

**OPTIMAL LIFECYCLE INVESTMENT FOR PENSION FUNDS  
WITH VARIABLE RATE OF RETURN AND TRANSACTION  
COSTS**

**BRIGHT O. OSU AND SILAS A. IHEDIOHA**

Department of Mathematics,  
Abia State University, P M B 2000, Uturu, Nigeria

**Abstract**

This paper concerns optimal *lifecycle investment and consumption* of a Constant Relative Risk Aversion (CRRA) investor, who has a finite investment horizon and is subject to the proportional transaction costs and a rate of return that is time varying, as a representative of pension plan participants. Attempt was made to maximize the investor's utility by trading between stock and money market account. The case of no transaction costs led to a Bernoulli equation, while a set of partial differential equations are derived when transaction costs are involved and closed form solution proffered. The investigation of the effects of the volatility of the risky asset showed that a zero value of the volatility resulted to the value function becoming zero and its unit value with the drift parameter  $\xi(t)$  equals the discount rate  $k$  ( $\xi(t) = k$ ) led to an indeterminate value function. The growth rate of the value functions in the sell and buy regions were obtained using precise conditions.

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Key Words : *Optimal portfolio selection, Pension funds, Constant rate of return, Transaction costs, CRRA.*

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