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GENERALISED MULTIPLE $L - H$ TRANSFORM

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Abstract

In this paper an integral transform involving the product of an exponentional function, multivariable H- function and the product of 'r' H-functions of one variable is established. Special cases include the result given by Prasad and Mourya [4] and Vasishta and Goyal [8].

1. Notations and Results Used

$(a_p) = (a_1, a_2, \cdots, a_p)$

${}_1(a_j, A_j)_p = (a_1, A_1), \cdots, (a_p, A_p)$

$(a_n) = a(a+1), \cdots, (a+n-1), \quad (a)_0 = 1$

$(a_p) = 1$ denote $a_1 = \cdots = a_p = 1.$

Srivasthava [7, p. 18, 19].

$$H_{0,1}^{1,0}\left[x \left| \begin{matrix} - \\ \\ (b,\beta) \end{matrix} \right.\right] = \beta^{-1} x^{\frac{b}{\beta}} e^{-x^1/\beta} \tag{1.1}$$

$$H_{1,1}^{1,1}\left[x \left| \begin{matrix} (1-a,1) \\ \\ (0,1) \end{matrix} \right.\right] = \Gamma a(1+x)^{-a} \tag{1.2}$$

$$H^{1,p}_{p,q+1}\left[-x\left|\begin{matrix}{}_1(1-a_j,\alpha_j)_p\\ \\ (0,1),{}_1(1-b_j,\beta_j)_q\end{matrix}\right.\right]={}_p\Psi_q\left[x\left|\begin{matrix}{}_1(a_j,\alpha_j)_p\\ \\ (b_j,\beta_j)_q\end{matrix}\right.\right]=\sum_{d=0}^{\infty}R_d x^d \tag{1.3}$$

where

$$R_d=\frac{\prod\limits_{j=1}^{p}\Gamma(a_j+\alpha_j d)}{\prod\limits_{j=1}^{q}\Gamma(b_j+\beta_j d)d!}. \tag{1.4}$$

Srivastava and Pande [9].

The H-function of several complex variables $(x_1,\cdots,x_r)$ is given by

$$\begin{aligned}H[x_1,\cdots,x_r] &= H^{0,n:(m_1,n_1);\cdots;(m_r,n_r)}_{p,q:(p_1,q_1);\cdots;(p_r,q_r)}\\ &\left[\begin{matrix}x_1\\ \cdots\\ x_r\end{matrix}\left|\begin{matrix}{}_1(a_j,\alpha_j^{(1)},\cdots,\alpha_j^{(r)})_p:(c_j^{(1)},C_j^{(1)})_{1,p_1}\cdots;(c_j^{(r)},C_j^{(r)})_{1,p_r}\\ \\ {}_1(b_j,\beta_j^{(1)},\cdots,\beta_j^{(r)})_q:(d_j^{(1)},D_j^{(1)})_{1,q_1};\cdots(d_j^{(r)},D_j^{(r)})_{1,q_r}\end{matrix}\right.\right]\\ &= \frac{1}{2\pi i}\int_{L_1}\cdots\int_{L_R}\theta(s_1,\cdots,s_r)\prod_{k=1}^{r}[\phi_k(s_k)x_k^{s_k}]ds_1,\cdots,ds_r.\end{aligned} \tag{1.5}$$

2. Result

$$\int_0^{\infty}\cdots\int_0^{\infty}e^{-\sum\limits_{i-1}^{r}|p_ix_i}\prod_{i=1}^{r}\left\{(x_i)^{-k_i}H_i[\lambda_i x^{\mu_i}]H[z_1x_1^{\sigma_1},\cdots,z_rx_r^{\sigma_r}]\prod_{i=1}^{r}dx_i\right\}$$
$$=H^{0,n,(m_1,n_1+1);\cdots;(m_r,n_r+1)}_{p,q:(p_1+1,q_1);\cdots,(p_r+1,q_r)}$$
$$\left[\begin{matrix}z_1p_1^{-\sigma_1}\\ \vdots\\ z_rp_r^{-\sigma_r}\end{matrix}\left|\begin{matrix}{}_1(a_j,\alpha_j^{(1)},\cdots,\alpha_j^{(r)})_p;(k_1-\mu_1d_1,\sigma_1),(c_j^{(1)},C_j^{(1)})_{1,p_1};\cdots,\\ (k_r-\eta_rd_r,\sigma_r),(c_j^{(r)},C_j^{(r)})_{1,p_r}(k_r-\mu_rd_r,\sigma_r),{}_1(c_j^{(r)},C_j^{(r)})_{p_r}\\ \\ {}_1(b_j,\beta_j^{(1)},\cdots,\beta_j^{(r)})_q:\quad(d_j^{(1)},D_j^{(1)})_{1,q_1};\cdots;(d_j^{(r)},D_j^{(r)})_{1.q_r}\ {}_1(d_j^{(r)},D_j^{(r)})_{q_r}\end{matrix}\right.\right]$$
$$\times\prod_{i=1}^{r_i}\left\{(p_i)^{k_i-\mu_id_i-1}\left[\sum_{d_i=0}^{\infty}R_{d_i}(\lambda_i)^{d_i}\right]\right\} \tag{2.1}$$

where

$$R_{d_1}=\frac{\prod\limits_{j=1}^{G_I}\Gamma(1-g_j^{(i)}+G_j^{(i)}d_i)}{\prod\limits_{j=1}^{W_i}\Gamma(1-h_j^{(i)}+H_j^{(i)}d_i)}\frac{(-1)^{d_i}}{d_i!}. \tag{2.2}$$

Provided

(1) $p_i > 0, k_i.0, \lambda_i > 0,\ Re(\sigma_i) > 0,\quad (1 \leq i \leq r)$

(2) $Re(1 - k_i + \sigma_i v_i) > 0,\quad (1 \leq i \leq r)$, where v_i is defined by

(3) $B_i > 0$ and $|arg\ z_i| < \frac{B_i \pi}{2},\quad (1 \leq i \leq r)$, where B_i is defined by

(4) the series occurring on the right hand side of (3.1) is assumed to be absolutely convergent.

(5) $1 + \sum_{j=1}^{w_i} H_j^{(i)} - \sum_{j=1}^{h_i} G_j^{(i)} > 0,\quad (1 \leq i \leq r)$.

Proof : To prove (2.1), first substitute the H-function of several complex variable in terms of contour integral of Mellin Barnes type given by (1.5)], and then change the order of $(x_1, \cdots, x_r)$ and $(s_1, \cdots, s_r)$ integrals.[which is justified due to their absolute convergence]. To evaluate the inner integral $(x_1, \cdots, x_r)$ integral, express each of the H-function of Fox in the series from using (2.2) and then change the order of integration and summation [which is justified due to the uniform convergence of the series and the absolute convergence of multiple integral thus involved]. Evaluate the inner $(x_1, \cdots, x_r)$ integral by taking the Laplace transform term by term then interpret the resulting contour integral by means of (1.5) to get the required result.
The integral transform is defined by

$$\phi(p_1, \cdots, p_r) = \prod_{i=1}^{r} p_i \int_0^\infty \cdots \int_0^\infty e^{-\sum_{i=1}^{r} \eta_i p_i x_i} \prod_{i=1}^{r} H_i[\lambda_i (p_i x_i)^{\mu_i}] \times$$

$$H[z_1 (p_1 x_1)^{\sigma_1}, \cdots, z_r (p_r x_r)^{\sigma_r}] \times f(x_1, \cdots, x_r) \prod_{i=1}^{r} dx_i \tag{2.3}$$

Where each $H_i[x_i]$ is the Fox's H-function and $H[z_1, \cdots, z_r]$ is the multivariable H-function defined by Srivastava and Panda [9], provided the integral (2.3) is absolutely convergent.

Special Cases

In (2.3) putting $(G_i) = 0, (W_i) = 0, (d_i) = 1$ and make $\lambda_i \to 0$ and rename the parameter, to get the result given by Prasad and Mourya [4. P.374].

In (2.3) putting $r = 2, \eta_1 = \alpha, \eta_2 = \beta, p_1 = p, q_1 = q, si_1 = \mu, \sigma_2 = \gamma, z_1 = \xi, z_2 = \eta, G_1 = G, W_1 = W, G_2 = r, W_2 = t$ and rename the parameter to get the following transform.

$$\phi(p,q) = pq\int_0^\infty\int_0^\infty e^{-\alpha px-\beta qy} H^{1,u}_{u,w+1}\left[\lambda_1(px)^{\mu_1}\,\middle|\,\begin{matrix} g_u, G_u \\ \\ (0,1),(h_w,H_w)\end{matrix}\right]$$

$$\times H^{1,s}_{z,t+1}\left[\lambda_2(qy)^{\mu_2}\,\middle|\,\begin{matrix} l_s, L_s \\ \\ (0,1),(k_t,K_t)\end{matrix}\right] \times H[\xi(px)^\mu, \eta(qy)]f(x,y)dx\cdot dy. \tag{2.4}$$

where $H[x,y]$ is defined by Mittal and Gupta (10), provided the integral (2.4) is absolutely convergent.

In (2.4) putting $u = w = r = t = 0, \mu_1 = \mu_2 = 1$, make $\lambda_1, \lambda_2 \to 0$, to get the transform given by Vasishta and Goyal [8. p.9].

In (2.4) putting $n_1 = p_1 = q_1 = 0$, to get:

$$\phi(p,q) = pq\int_0^\infty\int_0^\infty 9e^{-\alpha px-\beta qy} H^{1,u}_{u,w+1}\left[\lambda_1(px)^{\mu_1}\,\middle|\,\begin{matrix} g_u, G_u \\ \\ (01,),(h_2,H_w)\end{matrix}\right]$$

$$H^{1,s}_{s,t+1}\left[\lambda_2(gy)^{\mu_2}\,\middle|\,\begin{matrix} l_s, L_s \\ \\ (0,1),(k_t,K_t)\end{matrix}\right] \times H^{m_2,n_2}_{p_2,q_2}\left[xi(px)^\mu\,\middle|\,\begin{matrix} (c_{p_2}, C_{p_2}) \\ \\ (d_{q_2}, D_{q_2})\end{matrix}\right]$$

$$\times H^{m_3n_3}_{p_3,q_3}\left[\eta(qy)^\gamma\,\middle|\,\begin{matrix} (e_{p_3}, E_{p_3}) \\ \\ (f_{q_3}, F_{q_3})\end{matrix}\right] \times f(x,y)dx\cdot dy. \tag{2.5}$$

In (2.5) putting $u = w = r = t = 0, \mu_1 = \mu_2 = 1$, make $\lambda_1, \lambda_2 \to 0$, to get a known transform given by Vasishta and Goyal [8, p.12].

In (2.3) putting $r = 1, \eta = 0, p = q = 0, p_1 = p, \eta_1 = \alpha, \lambda_1 = \lambda, \mu_1 = \mu, \sigma_1 = \sigma, z_1 = z$, to get the following transform,

$$\phi(p) = p\int_0^\infty e^{-\alpha px} H^{1,u}_{u,w+1}\left[\lambda(px)^{\mu}\,\middle|\,\begin{matrix} (g_u, G_u) \\ \\ (0,1),(h_w,H_w)\end{matrix}\right]$$

$$\times H^{m_2,n_2}_{p_2,q_2}\left[z(px)^\sigma\,\middle|\,\begin{matrix} (c_{p_2}, C_{p_2}) \\ \\ (d_{q_2}, D_{q_2})\end{matrix}\right] \times f(x)dx \tag{2.6}$$

provided the integral exists.

In (2.6) putting $\alpha = 0, \mu = 2, \sigma = 1$, to get:

$$\phi(p) = p \int_0^\infty H^{1,u}_{u,w+1} \left[\lambda(px)^2 \middle| \begin{matrix} (g_u, G_u) \\ \\ (0,1), (h_w, H_w) \end{matrix} \right]$$

$$\times H^{m_2,n_2}_{p_2,q_2} \left[z(px) \middle| \begin{matrix} (c_{p_2}, C_{p_2}) \\ \\ (d_{q_2}, D_{q_2}) \end{matrix} \right] \times f(x)dx \tag{2.7}$$

In (2.7) putting $\lambda \to 0$ and rename the parameters, to get the transform given by Gupta and Mittal [3].

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