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MHD FLOW PAST A STRETCHING HEATED VERTICAL PLATE OF A NEWTONIAN CONDUCTING FLUID IN PRESENCE OF FREE CONVECTIION A NUMERICAL RE-INVESTIGATION

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Abstract

An analysis of steady flow of a viscous, incompressible electrically conducting fluid past a stretching semi-infinite non-conducting vertical plate with heat transfer has been discussed in presence of magnetic field. The basic governing equation for conservation of mass, momentum and the energy equation are reduced to a set non-linear coupled ordinary differential equation. The resulting set of similarity equation has been solved numerically by employing Runge-Kutta algorithm with Newton iteration in double precision with a systematic guessing the missing initial conditions using the shooting technique. The effects of Prandtl number, Grashoff number and the magnetic parameter on velocity, temperature have been discussed numerically. Further more, the effects of these existing physical parameters are discussed on skin-friction co-efficient and heat transfer co-efficient on the stretching heated vertical sheet.

1. Introduction

The free convection finds technical application in many areas such as containing machinery, atmosphere and oceanic circulation, power transformation etc.. The problem

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Key Words : *Electrically conducting Newtonian fluid, Stretching heated sheet, Magnetic field.*

of free convection flow and heat transfer problems of stretching sheet has attracted many investigators due to its wide range of practical application, i.e. in cooling of nuclear reactor, chemical engineering and aeronautics. Free convection flow is developed in a fluid when temperature causes density variations leading to buoyancy force acting on the fluid element. The free convection flow problems over infinite vertical disk with buoyancy forces has been studied by several investigators. One may refer the works of Kumari, Takhar and Nath (1993), Yan and Soon(1997), Kumar et.al(1995).

The heat transfer over a stretching surface is of interest in polymer extrusion processes where the object after passing through a die enters the fluid for cooling below a certain temperature. The rate of which such objects are cooled has are important bearingon the properties of final product. In the process of cooling the fluid, the momentum boundary layer for linear stretching of sheet was first studied by Crane (1970). A study on heat and mass transfer over a stretching surface with suction or blowing was carried out by Gupta and Gupta(1977). The same type of problem with inclusion of constant surface velocity and power- law temperature variations were studied by Soundalgekar and Ramamurthy (1980), Grubba and Bobba(1982) studied the power-law temperature variations in the case of a stretching continuous surface. Chen and Char (1998) investigated the effect of power-law temperature and power-law heat flux in the heat transfer characteristics of a continuous linear stretching surface. The problem of a stretching surface with constant surface temperature was analysed by Noor Afzal(1993).

Atul kumar singh(2001) analysed the MHD free convection and mass transfer flow with heat source and thermal diffusion. Moreover, an investigation was made on unsteady free convective MHD flow and heat transfer past a vertical porous plate with variable temperature by Sarangi and Jose(2005) Kandasamy et. al (2005) investigated heat and mass transfer on MHD flow over a vertical stretching surface with heat source and thermal stratification effects. The present study deals with the details investigation on MHD flow past a stretching heated vertical plate of a Newtonian conducting fluid in presence of free convection.

2. Formulation of the Problem

A steady two-dimensional nom-linear flow of an incompressible, viscous, electrically conducting and Boussinesq fluid flowing over a vertical heated stretching sheet in the presence of an uniform magnetic field has been considered in the region $y > 0$. According to the co-ordinate system, the x-axis is chosen parallel to the vertical heated sheet and y- axis is taken normal to it. Keeping the origin fixed, two equal and opposite forces are applied along the x-axis which results in stretching of the sheet. A transverse magnetic field of strength B_0 is applied parallel to the y-axis. The fluid properties are assumed to be constant in a limited temperature range. It is assumed that the induced magnetic field, the external electric field and the electric field due to the polarization of charges are negligible.

Furthermore, it is assumed that the temperature of the stretching sheet is T_w where $T_w > T_\infty$ where T_∞ is the temperature of the fluid far from the sheet and T is the temperature of the fluid. Under these conditions, the governing boundary layer equations of continuity, momentum and energy neglecting viscous and Joules dissipation under Boussinesq's approximation are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0. \tag{1}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}, \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} \tag{3}$$

subjects to the boundary conditions

$$\left.\begin{array}{l} u = ax, \quad v = 0, \qquad T = T_w(x) \quad \text{at } y = 0 \\[2ex] u = 0 \qquad T = T_\infty \quad \text{as} \quad y \to \infty \end{array}\right\} \tag{4}$$

As in Kandasamy et. al (2005), the following change of variables are introduced

$$\left.\begin{array}{l} \psi(x,y) = x\sqrt{\nu a}F(\eta), \qquad \eta(x,y) = \sqrt{\frac{a}{\nu}}y \\[2ex] T_w(x) = T_\infty + Cx\theta(\eta), \quad \theta(\eta) = \frac{T-T_\infty}{T_w-T_\infty}. \end{array}\right\} \tag{5}$$

The velocity components are given by

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}. \tag{6}$$

The equation of continuity (1) is identically satisfied which can be easily verified.

Similarity solution exists if we assume that $U(x) = ax$ and introduce the above non-dimensional change of variable i.e. using (5) and (6) in (2) and (3) give the non-linear coupled ordinary differential equationas

$$F''' + FF'' - F'^2 - MF' + G_r\theta = 0 \tag{7}$$

$$\theta'' + P_r F\theta' = 0, \tag{8}$$

subject to the boundary conditions

$$\left.\begin{aligned} &F(0) = 0,\ F'(0) = 1,\ \theta(0) = 0 \\ &F'(\infty) = 0, \quad \theta(\infty) = 0, \end{aligned}\right\} \tag{9}$$

where prime denotes differentiation with respect to η, $M = \frac{\sigma B_0^2}{\rho a}$ is the hydro-magnetic parameter $G_r = \frac{g\beta C}{a^2}$ is the buoyancy parameter i.e. Grashoff number and $P_r = \frac{\nu}{al}$is the prandtl number of the conducting fluid.

3. Numerical Solution

A direct numerical solution of the above non-linear coupled ordinary differential equations (7) and (8) subject to the boundary condition (9) cannot be obtained because to integrate a pair of fifth order non-linear ordinary differential equation by the standard numerical methods, five conditions must be prescribed at the initial point of integration. But here two (2) conditions are missing at the initial point of integration. The shooting method is a suitable one which can convert a boundary value problem to an initial value problem. For numerical computation we take the infinity boundary condition at a large but finite value of η where considerable variation of velocity and temperature field do not occur. The set of equations (7) and (8), thus obtained are solved numerically. Regarding missing boundary conditions we apply shooting technique. In this numerical technique the missing boundary conditions at the initial point are being guessed first suitably. The value of the dependent variable is then calculated at the terminal point by adopting fourth order Runge-Kutta method. The value of the dependent variable may over shoot or undershoot the given value at the terminal point. The process is then repeated in each case changing the value for the missing boundary condition at the initial point of integration until the calculated value of the dependent variables match with given value at the terminal point within a admissible tolerance viz. 0.0000001 algorithm of the procedure are given in Niyogi(2003). The variations of the velocity and temperature fields are found for different values of existing parameter and their behavior are discussed in the next section.

4. Results and Discussion

In order to get a clear insight of this physical problem, numerical results are displayed with the help of tables investigation. So a representative tabulated results are presented in the tables 1 to 12. The numerical values of the velocity, temperature, skin friction coefficients and heat transfer coefficients are computed for different existing flow parameters viz. magnetic parameter (M), Grashoff number (G_r) and prandtl number (P_r), and the results are presented in the tables 1 to 12 to show the inhibiting influence of various parameters on velocity profile, temperature distribution, skinfriction co-efficient and heat transfer co-efficient. Table-1 presents the velocity profile F' vs η for several magnetic parameter M. It is clear from the table-1 that the velocity F' of the conducting fluid decreases with the increase of magnetic parameter M. In the table-2, the values of F' vs η across the boundary layer have been presented for various Grashoff Number G_r with constant or uniform magnetic field. I.e. $M = 2$ and $P_r = 0.71$. It is observed from table-2 that the effect of buoyancy parameter G_r are to increase the velocity of the conducting fluid across the boundary layer. Table-3 and 4 list values of F' vs η and θ vs η respectively for several prandtl number with constant/ uniform magnetic field M and constant buoyancy parameter G_r. It is clear that the thickness of thermal boundary layer and thickness of thermal boundary layer both decrease sharply with the increase in Prandtl number P_r. Table -5 shows the buoyancy effect for different values of buoyancy parameter i.e. Grashoff number, which says that the dimensionless temperature of the conducting fluid increases with the with the increase in the value of Grashoff number G_r with uniform magnetic field and $P_r = 0.71$. The effects of the magnetic parameter on the temperature of the fluid θ vs η are shown in the table-6 with constant buoyancy parameter $G_r = 5$ and $P_r = 0.71$. It is noticed that the temperature of the conducting fluid increases with the increase in the value of magnetic parameter across the boundary layer. The value of skin-friction co-efficient $F''(0)$ on the stretching vertical sheet are presented in the tables 7, 8 and 9 for various magnetic parameter, buoyancy parameter (Grashoff number)and prandtl number respectively. It is inferred from table 7 and 9 that skin-friction co-efficient decreases with the increase of magnetic parameter (M) and buoyancy parameter (G_r) respectively. Again from table-8 it is observed that skin-friction co-efficient increases due to increase in the value of Grashoff number. The values of heat transfers co-efficient $-\theta'(0)$ on the stretching vertical sheet are presented in the tables 10, 11 and 12 for various prandtl number, buoyancy parameter (Grashoff number)and magnetic parameter respectively. It is noticed from table-10 that the heat transfer co-efficient $-\theta'(0)$ increases with the increase of Prandtl number P_r. It is observed from table 11 and 12 that the heat transfer co-efficient decreases due to increase in the buoyancy parameter (G_r) and magnetic parameter M respectively.

Table 1 : Dimensionless velocity distribution F' vs η for various values of magnetic parameter M when $P_r = 0.71$ and $G_r = 5$

η	$M = 0$	$M = 1$	$M = 3$	$M = 5$	$M = 10$
0.000000	1.000000	1.000000	1.000000	1.000000	1.000000
0.000000	1.010401	0.980423	0.929117	0.886500	0.804161
0.200000	0.984720	0.934300	0.849617	0.781107	0.654508
0.300000	0.928103	0.866021	0.763420	0.682322	0.538345
0.400000	0.845592	0.779663	0.672251	0.589115	0.446962
0.500000	0.742013	0.678981	0.577632	0.500783	0.374503
0.600000	0.621998	0.567420	0.480909	0.416879	0.317241
0.700000	0.489946	0.448141	0.383272	0.337156	0.273084
0.800000	0.350026	0.324049	0.285784	0.261525	0.241264
0.900000	0.206175	0.197827	0.189403	0.170029	0.163217
1.000000	0.005643	0.002438	0.0006893	0.0002185	0.000763

Table 2 : Dimensionless velocity distribution F' vs η for various values of G_r when $P_r = 0.71$ and $M = 2$

η	$G_r = 1$	$G_r = 2$	$G_r = 5$	$G_r = 7$	$G_r = 10$	$G_r = 15$
0.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
0.100000	0.851925	0.877510	0.953489	1.003592	1.077898	1.199862
0.200000	0.720951	0.763488	0.889568	0.972536	1.095316	1.296240
0.300000	0.604099	0.656504	0.811533	0.913330	1.063641	1.308862
0.400000	0.498882	0.555414	0.722348	0.831750	0.992959	1.255239
0.500000	0.403222	0.459299	0.624659	0.732870	0.892082	1.150603
0.600000	0.315378	0.367429	0.520817	0.621139	0.768654	1.008041
0.700000	0.233894	0.279229	0.412925	0.500453	0.629286	0.838707
0.800000	0.157547	0.194255	0.302868	0.374236	0.479693	0.652046
0.900000	0.085302	0.112168	0.192347	0.245513	0.324824	0.456047
1.000000	0.016287	0.032718	0.082919	0.116979	0.168994	0.257479

Table 3 : Dimensionless velocity distribution F' vs η for various values of P_r when $M = 2$ and $G_r = 5$

η	$P_r = 0.00741$	$P_r = 0.71$	$P_r = 1$	$P_r = 2$	$P_r = 7$	
0.000000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
0.100000	0.959943	0.955017	0.953879	0.953541	0.952728	0.952663
0.200000	0.902581	0.892647	0.890352	0.889669	0.888029	0.887901
0.300000	0.831186	0.816180	0.812709	0.811676	0.809194	0.809008
0.400000	0.748273	0.728483	0.723891	0.722523	0.719238	0.719004
0.500000	0.655467	0.631962	0.626483	0.624851	0.620928	0.620663
0.600000	0.553497	0.528564	0.522740	0.521008	0.516848	0.516577
0.700000	0.442288	0.419798	0.414621	0.413091	0.409444	0.409208
0.800000	0.321107	0.306768	0.303815	0.302982	0.301081	0.300936
0.900000	0.194107	0.194082	0.192378	0.191783	0.190215	0.188735
1.000000	0.097672	0.91069	0.82830	0.79789	0.070568	0.433617

Table 4 : Dimensionless temperature distribution θ vs η for various values of P_r when $M = 2$ and $G_r = 5$

η	$P_r = 0.00741$	$P_r = 0. - 025$	$P_r = 0.71$	$P_r = 1$	$P_r = 2$	$P_r = 7$
0.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
0.100000	0.899927	0.899763	0.892870	0.889806	0.879295	0.822091
0.200000	0.799860	0.799547	0.786391	0.780556	0.760646	0.654556
0.300000	0.699981	0.699373	0.681172	0.673125	0.645909	0.505622
0.400000	0.5997699	0.599257	0.577686	0.568187	0.536428	0.379372
0.500000	0.499753	0.499210	0.476263	0.466200	0.433001	0.276202
0.600000	0.399760	0.399238	0.377079	0.367396	0.335915	0.193990
0.700000	0.299792	0.299343	0.280159	0.271795	0.245010	0.129343
0.800000	0.199846	0.199519	0.185383	0.179214	0.159757	0.078568
0.900000	0.099921	0.0.099757	0.092487	0.089284	0.079330	0.038238
1.000000	0.005436	0.002665	0.001461	0.001066	0.000540	0.000012

Table 5 : Dimensionless temperature distribution θ vs η for various values of G_r when $M = 2$ and $P_r = .71$

η	$G_r = 1$	$G_r = 2$	$G_r = 5$	$G_r = 7$	$G_r = 10$	$G_r = 15$
0.000000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
0.100000	0.8911415	0.891576	0.892881	0.893706	0.895088	0.897335
0.200000	0.782902	0.783762	0.786362	0.788008	0.790762	0.795242
0.300000	0.675926	0.677206	0.681073	0.683523	0.687618	0.694283

η	$G_r = 1$	$G_r = 2$	$G_r = 5$	$G_r = 7$	$G_r = 10$	$G_r = 15$
0.400000	0.570784	0.572455	0.577512	0.580719	0.586073	0.594797
0.500000	0.467953	0.469960	0.476034	0.479897	0.486332	0.496843
0.600000	0.367838	0.370073	0.376847	0.381171	0.388359	0.400147
0.700000	0.270772	0.273060	0.280007	0.284467	0.291864	0.304067
0.800000	0.177033	0.179108	0.185428	0.189516	0.196283	0.207542
0.900000	0.086864	0.088332	0.092881	0.0955857	0.100757	0.109012
1.000000	0.000393	0.000789	0.002001	0.002826	0.004096	0.006287

Table 6 : Dimensionless temperature distribution θ vs η for various values of magnetic parameter M when $G_r = 5$ and $P_r = 0.71$.

η	$M = 0$	$M = 1$	$M = 2$	$M = 5$	$M = 7$	$M = 10$	$M = 15$
0.000000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
0.100000	0.906147	0.906516	0.906902	0.908174	0.909125	0.910725	0.913928
0.200000	0.811766	0.812505	0.813280	0.815829	0.817734	0.820940	0.827362
0.300000	0.716296	0.717484	0.718564	0.722388	0.725248	0.730062	0.739714
0.400000	0.619277	0.620738	0.622278	0.627325	0.631113	0.637500	0.650335
0.500000	0.520374	0.522143	0.524001	0.530152	0.534779	0.542608	0.558422
0.600000	0.419401	0.421380	0.423465	0.430410	0.435669	0.444625	0.462889
0.700000	0.316351	0.318366	0.3220499	0.327669	0.333157	0.342599	0.362163
0.800000	0.211424	0.213197	0.215086	0.221522	0.226526	0.235272	0.253843
0.900000	0.105052	0.106176	0.107386	0.111586	0.114927	0.120902	0.134072
1.000000	0.002092	0.002167	0.002245	0.002503	0.002696	0.003025	0.003703

Table 7 : Values of skin-friction co-efficient $F''(0)$ for various values of magnetic parameter M when $G_r = 5$ and $P_r = 0.71$

M	1	3	5	7	10	15
$F''(0)$	1.422821	0.993452	0.248986	-0.37798	$-.917684$	-2.54826

Table 8 : Values of skin-friction co-efficient $F''(0)$ for various values of buoyancy parameter G_r when $M = 2$ and $P_r = 0.71$

G_r	1	2	5	10	15	20
$F''(0)$	-1.349314	-0.850374	0.604168	2.90967	5.095994	7.186621

Table 9 : Values of skin-friction co-efficient $F''(0)$ for various values of prandtl number P_r when $M = 2$ and $G_r = 5$

$F''(0)$	0.619683	0.619276	0.604168	0.598175	0.579116	0.509960

Table 10 : Values of heat transfer co-efficient $-\theta'(0)$ for various values of prandtl number P_r when $M = 2$ and $G_r = 5$

P_r	0.00741	0.025	0.71	1	2	7
$-\theta'(0)$	1.000732	1.002472	1.071492	1.101428	1.207650	1.782764

Table 11 : Values of heat transfer co-efficient $-\theta'(0)$ for various values of Grashoff number G_r when $M = 2$ and $P_r = 0.71$

G_r	1	2	5	10	15	20
$-\theta'(0)$	1.08885	1.08454	1.084430	1.049416	1.026939	1.00407

Table 12 : Values of heat transfer co-efficient $-\theta'(0)$ for various values of magnetic parameter M $G_r = 5$ and $P_r = 0.71$

M	0	1	3	5	7	10
$-\theta'(0)$	-1.063705	1.067517	1.075633	1.084430	1.093936	1.109578

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