

## GENERALIZED MATRIC SPACE IN NON-EXPANSIVE FUNCTION FOR CONTINUOUS MAPS

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### Abstract

Following Lawvere, a generalized metric space (gms) is a set  $X$  equipped with a metric map from  $X^2$  to the interval of upper reals (approximated from above but not from below) from 0 to  $\infty$  inclusive, and satisfying the zero self-distance law and the triangle inequality.

We describe a completion of gms's by Cauchy filters of formal balls. In terms of Lawvere's approach using categories enriched over  $[0, \infty]$ , the Cauchy filters are equivalent to flat left modules.

The work investigates the powerlocales (lower, upper, Vietoris) of localic completions of generalized metric spaces. The main result is that all three are localic completions of generalized metric powerspaces, on the Kuratowski finite powerset. This is a constructive, localic version of spatial results of Bonsangue et al. and of Edalat and Heckmann.

The completion generalizes the usual one for metric spaces. For quasimetrics it is equivalent to the Yoneda completion in its netwise form due to Künzi and Schellekens and thereby gives a new and explicit characterization of the points of the Yoneda completion.

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