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FIXED POINT THEOREM IN $\mathcal{L}$-FM SPACE SATISFYING A CONTRACTIVE CONDITION OF INTEGRAL TYPE

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Abstract

1. Introduction

In 1965, the concept of fuzzy sets was introduced initially by Zadeh [13]. Since then, to use this concept in topology and analysis many authors have expansively developed the theory of fuzzy sets and applications. In particular, George and Veeramani [61] have introduced and studied a notion of fuzzy metric space with the help of continuous t-norms, which constitutes a slight but appealing modification of the one due to Kramosil and Michalek [89]. Deschrijver and Kerre [have shown that intuitionistic fuzzy sets can also be seen as L-fuzzy sets in the sense of Goguen [66]. Using the idea of L-fuzzy sets [66], Saadati et.al. [148] introduced the notion of L-fuzzy metric spaces with the help of continuous t-norms as a generalization of fuzzy metric space due to George and Veeramani [61] and intuitionistic fuzzy metric space due to Park and Saadati [121, 146, 147] and prove a common fixed point theorem for a pair of commuting mappings. Later on in [145] he introduce the notion of uniform continuity and equicontinuity in an L-

-fuzzy metric space and prove Uniform continuity theorem for L-fuzzy metric space and prove Ascoli. Arzela theorem for L-fuzzy metric space. In 2008 [144] he also prove fuzzy Banach and Edelstein fixed point theorems in generalized fuzzy metric spaces i.e., L-fuzzy metric spaces for modified definition of Cauchy sequence in George and Veeramanifs sense. In 2008 Efe [53] prove Bairefs theorem and uniform limit theorem for$L$-fuzzy metric spaces and in 2010 Shakeri, Ciric and Saadati [164] prove fixed point theorem in Partially Ordered $\mathcal{L}$-Fuzzy Metric Spaces which is an extension of Nieto and Rodriguez- Lopez [111,112] and Ran and Reurings [132].

Adibi et.al. [5] introduced the concept of compatible mappings and proved common fixed point theorems for four mappings satisfying some conditions in L-fuzzy metric spaces which results are further generalized by Saadati et.al.[149]. Huang et.al. [70] prove fixed point theorems for any even number of compatible mappings in complete L-fuzzy metric spaces.

Branciari [30] initiated the study of contractive conditions of integral type in 2002 and give integral version of Banach contraction principle which was further generalized by Rhoades [141]. Several common fixed point theorems for a family of four mappings satisfying some contractive conditions of integral type were established in [14, 47, 48] and [12].

In metric fixed point theory, various mathematicians weakened the notion of commutativity by introducing the notions of weak commutativity, compatibility and weak compatibility and produced a number of fixed point theorems using these notions. It is worth to mention that every pair of commuting self-maps is weakly commuting, each pair of weakly commuting self-maps is compatible and each pair of compatible self-maps is weak compatible but the reverse is not always true. The main object of this chapter is to prove common fixed point theorem in $\mathcal{L}$-fuzzy metric space for weakly compatible mappings satisfying integral type contractive condition and property (C).Which is a generalization of some results Adibi et. al. [5] for this first , we recall some definitions and known results that will be used in the sequel.

2. Preliminary

Definition 2.1 : Let $\mathcal{L} = (L, \leq_L)$ be a complete lattice, and U a nonempty set called a universe. An $\mathcal{L}$-fuzzy set $\mathcal{L}$ on U is defined as a mapping: $U \to L$. For each u in U,

$\mathcal{A}(u)$ represents the degree (in L) to which u satisfies $\mathcal{A}$.

Lemma 2.2 : Consider the set L^* and the operation $\leq_{L^*}$ defined by:

$$L^* = \{(x_1, x_2) : (x_1, x_2) \in [0,1]^2 \text{ and } x_1 + x_2 \leq 1\},$$

$$(x_1, x_2). \leq_{L^*} (y_1, y_2) \Leftrightarrow x_1 \leq y_1 \text{ and } x_2 \leq y_2,$$

for every $(x_1, x_2), (y_1, y_2) \in L^*$. Then $(L^*, \leq_{L^*})$ is a complete lattice.

Classically, a triangular norm T on $([0,1], \leq)$ is defined as an increasing, commutative, associative mapping $T : [0,1]^2 \rightarrow [0,1]$ satisfying $T(1,x) = x$, for all $x \in [0,1]$.

These definitions can be straightforwardly extended to any lattice $\mathcal{L} = (L, \leq_L)$. Define first

$$0_{\mathcal{L}} = \inf L \text{ and } 1_{\mathcal{L}} = \sup L.$$

Definition 2.3 : A triangular norm (t-norm) on $\mathcal{L}$ is a mapping $T : L^2 \rightarrow L$ satisfying the following conditions:

2.3 (i) $(\forall x \in L)(T(x, 1_{\mathcal{L}}) = x)$

2.3 (ii) $(\forall (x,y) \in L^2)(T(x,y) = T(y,x))$

2.3 (iii) $(\forall (x,y,z) \in L^3)(T(x, T(y,z)) = T(T(x,y), z))$

2.3(iv) $(\forall (x, x', y, y') \in L^4)(x \leq_L x' \text{ and } y \leq_L y' \Rightarrow T(x,y) \leq_L T(x'f, y'))$.

Definition 2.4 : A t-norm T on $\mathcal{L}$ is said to be continuous if for any $x, y \in \mathcal{L}$ and any sequence $\{x_n\}$ and $\{y_n\}$ which converge to x and y we have

$$\lim_{n \to \infty} T(x_n, y_n) = T(x, y).$$

Example 2.5 : $T(x,y) = \min(x,y)$ and $T(x,y) = xy$ are two continuous t- norms on [0 ,1]. A t-norm can also be defined recursively as an $(n+1)$ -ary operation $(\in N)$ by $T^1 = T$ and

$$T^n(x_1, x_2, \cdots, x_{n+1}) = T(T^{n-1}(x_1, x_2, \cdots, x_n), x_{n+1})$$

for $n \geq 2$ and $x_i \in L$, $1 \leq i \leq n+1$.

Definition 2.6 : A negation on $\mathcal{L}$ is any decreasing mapping $N : \mathcal{L} \rightarrow \mathcal{L}$ satisfying $N(0_{\mathcal{L}}) = 1_{\mathcal{L}}$ and $N(1_{\mathcal{L}}) = 0_{\mathcal{L}}$.

If $N(N(x)) = x$, for all $x \in \mathcal{L}$, then N is called an involutive negation is fixed.

If, for all $\in [0,1]$, $N_s(x) = 1 - x$, we say that Nsis the standard negation on $([0,1], \leq)$.

Definition 2.7 : The 3-tuple (X, M, T) is said to be an $\mathcal{L}$-fuzzy metric space if X is an arbitrary (non-empty) set, T is a continuous t-norm on $\mathcal{L}$ and M is an $\mathcal{L}$-fuzzy set on $X^2 \times (0, \infty)$ satisfying the following conditions for all $x, y, z \in X$ and $s, t > 0$.

2.7 (i) $M(x, y, t) >_L 0_{\mathcal{L}}$.

2.7 (ii) $M(x, y, t) = 1_{\mathcal{L}}$ for all $t > 0$ if and only if $x = y$

2.7 (iii) $M(x, y, t) = M(y, x, t)$

2.7 (iv) $T(M(x, y, t), M(y, z, s)) \leq_L M(x, z, t + s)$ for all $x, y, z \in X$ and $s, t > 0$

2.7 (v) $M(x, y, .) : (0, \infty) \to L$ is continuous and $\lim_{t\to\infty} M(x, y, t) = 1_{\mathcal{L}}$.

In this case M is called an $\mathcal{L}$ -fuzzy metric.

Definition 2.8 : Let (X, M, T) be an $\mathcal{L}$-fuzzy metric space, For $t \in (0, \infty)$ we define the open ball $B(x, r, t)$ with center $x \in X$ and radius $r \in L \setminus \{0_{\mathcal{L}}, 1_{\mathcal{L}}\}$ is defined by

$$B(x, r, t) = \{y \in X : M(x, y, t) >_L N(r)\}.$$

A subset $A \subseteq X$ is called open if for each $x \in A$, there exists $t > 0$ and $r \in L \setminus \{0_{\mathcal{L}}, 1_{\mathcal{L}}\}$, such that $B(x, r, t) \subseteq A$.

Let τ_M denote the family of all open subsets of X. Then τ_M is called the topology induced by the $\mathcal{L}$ -fuzzy metric M.

Lemma 2.9 : Let (X, M, T) be an $\mathcal{L}$-fuzzy metric space. Then, $M(x, y, t)$is non-decreasing with respect to t, for all $x, y \in X$.

Definition 2.10 : Let (X, M, T) be an $\mathcal{L}$-fuzzy metric space and $\{x_n\}$ be a sequence in X.

(1) $\{x_n\}$ is said to be convergent to a point $x \in X$ (denoted by $\lim\limits_{n\to\infty} x_n = x$) if $\lim\limits_{n\to\infty} M(x, x_n, t) = 1_{\mathcal{L}}$ for all $t > 0$.

(2) $\{x_n\}$ is called a Cauchy sequence if for each $\epsilon \in L \setminus \{0_{\mathcal{L}}\}$ and $t > 0$, there exists $n_0 \in \mathbf{N}$, such that $M(x_n, x_m, t) >_L N(\epsilon)$ for all $m \geq n \geq n_0$, $(n \geq m \geq n_0)$.

(3) A $\mathcal{L}$ -fuzzy metric in which every Cauchy sequence is convergent is said to be complete. Hence forth, we assume that T is a continuous t-norm on the lattice L, such that for every $\mu \in L \setminus \{0_{\mathcal{L}}, 1_{\mathcal{L}}\}$, there is a $\lambda \in L \setminus \{0_{\mathcal{L}}, 1_{\mathcal{L}}\}$, such that

$$T^{n-1}(N(\lambda), \cdots, N(\lambda)) \geq_L N(\mu).$$

Definition 2.11 : Let (X, M, T) be an $\mathcal{L}$-fuzzy metric space. M is said to be continuous functions on $X^2 \times (0, \infty)$ if

$$\lim_{n\to\infty} M(x_n, y_n, t_n) = M(x, y, t),$$

whenever a sequence $\{(x_n, y_n, t_n)\}$ in $X^2 \times\ (0, \infty)$ converges to a point $(x, y, t \in X^2 \times (0, \infty)$ i.e.

$$\lim_{n\to\infty} M(x_n, x, t) = \lim_{n\to\infty} M(y_n, , y, t) = 1_{\mathcal{L}}$$
$$\text{and} \lim_{n\to\infty} M(x, y, t_n) = M(x, y, t).$$

Lemma 2.12 : Let (X, M, T) be an $\mathcal{L}$ -fuzzy metric space. Then M is continuous functions on $X^2 \times (0, \infty)$.

Lemma 2.13 : Let (X, M, T) be an $\mathcal{L}$ -fuzzy metric space. If we define $E_{\lambda,M} : X^2 \to \mathbf{R}^+ \cup \{0\}$ by

$$E_{\lambda,M}(x, y) = \inf\{t > 0 : M(x, y, t) >_L LN(\lambda)\}$$

for all $\lambda \in L \setminus \{0_{\mathcal{L}}, 1_{\mathcal{L}}\}$ and $x, y \in X$, then

(1) For all $\mu \in L \setminus \{0_{\mathcal{L}}, 1_{\mathcal{L}}\}$ there exists $\lambda \in L \setminus \{0_{\mathcal{L}}, 1_{\mathcal{L}}\}$, such that

$$E_{\mu,M}(x_1, x_n) \leq E_{\lambda,M}(x_1, x_2) + E_{\lambda,M}(x_2, x_3) + \cdots + E_{\lambda,M}(x_{n-1}, x_n)$$

for all $x_1, x_2 \cdots , x_n \in X$.

(2) The sequence $\{x_n\}_{n\in N}$ is convergent to x w.r.t. $\mathcal{L}$ -fuzzy metric M if and only if $E_{\lambda M}(x_n, x) \to 0$.

Also the sequence $\{x_n\}_{n\in N}$ is Cauchy w.r.t. $\mathcal{L}$-fuzzy metric M if and only if it is Cauchy with $E_{\lambda,M}$.

We shall need the following lemma for proof of our main theorem:

Lemma 2.14 : Let (X, M, T) be a $\mathcal{L}$-fuzzy metric space. If

$$M(x_n, x_{n+1}, t) \geq_L M(x_0, x_1, k^n t)$$

for some $k > 1$ and for every $n \in \mathbf{N}$. Then sequence $\{x_n\}$ is a Cauchy sequence.

Definition 2.15 : We say that the $\mathcal{L}$-fuzzy metric space (X, M, T) has property (C), if it satisfies the following condition:

$$M(x, y, t) = C,$$

for all

$$t > 0 \Rightarrow C = 1_{\mathcal{L}}.$$

Definition 2.16 : Let S and T be two mappings from an $\mathcal{L}$-fuzzy metric space (X, M, T) into itself and $\{x_n\}$ be a sequence in X such that

$$li_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = z$$

for some $z \in X$. Then the mapping S and T are said to be compatible if

$$\lim_{n\to\infty} M(STx_n, TSx_n, t) = 1_{\mathcal{L}} \text{ for all } t > 0.$$

Definition 2.17 : Let S and T be mappings from an $\mathcal{L}$ -fuzzy metric space (X, M, T) into itself. The maps S and T are said to be weakly compatible if they commute at their coincidence points, i.e. if $S_p = T_p$ for some $p \in X$, then

$$ST_p = TS_p.$$

Proposition 2.18 : Self mappings S and T of an $\mathcal{L}$ -fuzzy metric space (X, M, T) are compatible then they are weakly compatible.
In fact Branciari give a following Integral contractive type condition.
For a given $\epsilon > 0$, there exists a real number $c \in (0, 1)$ and a locally Lebesgue-integrable function $g : [0, \infty) \to [0, \infty)$ Such that

$$\int_0^{d(fx,fy,t} g(t)dt \leq_c \int_0^{d(x,y)} g(t)dt \text{ and}$$

$$\in_0^\epsilon g(t)dt > 0 \quad \text{for all } x, y \in X \quad \text{and for each} \epsilon > 0.$$

Also, Branciari-Integral contractive type condition is a generalization of Banach contraction map if $g(t) = 1$ for all $t \geq 0$.

3. Main Result

Theorem 3.1 : Let A, B, S and T be self mappings of a compete $\mathcal{L}$ fuzzy metric space (X, M, T) which has property (C), satisfying:

3.1 (I) $A(X) \subseteq T(X)$, $B(X) \subseteq (X)$ and $T(X), S(X)$ are two closed subsets of X.

3.1 (II) The pairs (A, S) and (B, T) are weak compatible.

3.1 (III) $\int_0^{M(Ax,By,t)} \phi(t)dt \geq_L \int^{M(Sx,Ty,kt)M(Sx,sx,kt)M(Ty,Ty,kt)} \phi(t)dt$ for every $x, y \in X$ and some $k > 1$. Where $\varphi : R^+ \to R$ is a Lebesgue-integrable mapping which is summable, nonnegative and such that

$$\int_0^{\epsilon} \phi(t)dt > 0, \quad \epsilon > 0.$$

Then A, B, S and T have a unique common fixed point in X.

Proof : Let $x_0 \in X$ be an arbitrary point in X. By 3.1(I),there is $x_1, x_2 \in X$ such that

$$y_0 = Ax_0 = Tx_1,$$

$$y_1 = Bx_1 = Sx_2.$$

Inductively, construct sequences $\{y_n\}$ and $\{x_n\}$ in X such that

$$y_{2n} = Ax_{2n} = Tx_{2n+1}, y_{2n+1} = Bx_{2n+1} = Sx_{2n+2}$$

for $n = 0, 1, 2, \cdots$.

Now, we prove that $\{y_n\}$ is a Cauchy sequence.

Let $d_m(t) = M(y_m, y_{m+1}, t), t > 0$. Then, we have

$$\begin{aligned}
\int_0^{d_{2n}(t)} \phi(t)dt) &= \int^{M(y_{2n},y_{2n+1},t)M(y_{2n},y_{2n},t)(M(y_{2n+1},y_{2n+1},t))} \phi(t)dt \\
&= \int^{M(Ax_{2n},Bx_{2n+1},t)M(Ax_{2n}Ax_{2n},t)M(Bx_{2n+1},Bx_{2n+1},t)} \phi(t)dt \\
&\geq_L \int^{M(Sx_{2n},Tx_{2n+1},kt)*1*!} \phi(t)dt \\
&= \int^{M(y_{2n-1},y_{2n},kt)}]phi(t)dt \\
&= \int^{d_{2n-1}(kt)} \phi(t)dt.
\end{aligned}$$

Thus

$$d_{2n}(t) \geq_L d_{2n-1}(kt)$$

for every $m = 2n \in \mathbb{N}$ and $\forall t > 0$.

Similarly for an odd integer $m = 2n + 1$, we have $d_{2n+1}(t) \geq_L d_{2n}(kt)$.

Hence for every $n \in \mathbb{N}$, we have

$$d_n(t) \geq_L d_{n-1}(kt).$$

That is,

$$\int^{M(y_n,y_{n+1},t)} \phi(t)dt \;\; \geq \;\; _L\int^{M(y_{n-1},y_n,kt)M(y_{n-1},y_{n-1}ktM(y_n,y_n,kt))} \phi(t)dt \geq_L$$
$$\geq_L \int^{M(y_0,y_1,k^n t)} \phi(t)dt.$$

So, by Lemma 2.14, $\{y_n\}$ is Cauchy and the completeness of X implies $\{y_n\}$ converges to y in X. That is

$$\lim_{n\to\infty} y_n = y$$

$$\lim_{n\to\infty} y_{2n} = \lim_{n\to\infty} Ax_{2n} = \lim_{n\to\infty} Tx_{2n+1} = \lim_{n\to\infty} y_{2n+1} = \lim_{n\to\infty} Bx_{2n+1} = \lim_{n\to\infty} Sx_{2n+2} = y.$$

As $B(X) \subseteq S(X)$, there is $u \in X$ such that $Su = y$.

By (iii), we have

$$\int^{M(Au,Bx_{2n+1},t)} \phi(t)dt \geq_L$$
$$\int^{M(Su,Tx_{2n+1},kt)M(Su,Su\ kt)M(Tx_{2n+1},Tx_{2n+1},kt)} \phi(t)dt.$$

Since M is continuous, we get(whenever $n \to \infty$ in the above inequality):

$$\int_0^{M(Au,y,t)} \phi(t)dt7ge_L \int^{M(y,y,kt)*1*1} \phi(t)dt = 1_{\mathcal{L}}.$$

Thus $M(Au, y, t) = 1_{\mathcal{L}}$,

i.e. $Au = y$.

Therefore, $Au = Su = y$.

Since $A(X) \subseteq T(X)$, there is $v \in X$ such that $Tv = y$. Thus,

$$\int_0^{M(y,Bv,t)} \phi(t)dt \;\; = \;\; \int^{M(Au,Bv,t)M(Au,Au,t)M(Bv,Bv,t))} \phi(t)dt$$
$$\geq_L \int_0^{M(Su,Tv,kt)} \phi(t)dt$$
$$= 1_{\mathcal{L}}.$$

Hence $Tv = Bv = Su = y$.

Since (A, S) is weak compatible, we conclude that

$$ASu = SAu,$$

that is

$$Ay = Sy.$$

Also (B, T) is weak compatible then,

$$TBv = BTv$$

that is

$$Ty = By$$

We now prove that

$$Ay = y.$$

By 3.1(III), we have

$$\begin{aligned}\int^{M(Ay,y,t)} \phi(t)dt &= \int^{M(Ay,Bv,t)M(Ay,Ay,t)M(Bv,Bv,t)} \phi(t)dt. \\ &\geq_L \int_0^{M(Sy,Tv,kt)*1*1} \phi(t)dt \\ &\geq_L \int^{M(Ay,y,k^n,t)} \phi(t)dt.\end{aligned}$$

On the other hand, from Lemma 2.9 we have that

$$\int_0^{M(Ay,y,t)} \phi(t)dt \leq_L \int^{M(Ay,y,k^n t)} \phi(t)dt.$$

Hence,

$$M(Ay, y, t) = C \quad \text{for all} t > 0.$$

Since (X, M, T) has property (C) it follows that

$$C = 1_{\mathcal{L}},$$

i.e.,

$$Ay = y,$$

therefore

$$Ay = Sy = y.$$

Similarly we prove that

$$By = y.$$

By 3.1(III), we have

$$\begin{aligned}\int^{M(y,By,t)} \phi(t)dt &= \int^{M(Ay,By,t)M(Ay,Ay,t)M(By,By,t)} \phi(t)dt.\\ &\geq_L \int_0^{M(Sy,Ty,kt)} \phi(t)dt\\ &= \int^{M(y,By,kt)} \phi(t)dt\\ &\geq_L \int_0^{M(y,By,k^n t)} \phi(t)dt.\end{aligned}$$

On the other hand, from Lemma 2.9 we have that

$$\int_0^{M(y,By,t)} \phi(t)dt \leq_L \int_0^{M(y,by,k^n t)} \phi(t)dt.$$

Hence, $M(y, By, t) = C \quad \forall t > 0$.

Since (X, M, T) has property (C), it follows that $C = 1_{\mathcal{L}}$,

i.e. $By = y$.

Therefore

$$Ay = By = Sy = Ty = y.$$

i.e., y is a comman fixed point of A, B, S and T.

Uniqueness : Let x be another comman fixed point of A, B, S and T

i.e., $x = Ax = Bx = Sx = Tx$.

Hence

$$\begin{aligned}\int_0^{M(y,x,t)} \phi(t)dt &= \int^{M(y,Bx,t)M(y,y,tM(Bx,Bx,t))} \phi(t)dt.\\ &\geq_L \int^{M(Sy,Tx,kt)*1*1} \phi(t)dt\\ &= \int_0^{M(y,x,kt)} \phi(t)dt\\ &\geq_L \int_0^{M(y,x,k^n t)} \phi(t)dt.\end{aligned}$$

On the other hand, from Lemma 2.9 we have that

$$\int_0^{M(y,x,t)} \phi(t)dt \leq_L \int^{M(y,x,k^n t)} \phi(t)dt.$$

Hence,

$$M(y,x,t) = C \quad \forall \;\; t > 0.$$

Since (X, M, T) has property (C), it follows that $C = 1_{\mathcal{L}}$,

i.e. y= x.

Therefore, y is the unique comman fixed point of self maps A, B, S and T.

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