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THE MAGIC TRANSFORM : AN INTRODUCTION

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Abstract

The magic squares have attracted many researchers for their mysterious nature [1] since ancient ages. In this paper, different aspects of magic squares are studied using MATLAB. The magic squares have been imaged to grey scale images to categorise the patterns. A new transform has also been introduced which is to be called as 'Magic Transform'. After applying 'Magic Transform', the resultant matrices have been converted into grey scale images. The patterns have also been identified. In this paper, the magic square and magic transform are visualised which will be useful for encryption techniques of data security.

1. Introduction

The topic of Magic squares has attracted many researchers since 550 BC starting from V arahmihira,[2] an Indian Scholar from ancient days. Several attempts have been made by scholars to understand the beauty of magic squares. The magic squares also provide historical links between many civilizations. This paper aims to touch upon a new aspect of magic squares, by exploring the aesthetics of magic squares. This paper, introduces

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a transform over magic squares referred to as “Magic Transform” or “Magic Mapping”. Grey scale images of magic squares and its Magic Transform have also been studied. Then different patterns in greys have been observed and named. V arāhmihira [2] 550 BC had introduced a 4×4 magic square. Lee Sallows [3] has discussed the formation of 3×3 magic square using the formula of Edouard Lucas. He has also given the graphical representation of 3×3 magic square in real and complex plane.
W. S. Andrews [1] has defined, a magic square consists of a series of numbers so arranged in a square, that the sum of each row and column and of both the corner diagonals shall be the same amount which may be termed as ‘Magic Number’ (m) or ‘Summation’. The magic squares with the entries from 1 to n^2 have been considered. These magic squares may be called as ‘Basic Magic Squares’ or ‘Primary Magic Squares’. MATLAB has been used to generate the magic squares.

2. Patterns of Magic Squares

Magic squares of different sizes have been studied and after converting the magic squares into grey scale images, the following 3 patterns corresponding to $n = 4k$, $n = 4k + 1$ or $4k + 3$ and $n = 4k + 2$, where n is order of magic square, have been obtained.
Let n be the order of the magic matrix.

1. If the order of magic square is doubly even i.e. $n = 4k (\equiv 0 \ mod \ n)$, then the pattern observed looks like Chess board. So this pattern is named as ‘Chess Board’ pattern.

2. If the order of magic square is odd i.e. $n = 4k + 1$ or $4k + 3$ ($\equiv 1$ or $3 \ mod \ n$), then the pattern observed looks like diagonal strips. So this pattern is named as ‘Diagonal’ pattern.

3. If the order of magic square is singly even i.e. $n = 4k + 2$ ($\equiv 2 \ mod \ n$), then the pattern observed is a combination of chess board and diagonal strips. So this pattern is named as ‘Overlapping’ pattern.

For readers understanding, the grey scale images are considered for $n = 64$ ($\equiv 0 \ mod‘ \ 4$) as shown in Fig. 1, $n = 65$ ($\equiv 1 \ mod \ 4$) as shown in Fig. 2, $n = 66$ ($\equiv 2 \ mod \ 4$) as shown in Fig. 3 and $n = 67$ ($\equiv 3 \ mod \ 4$) as shown in Fig. 4. The grey scale images are obtained in each case are as below.

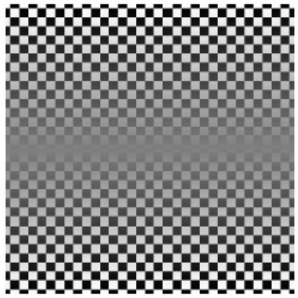

Figure 1: Chess Board image for magic matrix of order $n = 64$

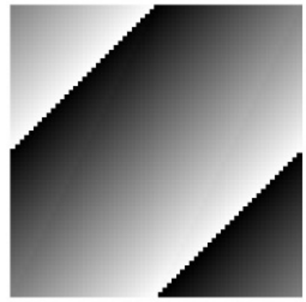

Figure 2: Diagonal image for magic matrix of order $n = 65$

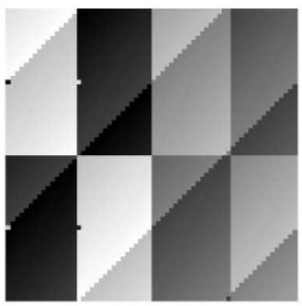

Figure 3: Overlapping image for magic matrix of order $n = 66$

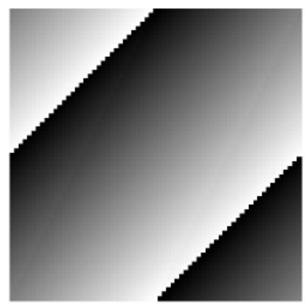

Figure 4: Diagonal image for magic matrix of order $n = 67$

3. Dasre-Gujarathi Transform : Magic Transform

As magic square has interesting patterns, here symmetric row/column transformations were applied to achieve beautiful patterns and this transformation to be termed as Magic Transform. The construction of transform is explained through the following algorithm. Algorithm for Dasre-Gujarathi Transform:

1. Let A be a magic square of order n. Let $k = 1$.
2. Perform the operation $B(i;j) = A((i+1)\ modn;\ (j+1)\ mod\ n)$.
3. Store first row and first column of matrix B in the k-th row and k-th column of matrix C starting from $C(k,k)$.

4. Reconstruct matrix A of order $(n-1)$ by eliminating first row and first column of matrix B.

5. Decrease n by 1 and increase k by 1.

6. Continue step (2) to step(5), $(n-1)$ times.

The algorithm 'Dasre-Gujarathi Transform' is first applied on Magic Squares hence we call it as "Magic Transform". The resultant matrix C is image of A under "Magic Transform". This is denoted as $C = DG(A)$ where C is transformed matrix after applying "Magic Transform" to magic matrix A. It has been observed that the resultant matrix $C = DG(A)$ is no more magic square. After applying magic transformation, the corresponding transformed matrix is converted to the grey scale image. Again 3 patterns were obtained as below.

1. For $n \equiv 0 \ mod\ 4$ a doubly even i.e. $n = 4k$, then the pattern observed looks like Kite. So this pattern is named as 'Kite' pattern.

2. For $n \equiv 1$ or $3 \ mod\ 4$ an odd i.e. $n = 4k+1$; $4k+3$, then the pattern observed looks like a Rose flower. So this pattern is named as 'Rose' pattern.

3. For $n \equiv 2 \ mod\ 4$ a singly even i.e. $n = 4k+2$, then the pattern observed is a combination of kite and rose flower. So this pattern is named as 'Overlapping' pattern.

Also the images are recorded after applying 'Magic Transformation' for the magic squares of order $n = 64$ ($\equiv\ 0 \ mod\ 4$) as shown in Fig. 5, $n = 65$ ($\equiv\ 1 \ mod\ 4$) as shown in Fig. 6, $n = 66$ ($\equiv\ 2 \ mod\ 4$) as shown in Fig. 7 and $n = 67$ ($\equiv\ 3 \ mod\ 4$) as shown in Fig. 8.

Figure 5: Kite image for magic matrix of order $n = 64$

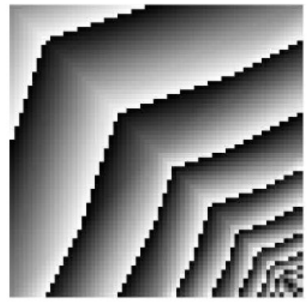

Figure 6: Rose image for magic matrix of order $n = 65$

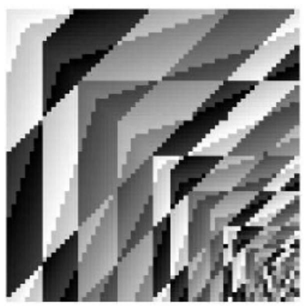

Figure 7: Overlapping image for magic matrix of order $n = 66$

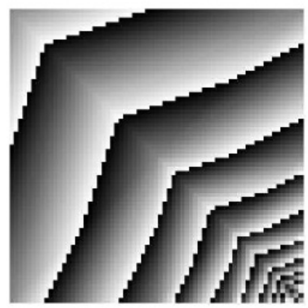

Figure 8: Rose image for magic matrix of order $n = 67$

4. Inverse Magic Transformation

After defining the Magic Transform by above algorithm, here the Inverse Magic Transform is defined to obtain the original Magic square. The construction of inverse transform is explained through the following algorithm.

Algorithm for Inverse Magic Transform :

1. Let A be a magic square of order n. Consider the transformed matrix $C = DG(A)$.
2. For $m = 2$, construct $P_{m\times m}$ a submatrix of $C_{n\times n}$ such that

$$P = P(i,j)_{m\times m} = C(r,s)_{r=n-m+1,n}^{s=n-m+1,n}.$$

3. Find $x = (i-1)\ mod\ m$ and $y = (j-1)\ mod\ m$. If x or y takes value zero then replace it by m.

4. Define $Q(i,j) = P(x,y)$.

5. Replace the submatrix P by matrix Q in matrix C.

6. Increase the value of m by 1 and repeat step (3) to step (6) till $m = n$.

By applying above algorithm, $DG^{-1}(C) = A$ is obtained, which shows that the transformation is invertible.

5. Experimental Observations

1. After applying Magic transform to Magic square of any order, the resultant matrix is not magic square.

2. After applying Magic transform to Magic square of any order, the sum of first row, first column and principle diagonal of transformed matrix is same as magic number.

3. Every magic square matrix has symmetric patterns before and after applying the magic transformation.

6. Conclusion

In this paper a new transform has been presented which links the traditional magic square to the area of Image Processing. The magic transform is introduced with its visualisation. Here the magic squares are visualised first time. Depending on the visualisation, the Magic Squares are categorised in 3 categories. Surprisingly these 3 categories are preserved after Magic Transform. Also this transform can be used for image encryption in data security.

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References

[1] Andrews W. S., Magic Squares and Cubes, Dover Publications Inc., New York. (1960).

[2] Takoa Hayashi, Varāhamihira's pandiagonal magic square of the order four, Historia Mathematica, 14(2) (1987), 159-166.

[3] Lee Sallows, The Lost Theorem, The Mathematical Intelligencer, 19(4) (1997), 51-54.