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ON THE r-INTEGRALS INVOLVING THE I-FUNCTIONS OF SEVERAL VARIABLES

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Abstract

The object of this paper is to obtain the r-integrals involving the I-functions of several variables. On specializing the parameters similar results can be derived in the case of I-functions of two variables and H functions of r and two-variables which include the result proved by Prasanth and Nambisan [2, p.102].

1. Introduction

Notations used:

$_1(a_j;\alpha_j,A_j)_p$ stands for $(a_1;\alpha_1,A_1),(a_2;\alpha_2,A_2),\cdots,(a_p;\alpha_p,A_p)$.

The generalized Fox's H-function, namely I-function of r-variables introduced by Prathima, Nambisan and Santha Kumari [3, p.38] is defined and represented in the following manner:

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$$I[z_1,\cdots,z_r] = I^{0,N:m_1,n_1;\cdots;m_r,n_r}_{P,Q:p_1,q_1;\cdots;p_r,q_r}$$

$$\left[\begin{array}{c|c} z_1 & {}_1(a_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)};A_j)_P:\ {}_1(c_j^{(1)},\gamma_j^{(1)};C_j^{(1)})_{p_1};\cdots;\ {}_1(c_j^{(r)},\gamma_j^{(r)};C_j^{(r)})_{p_r} \\ \vdots & \\ z_r & {}_1(b_j;\beta_j^{(1)},\cdots,\beta_j^{(r)},B_j)_Q:\ {}_1(d_j^{(1)},\delta_j^{(1)};D_j^{(1)})_{q_1};\cdots;\ {}_1(d_j^{(r)},\delta_j^{(r)};D_j^{(r)})_{q_r} \end{array}\right]$$

$$= \frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\theta_1(s_1)\cdots\theta_r(s_r)\ \ \phi(s_1,\cdots,s_r)\ z_1^{s_1}\cdots z_r s^r ds_1\cdots ds_r, \qquad (1.1)$$

where $\phi(s_1,\cdots,s_r)$ and $\theta_i(s_i), i=1,2,\cdots,r$ are given by

$$\phi(s_1,\cdots,s_r) = \frac{\prod\limits_{j=1}^{N}\Gamma^{A_j}\left(1-a_j+\sum\limits_{i=1}^{r}\alpha_j^{(i)}s_i\right)}{\prod\limits_{j=1}^{Q}\Gamma^{B_j}\left(1-b_j+\sum\limits_{i=1}^{r}\beta_j^{(i)}s_i\right)\prod\limits_{j=N+1}^{P}\Gamma^{A_j}\left(a_j-\sum\limits_{i=1}^{r}\alpha_j^{(i)}s_i\right)}, \qquad (1.2)$$

$$\theta_i(s_i) = \frac{\prod\limits_{j=1}^{m_i}\Gamma^{D_j^{(i)}}(d_j^{(i)}-\delta_j^{(i)}s_i)\prod\limits_{j=1}^{n_i}\Gamma^{C_j^{(i)}}(1-c_j^{(i)}+\gamma_j^{(i)}s_i)}{\prod\limits_{j=m_i+1}^{q_i}\Gamma^{D_j^{(i)}}(1-d_j^{(i)}+\delta_j^{(i)}s_i)\prod\limits_{j=n_i+1}^{p_r}\Gamma^{C_j^{(i)}}(cj^{(i)}-\gamma_j^{(i)}s_i)}. \qquad (1.3)$$

Also $z_i \neq 0$ $(i=1,\cdots,r)$, $\omega=\sqrt{-1}, m_j, n_j, p_j, q_j$ $(j=1,\cdots,r)$, N, P, Q are non-negative integers such that $0 \leq N \leq P$, $Q \geq 0$, $0 \leq m_j \leq q_j$, $0 \leq n_j \leq p_j$ $(j=1,2,\cdots,r)$ (not all zero simultaneously). $\alpha_j^{(i)}$ $(j=1,2,\cdots,P, i=1,2,\cdots,r)$, $\beta_j^{(i)}$ $(j=1,2,\cdots,Q, i=1,2,\cdots,r)$, $\gamma_j^{(i)}$ $(j=1,2,\cdots,p_i, i=1,2,\cdots,r)$ and $\delta_j^{(i)}$ $(j=1,2,\cdots,q_i, i=1,2,\cdots,r)$ are positive numbers. a_j $(j=1,2,\cdots,P)$, b_j $(j=1,2,\cdots,Q)$, $c_j^{(i)}$ $(j=1,2,\cdots,p_i, i=1,2,\cdots,r)$ and $d_j^{(i)}$ $(j=1,2,\cdots,q_i, i=1,2,\cdots,r)$are complex numbers.

The exponents A_j $(j=1,2,\cdots,P)$, B_j $(j=1,2,\cdots,Q)$, $C_j^{(i)}$ $(j=1,2,\cdots,p_i, i=1,2,\cdots,r)$ and $D_j^{(i)}$ $(j=1,2,\cdots,q_i, i=1,2,\cdots,r)$ of various gamma functions may take integer values. The I-functioi of r-variables is analytic if

$$\Psi_i = \sum_{j=1}^{P}A_j\alpha_j^{(i)} - \sum_{j=1}^{Q}B_j\beta_j^{(i)} + \sum_{j=1}^{p_i}C_j^{(i)}\gamma_j^{(i)} - \sum_{j=1}^{q_i}D_j^{(i)}\delta_j^{(i)} \leq 0, i=1,2,\cdots,r.$$

The integral (1.1) converges absolutely if $|arg(z_i)| < \frac{1}{2}\pi\Delta_i, i=1,2,\cdots,r$ where

$$\Delta_i = -\sum_{j=N+1}^{P}A_j\alpha_j^{(i)} - \sum_{j=1}^{Q}B_j\beta_j^{(i)} + \sum_{j=1}^{m_i}D_j^{(i)}\delta_j^{(i)} - \sum_{j=m_i+1}^{q_i}D_j^{(i)}\delta_j^{(i)}$$

$$+\sum_{j=1}^{n_i} C_j^{(i)}\gamma_j^{(i)} - \sum_{j=n_i+1}^{p_i} C_j^{(i)}\gamma_j^{(i)} > 0. \tag{1.4}$$

The $\overline{I}$- function of r-variables is defined and represented in the following manner:

$$\overline{I}[z_1,\cdots,z_r] = I^{0,N:m_1,n_1;\cdots;m_r,n_r}_{P,Q:p_1,q_1;\cdots;p_r,q_r}$$
$$\left[\begin{array}{c|c} z_1 & {}_1(a_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)};A_j)_P : {}_1(c_j^{(1)},\gamma_j^{(1)};C_j^{(1)})_{p_1};\cdots;{}_1(c_j^{(r)},\gamma_j^{(r)};C_j^{(r)})_{p_r} \\ \vdots & {}_1(b_j;\beta_j^{(1)},\cdots,\beta_j^{(r)},B_j)_Q : {}_1(d_j^{(1)},\delta_j^{(1)};1)_{m_1};\ {}_{m_1+1}(d_j^{(1)},\delta_j^{(1)};D_j^{(1)})_{q_1} \\ z_r & \cdots;\ {}_1(d_j^{(r)},\delta_j^{(r)};1)_{m_r},{}_{m_r+1}(d_j^{(r)},\delta_j^{(r)};D_j^{(r)})_{q_r} \end{array}\right]$$

$$= \frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r} \overline{\theta}_1(s_1)\cdots\overline{\theta}_r(s_r)\ \ \phi(s_1,\cdots,s_r)\ z_1^{s_1}\cdots z_r^{s_r}ds_1\cdots ds_r, \tag{1.5}$$

where

$$\overline{\theta}_i(s_i) = \frac{\prod_{j=1}^{m_i}\Gamma(d_j^{(i)}-\delta_j^{(i)}s_i)\prod_{j=1}^{n_i}\Gamma^{C_j^{(i)}}(1-c_j^{(i)}+\gamma_j^{(i)}s_i)}{\prod_{j=m_i+1}^{q_i}\Gamma^{D_j^{(i)}}(1-d_j^{(i)}+\delta_j^{(i)}s_i)\prod_{j=n_i+1}^{p_i}\Gamma^{C_j^{(i)}}(c_j^{(i)}-\gamma_j^{(i)}s_i)}.$$

The integral converges absolutely of $|arg(z_i)| < \frac{1}{2}\pi\Delta_i', i=1,2,\cdots,r$ where

$$\Delta_i' = -\sum_{j=N+1}^{P} A_j\alpha_j^{(i)} - \sum_{j=1}^{Q} B_j\beta_j^{(i)} + \sum_{j=1}^{m_i}\delta_j^{(i)} - \sum_{j=m_i+1}^{q_i} D_j^{(i)}\delta_j^{(i)} + \sum_{j=1}^{n_i} C_j^{(i)}\gamma_j^{(i)} - \sum_{j=n_i+1}^{p_i} C_j^{(i)}\gamma_j^{(i)} \geq 0$$

$$f(p) \doteq f(t) = f(p) = p\int_0^\infty e^{-pt}f(t)dt \tag{1.6}$$

provided the integral is convergent and $Re(p) > 0$.

$$\int_0^\infty e^{-pt}t^{x-1}dt = \frac{\Gamma(x)}{p^x},\ \ Re(p) > 0. \tag{1.7}$$

2. r-integrals Involving the I-functions of Several Variables

If $f(p) \doteq$ f(t) and

$$\psi(t)=\frac{1}{a_1^{s_1}a_2^{s_2}\cdots a_r^{s_r}}t^{-s_1-s_2\cdots-s_r}\overline{I}^{0,N:m_1,n_1+1;\cdots;m_r,n_r+1}_{P,Q:p_1+1,q_1;\cdots;p_r+1,q_r}$$
$$\left[\begin{array}{c|c} & {}_1(a_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)};A_j)_P:(1-s_1,1;1),\ {}_1(c_j^{(1)},\gamma_j^{(i)};C_j^{(1)})_{p_1}; \\ 1/a_1t & \cdots;(1-s_r,1;1),\ {}_1(c_j^{(r)},\gamma_j^{(r)};C_j^{(r)})_{p_r} \\ \vdots & \\ 1/a_rt & {}_1(b_j;\beta_j^{(1)},\cdots,\beta_j^{(r)};B_j)_Q:\ {}_1(d_j^{(1)},\delta_j^{(1)};1)_{m_1},\ {}_{m_i+1}(d_j^{(1)},\delta_j^{(1)};D_j^{(1)})_{q_1}; \\ & \cdots;\ {}_1(d_j^{(r)},\delta_j^{(r)};1)_{m_r},\ {}_{m_r+1}(d_j^{(r)},\delta_j^{(r)};D_j^{(r)})_{q_r} \end{array}\right]f(t), \quad (2.1)$$

then

$$\int_0^\infty\cdots\int_0^\infty z_1^{s_1-1}\cdots z_r^{s_r-1}\overline{I}[z_1\cdots z_r](a_1z_1+\cdots+a_rz_r+c)^{-1}f(a_1z_1+\cdots+a_rz_r)dz_1\cdots dz_r$$
$$=\frac{\psi(c)}{c}. \tag{2.2}$$

Provided

(i) $Min\left\{Re\left(s_i+\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)\right\}>0,\quad j=1,2,\cdots,m_i;i=1,2,\cdots,r$

(ii) $|arg(z_i)|<\frac{1}{2}\pi\Delta_i',i=1,2,\cdots,r$ where

$$\begin{aligned}\Delta_i &= -\sum_{j=n+1}^{P}A_j\alpha_j^{(i)}-\sum_{j=1}^{Q}B_j\beta_j^{(i)}+\sum_{j=1}^{m_i}\delta_j^{(i)}-\sum_{j=m_i+1}^{q_i}D_j^{(i)}\delta_j^{(i)}\\ &\quad+\sum_{j=1}^{n_i}C_j^{(i)}\gamma_j^{(i)}-\sum_{j=n_i+1}^{p_i}C_j^{(i)}\gamma_j^{(i)}>0\end{aligned}$$

(iii)

$$\begin{aligned}\Psi_i &= \sum_{j=1}^{P}A_j\alpha_j^{(i)}-\sum_{j=1}^{Q}B_j\beta_j^{(i)}+\sum_{j=1}^{p_i}C_j^{(i)}\gamma_j^{(i)}-\sum_{j=1}^{m_i}\delta_j^{(i)}\\ &\quad-\sum_{j=m_i+1}^{q_i}D_j^{(i)}\delta_j^{(i)}\leq 0,i=1,2,\cdots,r.\end{aligned}$$

(iv) $Min\{Re(a_1),Re(a_2),\cdots,Re(a_r),Re(c)\}>0.$

Proof :

$$
\begin{aligned}
LHS &= \int_0^\infty \cdots \int_0^\infty z_1^{s_1-1} \cdots z_r^{s_r-1} \overline{I}[z_1 \cdots z_r](a_1 z_1 + \cdots + a_r z_r + c)^{-1} \\
&\quad f(a_1 z_1 + \cdots + a_r z_r + c) dz_1 \cdots dz_r \\
&= \int_0^\infty \cdots \int_0^\infty z_1^{s_1-1} \cdots z_r^{s_r-1} \left\{ \left(\frac{1}{2\pi i}\right)^r \int_{L_1} \cdots \int_{L_r} \overline{\theta}_1(\lambda_1) \cdots \overline{\theta}_r(\lambda_r) \right. \\
&\quad \left. \phi(\lambda_1, \cdots, \lambda_r) z_1^{\lambda_1} \cdots z_r^{\lambda_r} d\lambda_1 \cdots d\lambda_r \right\} \\
&\quad \left\{ \int_0^\infty e^{-(a_1 z_1 + \cdots + a_r z_r + c)t} f(t) dt \right\} dz_1 \cdots dz_r.
\end{aligned}
$$

Now changing the order of integration, which is justified under the given conditions, and evaluating the inner integral, LHS is

$$
\begin{aligned}
&= \int_0^\infty e^{-ct} \left\{ \left(\frac{1}{2\pi i}\right)^r \int_{L_r} \cdots \int_{L_r} \overline{\theta}_1(\lambda_1) \cdots \overline{\theta}_r(\lambda_r) \phi(\lambda_1, \cdots, \lambda_r) \right. \\
&\left. \left[\int_0^\infty \cdots \int_0^\infty z_1^{\lambda_1+s_1-1} \cdots z_r^{\lambda_r s_r-1} e^{-(a_1 z_1 + \cdots + a_r z_r)t} dz_1 \cdots dz_r \right] d\lambda_1 \cdots d\lambda_r \right\} f(t) dt \\
&= \int_0^\infty e^{-ct} \left\{ \left(\frac{1}{2\pi i}\right) \int_{L_1} \cdots \int_{L_r} \overline{\theta}_1(\lambda_1) \cdots \overline{\theta}_r(\lambda_r) \phi(\lambda_1, \cdots, \lambda_1) \right. \\
&\left. \left[\frac{\Gamma(\lambda_1 + s_1)}{(a_1 t)^{\lambda_1+s_1}} \cdots \frac{\Gamma(\lambda_r + s_r)}{(a_r t)^{\lambda_r+s_r}} \right] d\lambda_1 \cdots d\lambda_r \right\} f(t) dt \\
&= \int_0^\infty e^{-ct} \left\{ \left(\frac{1}{2\pi i}\right)^r \int_{L_1} \cdots \int_{L_r} \overline{\theta}_1(\lambda_1) \cdots \overline{\theta}_r(\lambda_r \phi(\lambda_1, \cdots \lambda_r) \Gamma(\lambda_1 + s_1) \cdots \Gamma(\lambda_r + s_r) \right. \\
&\left. \left(\frac{1}{a_1 t}\right)^{\lambda_1} \cdots \left(\frac{1}{a_r t}\right)^{\lambda_r} d\lambda_1 \cdots d\lambda_r \right\} \left(\frac{1}{a_1 t}\right)^{s_1} \cdots \left(\frac{1}{a_r t}\right)^{s_r} f(t) dt \\
&= \int_0^\infty e^{-ct} \psi(t) dt, \quad \text{on using (1.5), (1.7) and (2.1)} \\
&= \frac{\psi(c)}{c}, \quad \text{on using (1.6).}
\end{aligned}
$$

Special Cases

1. Taking $r = 2$, the result reduces to the double integral involving I -function of 2 variables.

If $f(p) \doteq f(t)$ and

$$\psi(t) = \frac{1}{a_1^{s_1} a_2^{s_2}} t^{-s_1-s_2} \overline{I}^{0,N:m_1,n_1+1;m_2,n_2+1}_{P,Q:p_1+1,q_1;p_2+1,q_2}$$

$$\left[\begin{array}{c|c} 1/a_1 t & {}_1(a_j;\alpha_j^{(1)},\alpha_j^{(2)};A_j)_P : (1-s_1,1;1),\ {}_1(c_j^{(1)},\gamma_j^{(1)};C_j^{(1)})_{p_1}; \\ & (1-s_2,1),\ {}_1(c_j^{(2)},\gamma_j^{(2)};C_j^{(2)})_{p_2} \\ 1/a_2 t & {}_1(b_j;\beta_j^{(1)},\beta_j^{(2)};B_j)_Q :\ {}_1(d_j^{(1)},\delta_j^{(1)},D_j^{(1)})_{q_1};\ {}_1(d_j^{(2)},\delta_j^{(2)};D_j^{(2)})_{q_2} \end{array} \right] f(t),$$

then

$$\int_0^\infty \int_0^\infty z_1^{s_1-1} z_r^{s_2-1} \overline{I}[z_1 \cdots z_2](a_1 z_1 + a_2 z_2 + c)^{-1} \phi(a_1 z_1 + a_2 a_2 + c) dz_1 dz_2$$

$$= \frac{\psi(c)}{c}. \tag{2.3}$$

Provided

$$Min\left\{Re\left(s_i + \frac{d_j^{(i)}}{\delta_j^{(i)}}\right)\right\} > 0, \quad j = 1,2,\cdots,m_i; i = 1,2$$

$|arg(z_i)| < \frac{1}{2}\pi\Delta'_i, i = 1,2$, where

$$\begin{aligned} \Delta'_i &= -\sum_{j=n+1}^{P} A_j\alpha_j^{(i)} - \sum_{j=1}^{Q} B_j\beta_j^{(i)} + \sum_{j=1}^{m_i} \delta_j^{(i)} - \sum_{j=m_i+1}^{q_i} D_j^{(i)}\delta_j^{(i)} \\ &\quad + \sum_{j=1}^{n_i} C_j^{(i)}\gamma_j^{(i)} - \sum_{j=n_i+1}^{p_i} C_j^{(i)}\gamma_j^{(i)} > 0. \end{aligned}$$

$$\begin{aligned} \Psi_i &= \sum_{j=1}^{P} A_j\alpha_j^{(i)} - \sum_{j=1}^{Q} B_j\beta_j^{(i)} + \sum_{j=1}^{p_i} C_j^{(i)}\gamma_j^{(i)} - \sum_{j=1}^{m_i} \delta_j^{(i)} \\ &\quad - \sum_{j=m_i+1}^{q_i} D_j^{(i)}\delta_j^{(i)} \le 0, i = 1,2. \end{aligned}$$

$Min\{Re(a_1), Re(a_2), Re(c)\} > 0$.

2. When all the exponents are equal to unity, (2.2) reduces to the r-integrals involving the H-functions of several variables given by Prasanth and Nambisan [2,p-102].

If $f(p) \doteq f(t)$ and

$$\psi(t) = \frac{1}{a_1^{s_1} a_2^{s_2} \cdots a_r^{s_r}} t^{-s_1-s_2-s_3} H^{0,N:m_1;\cdots;m_r,n_r+1}_{P,Q:p_1+1,q_1;\cdots;p_r+1,q_r}$$

$$\left[\begin{array}{c|c} \begin{array}{c} 1/a_1 t \\ \vdots \\ 1/a_r t \end{array} & \begin{array}{c} {}_1(a_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)})_P : (1-s_1,1),\ {}_1(c_j^{(1)},\gamma_j^{(1)})_{p_1}; \\ \cdots,(1-s_r,1),\ {}_1(c_j^{(r)},\gamma_j^{(r)})_{p_r} \\ {}_1(b_j;\beta_j^{(1)},\cdots,\beta_j^{(r)})_Q :\ {}_1(d_j^{(1)},\delta_j^{(1)})_{q_1};\cdots;\ {}_1(d_j^{(r)},\delta_j^{(r)})_{q_r} \end{array} \end{array}\right] f(t),$$

then

$$\int_0^\infty \cdots \int_0^\infty z_1^{s_1-1}\cdots z_r^{s_r-1} H[z_1\cdots z_2](a_1z_1+\cdots+a_rz_r+c)^{-1}\phi(a_1z_1+\cdots a_ra_r+c)dz_1\cdots dz_r$$

$$= \frac{\psi(c)}{c}. \tag{2.4}$$

Provided

$$Min\left\{Re\left(s_i+\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)\right\} > 0, \quad j=1,2,\cdots,m_i; i=1,2,\cdots,r$$

$|arg(z_i)| < \frac{1}{2}\pi\Delta_i', i=1,2,\cdots,r$ where

$$\Delta_i = -\sum_{j=n+1}^{P}\alpha_j^{(i)} - \sum_{j=1}^{Q}\beta_j^{(i)} + \sum_{j=1}^{m_i}\delta_j^{(i)} - \sum_{j=m_i+1}^{q_i}\delta_j^{(i)} + \sum_{j=1}^{n_i}\gamma_j^{(i)} - \sum_{j=n_i+1}^{p_i}\gamma_j^{(i)} > 0.$$

$$\Psi_i = \sum_{j=1}^{P}\alpha_j^{(i)} - \sum_{j=1}^{Q}\beta_j^{(i)} + \sum_{j=1}^{p_i}\gamma_j^{(i)} - \sum_{j=1}^{m_i}\delta_j^{(i)} \le 0, i=1,2,\cdots,r.$$

$Min\{Re(a_1), Re(a_2),\cdots,Re(a_r),Re(c)\} > 0.$

3. When $r=2$, (2.4) reduces to corresponding result for H function of 2 variables.

If $f(p) \doteq f(t)$ and

$$\psi(t) = \frac{1}{a_1^{s_1}a_2^{s_2}} t^{-s_1-s_2} H^{0,N:m_1,n_1+1;m_2,n_2+1}_{P,Q:p_1+1,q_1;p_2+1,q_2}$$

$$\left[\begin{array}{c|c} \begin{array}{c} 1/a_1 t \\ \vdots \\ 1/a_2 t \end{array} & \begin{array}{c} {}_1(a_j;\alpha_j^{(1)},\alpha_j^{(2)})_P : (1-s_1,1),\ {}_1(c_j^{(1)},\gamma_j^{(1)})_{p_1};(1-s_2,1),\ {}_1(c_j^{(2)},\gamma_j^{(2)})_{p_2} \\ \\ {}_1(b_j;\beta_j^{(1)},\beta_j^{(2)})_Q :\ {}_1(d_j^{(1)},\delta_j^{(1)})_{q_1};\ {}_1(d_j^{(2)},\delta_j^{(2)})_{q_2} \end{array} \end{array}\right] f(t),$$

then

$$\int_0^\infty \int_0^\infty z_1^{s_1-1} z_2^{s_2-1} H[z_1, z_2](a_1 z_1 + a_2 z_2 + c)^{-1} f(a_1 z_1 + a_2 z_2 + c) dz_1 dz_2$$

$$= \frac{\psi(c)}{c}. \tag{2.5}$$

Provided

$$Min \left\{ Re \left(s_i + \frac{d_j^{(i)}}{\delta_j^{(i)}} \right) \right\} > 0, \quad j = 1, 2, \cdots, m_i; i = 1, 2.$$

$|arg(z_i)| < \frac{1}{2}\pi\Delta_i'$, where

$$\Delta_i = -\sum_{j=n+1}^{P} \alpha_j^{(i)} - \sum_{j=1}^{Q} \beta_j^{(i)} + \sum_{j=1}^{m_i} \delta_j^{(i)} - \sum_{j=m_i+1}^{q_i} \delta_j^{(i)} + \sum_{j=1}^{n_i} \gamma_j^{(i)} - \sum_{j=n_i+1}^{p_i} \gamma_j^{(i)} > 0.$$

$$\Psi_i = \sum_{j=1}^{P} \alpha_j^{(i)} - \sum_{j=1}^{Q} \beta_j^{(i)} + \sum_{j=1}^{p_i} \gamma_j^{(i)} - \sum_{j=1}^{m_i} \delta_j^{(i)} \leq 0, i = 1, 2.$$

$Min\{Re(a_1), Re(a_2), \cdots, Re(c)\} > 0.$

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