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PROPER HOMOTHETIC VECTOR FIELD FOR MAXIMAL SYMMETRIC TRANSVERSE SPACES

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Abstract

We study homothetic vector fields for maximal symmetric transverse spaces using direct integration technique. It is shown that above space-time admits proper homothetic vector field.

1. Introduction

Many authors have been examined different types of symmetries and its significant role in the Einstein's theory of general relativity [1,2]. Due to lack of linearity property, unlike Killing, affine symmetry, conformal symmetry, homothetic symmetry is difficult

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to study [3-6]. Homothetic vector field is very important to discuss the solution of Einstein'fs field equations (EFE). Here four dimensional, connected, Hausdorff space-time manifold with Lorentz metric g of signature $(+,-,-,-)$ is represented by M. For any vector field X on M can be decomposed as

$$X_{a;b} = \frac{1}{2}h_{ab} + F_{ab}, \tag{1.1}$$

where $h_{ab}\ (= h_{ba}) = \mathcal{L}_X g_{ab}$ and $F_{ab} = -F_{ba}$ are symmetric and skew-symmetric tensor on manifold M respectively. If

$$h_{ab} = Cg_{ab}, \quad C \in R \tag{1.2}$$

then X satisfies the equivalent conditions

$$g_{ab,c}X^c + g_{cb}X^c_{,a} + g_{ac}X^c_{,b} = 2Cg_{ab}, \quad (a,b,c = 0,1,2,3). \tag{1.3}$$

Then X is called a homothetic vector field on M. If X is homothetic and C is non-zero Homothetic constant, then it is called proper homothetic and $C = 0$ it is Killing. The usual notations for covariant, partial and Lie derivative are denoted by semicolon, a comma, and the symbol $\mathcal{L}$ respectively.
In this paper we investigate the homothetic vector field of maximal symmetric transverse spaces using direct integration technique.

2. Homothetic Field Equations and Their Results

Consider maximal symmetric transverse spaces [7] in usual coordinates (t,r,θ,ϕ) with line element

$$ds^2 = e^{A(r)}dt^2 - e^{B(r)}dr^2 - r^2(d\theta^2 + f_k(\theta)d\phi^2), \tag{2.1}$$

where

$$f_k(\theta) = \begin{cases} \sin^2\theta, & \text{when} \quad k = 1, \\ \theta^2, & \text{when} \quad k = 0, \\ \sinh^2\theta, & \text{when} \quad k = -1. \end{cases}$$

Using equation (1.3), we obtain the following homothetic field equations for (2.1)

$$A_{,1}X^1 + 2X^0_{,0} = 2C \tag{2.2}$$

$$e^{A}X^{0}_{,1} - e^{B}X^{1}_{,0} = 0 \tag{2.3}$$

$$e^{A}X^{0}_{,2} - r^{2}X^{2}_{,0} = 0 \tag{2.4}$$

$$e^{A}X^{0}_{,3} - r^{2}f_{k}X^{3}_{,0} = 0 \tag{2.5}$$

$$B_{,1}X^{1} + 2X^{1}_{,1} = 2C \tag{2.6}$$

$$e^{B}X^{1}_{,2} + r^{2}X^{2}_{,1} = 0 \tag{2.7}$$

$$e^{B}X^{1}_{,3} + r^{2}f_{k}X^{3}_{,1} = 0 \tag{2.8}$$

$$X^{1} + rX^{2}_{,2} = rC \tag{2.9}$$

$$X^{2}_{,3} + f_{k}X^{3}_{,2} = 0 \tag{2.10}$$

$$X^{1} + rX^{3}_{,3} = rC \tag{2.11}$$

Using equations (2.3),(2.4),(2.5) and (2.10), we get,

$$X^{0} = e^{-A}r^{2}\sqrt{f_{k}}\int [E^{1}_{,0}t + E^{1}]d\phi + E^{3}(t,r,\theta), \tag{2.12}$$

$$X^{1} = e^{-B}r^{2}\sqrt{f_{k}}\{\int [(\frac{2}{r} - A_{,1})\int [E^{1}_{,0}t + E^{1}]d\phi]dt + \int [E^{1}_{,0}t + E^{1}]_{,1}d\phi + E^{3}_{,1}t + E^{5}, \tag{2.13}$$

$$X^{2} = \int \left\{ \frac{f_{k,2}}{2f_{k}} \int [E^{1}_{,0}t + E^{1}]d\phi \right\} dt + e^{-A}r^{-2}\int E^{3}_{,2}dt + E^{4}, \tag{2.14}$$

$$X^{3} = \frac{E^{1}t}{\sqrt{f_{k}}} + E^{2} \tag{2.15}$$

where $E^{1}(t,r,\phi), E^{2}(r,\theta,\phi), E^{3}(t,r,\theta), E^{4}(r,\theta,\phi)$ and $E^{5}(r,\theta,\phi)$ are functions of integration. In order to find $E^{1}(t,r,\phi), E^{2}(r,\theta,\phi), E^{3}(t,r,\theta), E^{4}(r,\theta,\phi)$ and $E^{5}(r,\theta,\phi)$ we need to solve remaining six equations. Due to lengthy calculations here we will present results only. It follows that there exist one possibility when above space-times (2.1) admit proper homothetic vector fields as

$$X^{0} = Ct + c_{1}, \quad X^{1} = Cr, \quad X^{2} = c_{2}, \quad X^{3} = c_{3}$$

where c_1, c_2, c_3 are constants.

In this case the space-time (2.1) takes the form

$$ds^{2} = \alpha dt^{2} - \beta dr^{2} - r^{2}(d\theta^{2} + f_{k}(\theta)d\phi^{2}),$$

where α and β are non-zero constants. The proper homothetic vector fields in this case are

$$X^0 = Ct, \quad X^1 = Cr, \quad X^2 = c_4, \quad X^3 = c_5$$

where $c_4, c_5 \in R$.

3. Conclusion

In this paper we have studied maximal symmetric transverse spaces according to their proper homothetic vector field using direct integration technique. After examine all possibilities we found that there is existence of proper homothetic vector fields and we get new metric for which homothetic vector fields have been determined.

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