cardinality of a degree equitable dominating set.

A special case of domination called a locating domination is defined by Rall and Slater [8]. A dominating set S in a graph G is called a **locating dominating set** in G, if for any two vertices $v, w \in V(G) - S$, $N_G(v) \cap S$ and $N_G(w) \cap S$ are distinct. The **locating domination number** $\gamma_L(G)$ of G is defined as the minimum number of vertices of a locating dominating set in G.

Muthammai and Meenal [3] introduced the concept of co-isolated locating dominating set. A locating dominating set $S \subseteq V(G)$ is called a **co-isolated locating dominating set**, if $\langle V - S \rangle$ contains atleast one isolated vertex. The minimum cardinality of a co-isolated locating dominating set is called the **co-isolated locating domination number** and is denoted by $\gamma_{cild}(G)$.

A dominating set with $\gamma(G)$ number of vertices is called a γ-set of G. Similarly, γ_{cild}-set, γ_d^e-set and γ_{clid}^e-set are defined.

In this paper, we introduce the concept of degree equitable locating dominating set and it is studied for the connected graphs.

2. Prior Results

Let G be a graph on p vertices and q edges.

Theorem 2.1 [3] : For every non-trivial graph G, $1 \leq \gamma_{cild}(G) \leq p - 1$.

Theorem 2.2 [3] : $\gamma_{cild}(G) = 1$ if and only if $G \cong K_2$.

Theorem 2.3 [3] : $\gamma_{cild}(K_p) = p - 1$, where K_p is a complete graph.

Theorem 2.4 [4] : For the path P_p and cycle $C_p (p \geq 3)$,

$$\gamma_{cild}(P_p) = \gamma_{cild}(C_p) = \left\lceil \frac{2p}{5} \right\rceil.$$

Theorem 2.5 [4] : For the Wheel W_p $(p \geq 6)$,

$$\gamma_{cild}(W_p) = \left\lceil \frac{2p+1}{5} \right\rceil.$$

Theorem 2.6 [4] : For the Triangular Snake Graph T_p $(p \geq 6)$,

$$\gamma_{cild}(T_p) = \left\lfloor \frac{2p+5}{5} \right\rfloor.$$

Theorem 2.7 [5] : If S is a co-isolated locating dominating set of $G(V, E)$ with $|S| = k$, then $V(G) - S$ contains atmost $pC_1 + pC_2 + \cdots + pC_k$ vertices.

Theorem 2.8 [7] : For the complete graph K_p, $\gamma_d^e(K_p) = 1$.

Theorem 2.9 [7] : For the path P_p and cycle C_p $(p \geq 3)$,

$$\gamma_d^e(P_p) = \gamma_d^e(C_p) = \left\lceil \frac{p}{3} \right\rceil.$$

3. Main Results

3.1 Preliminary Definitions

Definition 3.1.1 : A dominating set $S \subseteq V$ is called a degree equitable co-isolated locating dominating set, if it is both degree equitable and co-isolated locating dominating set. The degree equitable locating domination number γ_{cild}^e is the minimum cardinality of a degree equitable co-isolated locating dominating set.

Remark 3.1.2 : Since every degree equitable co-isolated locating dominating set is a degree equitable dominating set, co-isolated locating dominating set, locating dominating set and also a dominating set,

$$\gamma_d^e(G) \leq \gamma_{cild}^e(G), \ \gamma_{cild}(G) \leq \gamma_{cild}^e(G), \gamma_L(G) \leq \gamma_{cild}^e(G), \gamma_L(G) \leq \gamma_{cild}^e(G).$$

Illustration 3.1.3 : Consider the graph G as shown in Figure 3.1.4.

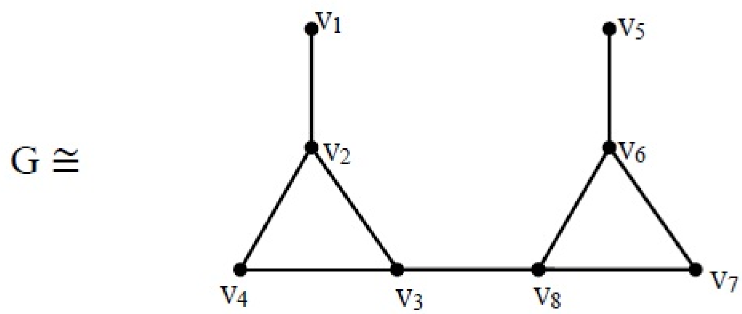

Figure 3.1.4.

$\{v_2, v_5\}$ is a $\gamma-$ set of G and $\gamma(G) = 2$.

$\{v_2, v_3, v_6, v_8\}$ is a γ_{cild} - set of G and $\gamma_{cild}(G) = 4$.

$\{v_2, v_3, v_6, v_8\}$ is a γ_d^e-set of G and $\gamma_d^e(G) = 4$.

$\{v_1, v_3, v_5, v_7, v_8\}$ is a γ_{cild}^e-set of G and $\gamma_{cild}^e(G) = 5$.

Therefore $\gamma(G) \leq \gamma_{cild}(G) \leq \gamma_{cild}^e(G)$ and $\gamma(G) \leq \gamma_d^e(G) \leq \gamma_{cild}^e(G)$..

3.2 Existence of Degree Equitable Co-isolated Locating Dominating Sets

There are certain classes of graphs for which degree equitable co-isolated locating dominating sets does not exist.

Illustration 3.2.1 : For the Bipartite Graph $K_{m,n}$ with $|m - n| \geq 2$, γ_{cild}^e-sets does not exist. In particular for the star $K_{1,p}$ with $p \geq 3$, $|deg(u) - deg(v)| = p - 1 \geq 1$ for all $u, v \in V(K_{1,p})$ and $uv \in E(K_{1,p})$.

Definition 3.2.2 : A subdivision of an edge uv in a graph G is obtained by removing the edge uv, adding a new vertex w and adding edges uw and wv. The vertex w is called the subdivided vertex. The Subdivision graph $S(G)$ of a graph G is the graph obtained from G by subdividing each edge of G exactly once.

A wounded spider is the graph obtained by subdividing atmost $p - 1$ edges of a star $K_{1,p}$ exactly once.

Theorem 3.2.3 : Let G be a connected graph G with minimum degree $\delta(G) \geq 4$. Then for $\gamma_{cild}^e(S(G))$ does not exist.

Proof : Let G be a connected graph with $\delta(G) \geq 4$. Then for any vertex $u \in V(G)$, $deg(u) \geq \delta(G) \geq 4$. From the definition of the subdivision graph $S(G)$, it follows that in between any two vertices there exists a subdivided vertex say v, such that $deg(v) = 2$. For any edge $wv \in E(S(G))$ and $w \in V(S(G))$, $|deg(w)-deg(v)| \geq 4-2 = 2$. Therefore, the degree equitable dominating set does not exist. Hence $\gamma^e_{cild}(S(G))$ does not exist.

Example 3.2.4 : In particular $\gamma^e_{cild}(S(K_5))$ does not exist.

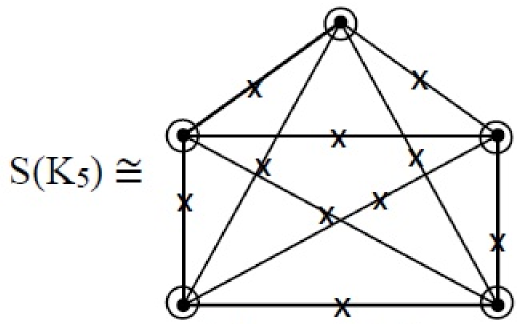

Figure 3.2.5.

The vertices marked with 'x' are subdivided vertices and the vertices marked with'O' are the vertices of a minimum co-isolated locating dominating set. Hence $\gamma^e_{cild}(S(K_5)) = 5$. But $\gamma^e_{cild}(S(K_5))$ does not exist.

3.3. Bounds for $\gamma^e_{cild}(G)$

Proposition 3.3.1 : For any connected graph $G, \gamma^e_{cild}(G) \leq p-1$, if $\gamma^e_{cild}(G)$ exists.

Theorem 3.3.2 : For a connected graph G, $1 \leq \gamma^e_{cild}(G) = 1$ if and only if $G \cong K_2$.

Proof : Let $G \cong K_2$. Then from the Definition 3.3.1. it follows that $\gamma^e_{cild}(G) = 1$. Conversely, let $\gamma^e_{cild}(G) = 1$ and let S be a minimum degree equitable co-isolated locating dominating set. Then $|S| = 1$. Let $S = \{u\}$, where $u \in V(G)$. Since S is a co-isolated locating dominating set, it follows from Theorem 2.4., that $|V-S| \leq 2^1 - 1 = 1$. Therefore $V-S$ also contains only one vertex say v. For G to be connected $uv \in E(G)$ which implies that $G \cong K_2$.

Remark 3.3.3 : . For any graph G,$\gamma^e_{cild}(G) \leq 2q-p+1$.

Theorem 3.3.4 : For any connected graph $G, \gamma^e_{cild}(G) \geq \min\{\gamma^e_d(G), \gamma_{cild}(G)\}$.

Proof : Since every degree equitable co-isolated locating dominating set is a equitable dominating set,

$$\gamma^e_{cild} \geq \gamma^e_d(G). \tag{1}$$

Also every degree equitable co-isolated locating dominating set is a co-isolated locating dominating set and hence

$$\gamma^e_{cild} \geq \gamma_{cild}(G). \quad (2)$$

Combining (1) and (2), the result is obtained.

Theorem 3.3.5 : If G is a regular graph or a bipartite graph $K_{m,m+1}$ for some m, then $\gamma^e_{cild}(G) = \gamma_{cild}(G)$.

Proof : Let G be a regular graph. Let S be a γ_{cild}-set of G. Each vertex of G has same degree. Therefore $|deg(u) - deg(v)| = 0 \leq 1$ for any $u, v \in V(G)$. Then S is also a degree equitable co-isolated locating dominating set of G. Therefore $\gamma^e_{cild}(G) = \gamma_{cild}(G)$. Suppose G is a bipartite graph $K_{m,m+1}$ where $m \geq 2$ then every vertex of G has degree either m or $m+1$ where m is a positive integer. Let S be a γ_{cild}-set of G. Then $|S| = \gamma_{cild}(G)$. Let $u \in V - S$, then their exist a vertex $v \in S$ such that $uv \in E(G)$. Also $deg(u) = m$ or $m+1$ and $deg(v) = k$ or $k+1$. Therefore $|deg(u) - deg(v)| = 0$ or $1 \leq 1$. Therefore S is a degree equitable co-isolated locating dominating set. Hence $\gamma^e_{cild} \leq \gamma_{cild}(G)$. But by Theorem 3.3.4., $\gamma_{cild}(G) \leq \gamma^e_{cild}(G)$. It follows that $\gamma^e_{cild}(G) = \gamma_{cild}(G)$. This completes the proof of the theorem.

Corollary 3.3.6 : For a cycle $C_p (p \geq 6), \gamma^e_{cild}(C_p) = \left\lceil \frac{2p}{5} \right\rceil$.

Proof : Since the cycle C_p is a 2-regular graph and $(C_p) = \gamma_{cild}(C_p)$ and by Theorem 2.7., $\gamma_{cild}(C_p) = \left\lceil \frac{2p}{5} \right\rceil$. Hence $\gamma^e_{cild}(C_p) = \left\lceil \frac{2p}{5} \right\rceil$.

Corollary 3.3.7 : For a complete graph $K_p (p \geq 2)$, $\gamma^e_{cild}(K_p) = p - 1$.

Proof : Since the complete graph K_p is a $(p-1)$ regular graph. By Theorem 3.3.5., $\gamma^e_{cild}(K_p) = \gamma_{cild}(K_p)$ and by Theorem 2.3, $\gamma_{cild}(K_p) = p - 1. \gamma^e_{cild}(K_p) = p - 1$.

Remark 3.3.8 : The converse of the Theorem 3.3.5 is not necessarily true. For Example consider the graph G given in Figure 3.3.9.

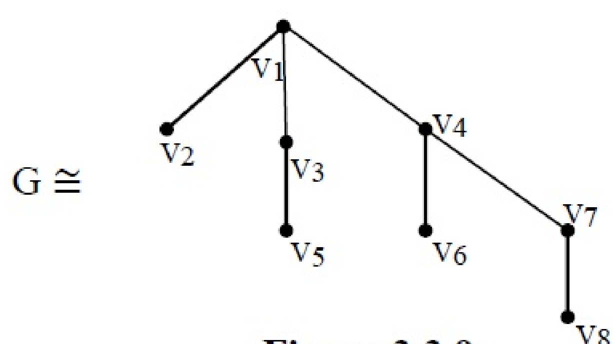

Figure 3.3.9.

The set $\{v_1, v_3, v_4, v_7\}$ is a γ_{cild}- set of G and the set $\{v_2, v_3, v_6, v_8\}$ is a γ^e_{cild}-set of G. Therefore $\gamma_{cild}(G) = \gamma^e_{cild}(G) = 4$. But G is not a regular graph.

Theorem 3.3.10 : If G is a connected graph with $\Delta(G) = \delta(G) + 1$ then $\gamma_{cild}(G) \leq \gamma^e_{cild}(G)$, where $\delta(G)$ and $\Delta(G)$ are the minimum and maximum degree of the graph G.

Proof : Let G be a connected graph with $\Delta(G) = \delta(G) + 1$. Then $\delta(G) \leq deg(u) \leq \Delta(G)$ where $u \in V(G)$. For any two adjacent vertices $u, v \in V(G)$, $deg(u) - deg(v) \leq \Delta(G) - \delta(G) \leq \delta(G) + 1 - \delta(G) \leq 1$. Therefore every co-isolated locating dominating set is a degree equitable co-isolated locating dominating set. Hence $\gamma^e_{cild}(G) = \gamma_{cild}(G)$.

Corollary 3.3.11 : For a path $P_p (p \geq 6)$, $\gamma^e_{cild} = \left\lceil \frac{2p}{5} \right\rceil$.

Proof : For a path P_p, $\delta(P_p) = 1$ and $\Delta(P_p) = 2$. Therefore $\Delta(P_p) = \delta(P_p) + 1$. Hence by Theorem 3.3.10, $\gamma^e_{cild}(P_p) = \gamma_{cild}(P_p)$. By Theorem 2.4., $\gamma_{cild}(P_p) = \left\lceil \frac{2p}{5} \right\rceil$. Therefore $\gamma^e_{cild}(P_p) = \left\lceil \frac{2p}{5} \right\rceil$.

3.4 γ^e_{cild}- for Some Particular Graphs

Definition 3.4.1 : The Triangular snake graph T_p is obtained from the path P_p by replacing each edge of the path by a triangle C_3.

Theorem 3.4.2 : For a Triangular Snake Graph $T_p (p \geq 3)$, $\gamma^e_{cild}(T_p) = \left\lceil \frac{2p-2}{5} \right\rceil$.

Proof : Let $V(T_p) = \{v_1, v_2, \cdots, v_p\}$, where p is odd. Let $A = \{v_1, v_3, v_5, c, v_p\}$ be the set of vertices of the underlying path $P_{\frac{p+1}{2}}$ of T_p. For $u \in A$ and $u \neq v_1$ or v_p, $deg_{T_p}(u) = 4$ and $deg_{T_p} = deg_{T_p}(v_1) = 2$. Let $B = \{v_2, v_4, v_6, \cdots, v_{p-1}\}$ and $deg_{T_p}(v) = 4$ for all $v \in B$. Also $\langle B \rangle$ is independent and the vertex v_{2i} of B is adjacent to v_{2i-1} and v_{2i+1} in A, for $i = 1, 2, \cdots, \frac{p-1}{2}$. For any edge $uv \in E(T_p)$, where vertex $u \in A$, $u \neq v_1$ or v_p and $v \in B$, $deg_{T_p}(u) - deg_{T_p}(v) = 4 - 2 = 2 \leq 1$. Hence all the vertices of B are to be included in the γ^e_{cild}-set of T_p. By Theorem 2.6., $\gamma^e_d(P_p) = \left\lceil \frac{p}{3} \right\rceil$. Therefore

$$
\begin{aligned}
\gamma^e_{cild}(T_p) &= \gamma^e_{cild}(\langle A - \{v_1, v_2\} \rangle) + |B| \\
&= \left\lceil \frac{\frac{p-1}{2}}{3} \right\rceil + \frac{p-1}{2} \\
&= \left\lceil \frac{3p - 3 + p - 1}{6} \right\rceil \\
&= \left\lceil \frac{4p - 4}{6} \right\rceil \\
\gamma^e_{cild}(T_p) &= \left\lceil \frac{2p - 2}{3} \right\rceil .
\end{aligned}
$$

Example 3.4.3 : The triangular snake graph T_{23} is given in Figure 3.4.4.

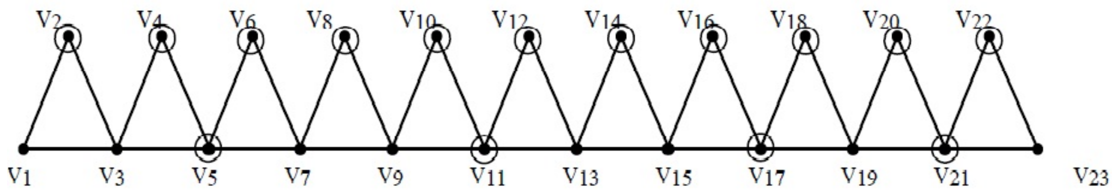

Figure 3.4.4

The set $S = \{v_2, v_4, v_6, v_8, v_{10}, v_{12}, v_{14}, v_{16}, v_{18}, v_{20}, v_{22}, v_5, v_{11}, v_{17}, v_{21}\}$ is a γ^e_{cild}-set of T_{23}. Also,

$$\begin{aligned}\gamma^e_{cild}(T_{23}) &= \left\lceil \frac{\frac{p-1}{2}}{3} \right\rceil + \frac{p-1}{2} \\ &= \left\lceil \frac{11}{3} \right\rceil + \frac{22}{2} \\ &= 15 = |S|.\end{aligned}$$

Hence, $\gamma^e_{cild}(T_{23}) = 15$.

Definition 3.4.5 : The **join of two graphs** G and H is the graph $G + H$ with $V(G + H) = V(G) \cup V(H)$ and $E(G + H) = E(G) \cup E(H) \cup \{uv :\in V(G), v \in V(H)\}$. The **Wheel** W_p with p vertices is defined to be the join of K_1 and C_{p-1}. The vertex corresponding to K_1 is known as apex (or central vertex) while the vertices corresponding to C_{p-1} are known as rim vertices.

Theorem 3.4.6 : For the Wheel graph $W_p(p \geq 6)$, $\gamma^e_{cild}(W_p) = \left\lceil \frac{2p+2}{5} \right\rceil$.

Proof : Let $V(W_p) = \{v_1, v_2, v_3, \cdots, v_p\}$, where v_1 is the central vertex and $deg(v_1) = p - 1$ and the remaining vertices $v_2, v_3, \cdots, v_p$ are of degree 3 and $\langle v_2, v_3, \cdots, v_p \rangle \cong C_{p-1}$. Then $deg(v_1) - deg(v_i) = p - 1 - 3 \geq 6 - 4 \geq 2$ for all $i = 2, 3, \cdots, p$. Also, $|deg(v_i) - deg(v_j)| = 0 < 1$ for $2 \leq i, j \leq p$ and $i \neq j$.

Case(i): $p \equiv 0, 1, 3$ **(mod 5)**

Let S be a γ_{cild}-set of W_p. By Theorem 2.8., $\gamma_{cild}(W_p) = \left\lceil \frac{2p+1}{5} \right\rceil$. Also each vertex in $V - S$ is adjacent to a vertex other than the central vertex of W_p. Hence S will also be a γ^e_{cild}- set of W_p. Therefore $\gamma^e_{cild}(W_p) = \gamma_{cild}(W_p) = \left\lceil \frac{2p+1}{5} \right\rceil$.

Case(ii) : $p \equiv 2, 4$ **(mod 5)**

Let S be a γ_{cild}-set of W_p. The set $S \cup \{v_p\}$ is a γ^e_{cild}-set of W_p. Therefore $\gamma^e_{cild}(W_p) = \gamma_{cild}(W_p) + 1 = \left\lceil \frac{2p+1}{5} \right\rceil + 1$. Hence the theorem.

Remark 3.4.7 : $\gamma^e_{cild}(W_4) = \gamma^e_{cild}(W_5) = 3$.

4. Conclusion

In this paper, a new domination combining the concepts of Degree equitable domination and co-isolated locating domination called degree equitable co-isolated domination is given and its corresponding number is studied for some standard graphs. Also its lower and upper bounds are found.

5. Open Problems

- To characterize all the connected graphs G for which $\gamma^e_{cild}(G)$ does not exist.
- To establish the sufficient condition for Theorem 3.3.5.

Acknowledgement

This research paper is not published or submitted elsewhere for possible publication and the authors assure this acknowledgement.

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