International J. of Pure & Engg. Mathematics (IJPEM)
ISSN 2348-3881, Vol. 3 No. II (August, 2015), pp. 77-92

RELIABILITY AND AVAILABILILTY CHARACTERISTICS OF A TWO-UNIT STANDBY REDUNDANT SYSTEMBY LINEAR DIFFERENTIAL EQUATION (L. D. E.) SOLUTION TECHNIQUE

PRADEEP K. JOSHI[1], RAVINDRA SEN[2] AND CHITARANJAN SHARMA[3]
[1] Associated Professor, IPS Academy, Indore, India
[2] Assistant Professor,
Shri Vaishnav Institutes of Technology, Indore, India
[3] Professor, Govt. Holkar Science College, Indore, India

Abstract

Significance of switching for preventive maintenance in two similar states of the system is studied comparatively using different failure rates. Also the system is facilitated with switching for preventive maintenance in second case. Preventive maintenance is the replacing components or subsystems before they fail in order to promote continuous system operation. In this paper Reliability and Availability characteristics of two different series system configurations are calculated, results are derived with the help of linear differential equation (L.D.E.) solution technique which is rarely used in the study of such systems. The method L. D. E solution is used to calculate the Availability and MTSF characteristics. The failure and preventive maintenance times are assumed to have exponential distribution. The failure times of a component are assumed to be exponentially distributed with parameter $\alpha', \alpha, \beta', \beta$. The preventive maintenance approach rate is also assumed exponential with parameter λ', λ. The mean-time-to failure MTTF and the steady-state availability, $A_T(\infty)$ is derived for the two system using linear first order differential

---

Key Words : *Availability, Reliability, Mean time to system failure, Preventive maintenance, Linear differential equations.*
2010 AMS Subject Classification : 93A30, 90B25.

equations. We observed significant change in performance of the system on facilitation of switching for preventive maintenance.

1. Introduction

Many authors have studied the two-similar unit redundant system or two-dissimilar unit redundant system with two or three states to calculate various reliability and availability measures, but no attention was paid to the reliability evaluation using preventive maintenance. Repairable systems receive maintenance actions that restore/renew system components when they fail. These actions change the overall makeup of the system. The standby unit support increases the reliability of the system. On the failure of the operating unit, a standby unit is switched on by switching device for preventive maintenance. A considerable papers has been published on Statistical Analysis of two-unit redundant systems during the last four decades. Several authors have studied such systems assuming only " two" states of operations, namely operative and failed [7, 8]. Many authors [4, 10] have studied the two unit redundant systems with two types of repair. [3] Have studied stochastic analysis of a two-unit parallel system with partial and catastrophic failure and preventive maintenance. In [9] evaluate the reliability and availability of two different systems by using linear first order differential equations. [11] has studied Cost Analysis of Two-Dissimilar-Unit Cold Standby System with three States and Preventive Maintenance using Linear First Order Differential Equations.

Many authors [12-15] have studied the stochastic behavior of two-unit cold standby redundant system. Models have been formulated to treat many situations and obtain various reliability parameters by using the theories of regenerative process. Markov renewal process and semi-Markov process.

Author [3, 4, 5] has studied the system of two unit stand by redundant system with preventive maintenance, inspection and two types of repair. Authors of [2] has discussed about the importance of switching for preventive maintenance. Authors [1] has discussed about the solution of linear differential equation technique to resolve and to obtain reliability and availability characteristics. This technique is very significant in the case as this will prove a good lead to tedious and lengthy method discussed for same cause in various research papers [3, 4, 5, 6]. Importance of switching for preventive maintenance is visible from the calculations made in the paper. The concept of switching does not provide great and large results but is very significant providing availability of the system as it is clear from the calculation performance.

The purpose of this paper is to study the Reliability, MTTF, Steady-state availability of a two-dissimilar-unit stand by redundant system using Linear Differential Equation techniques. A Graphical representation of improvement in availabilities of system performance is also discussed.

2. Model Description and Assumptions

The following assumptions are common for both the systems-

(1) The system consists of a single unit having two dissimilar parallel components, say A & B.

(2) The System remains operative even if a single component operates.

(3) The failure of a component changes the life time parameter of the other.

(4) Upon failure each component can be replaced with a similar component with both the component (When failed) can be replaced simultaneously.

(5) After Replacement of each component, the system is as good as new. But in the second system we assume that units at first and second state switches for preventive maintenance.

(6) Preventive maintenance (e.g. overhaul, inspection, minor repairs, etc.) is provided to this system at random epochs when the system is in the state S_0. Where both the components are normal.

(7) The system is down when both system failed.

(8) For constructing the model having probabilistic structure of the system, is it assumed that the failure and preventive maintenance times have exponential distribution.

Symbols and State Definition of the system

Symbols : The common symbols to both the systems are:

$\alpha;$	conditional failure rate of operative unit B_N
v'	conditional failure rate of operative unit A_N
α	unconditional failure rate of operative unit A_N
β	unconditional failure rate of operative unit B_N
β'	replacement rate of failed unit A_F
δ'	replacement rate of failed unit B_F
θ	failure rate of unit B while unit A is already failed
θ'	failure rate of unit A while unit B is already failed
μ	replacement rate of failed units A, B
λ	rate of taking unit A into preventive maintenance
η	completion of preventive maintenance time for unit A
λ'	rate of taking unit B into preventive maintenance
η'	completion of preventive maintenance time for unit B
$A_{Ti}(\infty)$	system steady state availability, $i = 1, 2$
E_i	expected time to reach an absorbing state, $i = 1, 2$

For Second System

γ	Rate of switching to state 2 for preventive maintenance.
γ'	Rate of switching to state 1 for preventive maintenance.
$P_{n,i}(t)$	Probability that exactly 'n' components are working at time $t, (t \geq 0)$ at state S_i.
$u(t)$	pdf of time for taking a unit into preventive maintenance i.e, $u(t) = \lambda \exp(-\lambda t), \lambda t > 0$
$v(t)$	pdf of preventive maintenance i.e, $v(t) = \eta \exp(-\eta t), \eta t > 0$

State Definition :

A unit can be in one of the following states at time t

A_N	Component A in normal mode and operative
B_N	Component B in normal mode and operative
A_F	Component A in failure mode and needs replacement
B_F	Component B in failure mode and needs replacement
A_{Np}	Component A in normal mode and under preventive maintenance
B_{Np}	Component B in normal mode and under preventive maintenance

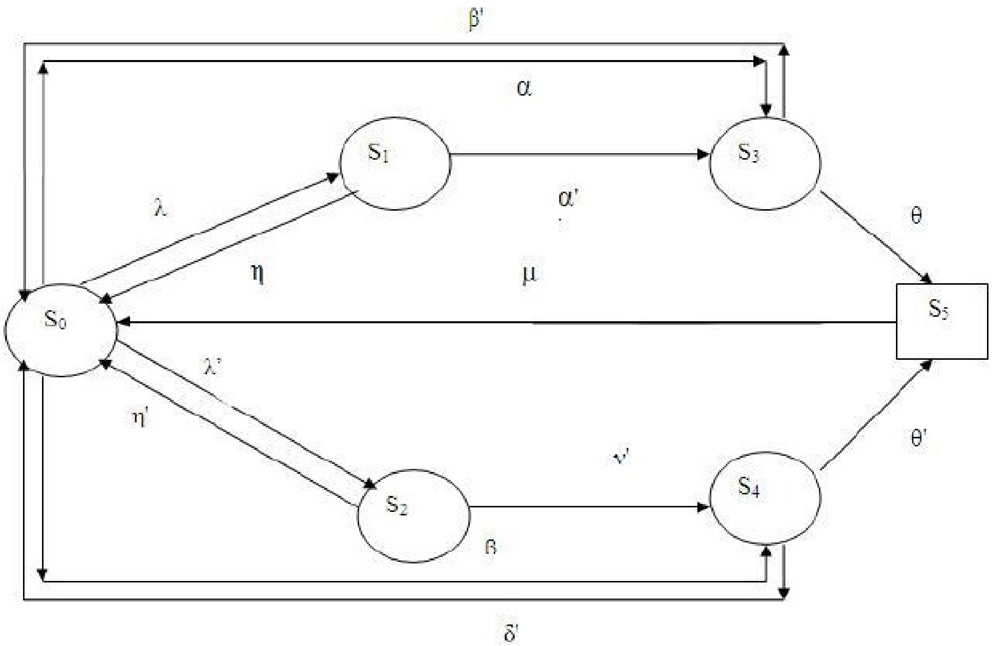
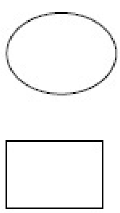


Figure 1. State Transition Diagram for System 1



- **UP State**

- **Down State**

3. Case (i) : States of the System for System 1

(a) Up states: $S_0 = (A_N, B_N)$, $S_1 = (A_{Np}, B_N)$, $S_2 = (A_N, B_{Np})$, $S_3 = (A_N, B_F)$, $S_4 = (A_F, B_N)$

(b) Downstate: $S_5 = (A_F, B_F)$.

3.1 Mean Time to System Failure (MTSF1)

Now we calculate, mean time to system failure (MTSF) for the proposed system1 using the above set of assumptions and Linear Differential Equation techniques. For figure-1, let $P_{n,i}(t)$ be the Probability that exactly n component are working at time t, (t . 0) at state Si,. If we let P(t) denote the probability row vector at time t, then the initial conditions for this problem are

$$P(0) = [P_{2,0}(0),\ P_{1,1}(0),\ P_{1,2}(0),\ P_{1,3}(0),\ P_{1,4}(0),\ P_{0,5}(0)] = (1, 0, 0, 0, 0) \tag{1}$$

Using, the method of Linear Differential Equation, we get following differential equations :

$$\frac{dP_{2,0}}{sdt} = -(\lambda + \lambda' + \beta + \alpha)P_{2,0} + \eta P_{1,1} + \eta P_{1,2} + \beta' P_{1,3} + \delta' P_{1,4} + \mu P_{0,5} \tag{2}$$

$$\frac{dP_{1,1}}{dt} = \lambda P_{2,0} - (\alpha' + \eta)P_{1,1} + 0P_{1,2} + 0P_{1,3} + 0P_{1,4} + 0P_{0,5} \tag{3}$$

$$\frac{dP_{1,2}}{dt} = \lambda' P_{2,0} + 0P_{1,1} - (\eta' + v')P_{1,2} + 0P_{1,3} + 0P_{1,4} + 0P_{0,5} \tag{4}$$

$$\frac{dP_{1,3}}{dt} = \alpha P_{2,0} + \alpha' P_{1,1} + 0P_{1,2} - (\beta' + \theta)P_{1,3} + 0P_{1,4} + 0P_{0,5} \tag{5}$$

$$\frac{dP_{1,4}}{dt} = \beta P_{2,0} + 0P_{1,1} + v' P_{1,2} + 0P_{1,3} - (\delta' + \theta')P_{1,4} + 0P_{0,5} \tag{6}$$

$$\frac{dP_{0,5}}{dt} = 0P_{2,0} + 0P_{1,1} + 0P_{1,2} + \theta P_{1,3} + \theta' P_{1,4} - \mu P_{0,5}. \tag{7}$$

This can be written in the matrix form as

$$\dot{P} = QP \tag{8}$$

where,

$$Q = \begin{bmatrix} -(\lambda + \lambda' + \beta + \alpha) & \eta & \eta' & \beta' & \delta' & \mu \\ \lambda & -(\eta + \alpha') & 0 & 0 & 0 & 0 \\ \lambda' & 0 & -(\eta' + v') & 0 & 0 & 0 \\ \alpha & \alpha' & 0 & -(\theta + \beta') & 0 & 0 \\ \beta & 0 & v' & 0 & -(\delta' + \theta') & 0 \\ 0 & 0 & 0 & \theta & \theta' & -\mu \end{bmatrix} \tag{9}$$

To evaluate the transient solution is too complex. Therefore, we will restrict ourselves in calculating the $MTSF_1$, we take the transpose matrix of Q and delete the rows and columns for the absorbing states. The new matrix is called A. the expected time to reach an absorbing state E_1, is calculated from

$$E_1[T_{p(0)\to p(\text{absorbing})}] = P(0)(-A^{-1})\begin{pmatrix}1\\1\\1\\1\\1\\1\end{pmatrix} \tag{10}$$

where,

$$A = \begin{bmatrix} -(\lambda+\lambda'+\beta+\alpha) & \lambda & \lambda' & \alpha & \beta \\ \eta & -(\eta+\alpha') & 0 & \alpha' & 0 \\ \eta' & 0 & -(\eta+v') & 0 & v' \\ \beta & 0 & 0 & -(\theta+\beta') & 0 \\ \delta' & 0 & 0 & 0 & -(\delta'+\beta') \end{bmatrix} \tag{11}$$

and

$$\int_0^\infty e^{At}dt = -A^{-1}. \tag{12}$$

We obtain the following explicit expression for the $MTSF_1$

$$E[T_{p(0)\to p(\text{absorbing})}] = MTSF_1 = P(0)(A^{-1})\begin{pmatrix}1\\1\\1\\1\\1\end{pmatrix} =$$

$$\frac{\begin{array}{c}[\{(\eta+\alpha'+\lambda)(v'+\eta')+\lambda'(\eta+\alpha')\}(\beta'+\theta)+(\eta\beta+\alpha'\lambda+\beta\alpha')(v'+\eta')] \\ (\delta'+\theta')+(v'\lambda'+\alpha+\eta'\alpha)(\alpha'+\eta)(\beta+\theta)\end{array}}{\theta'(\alpha'+\eta)(\beta'+\theta)\{v'(\lambda'+\alpha)+\eta'\alpha\}+(\delta+\theta')\{\eta\beta\theta(\eta'+v')+\theta\alpha'(v'\beta+\lambda v'+\lambda\eta')\}}. \tag{13}$$

3.2 Availability Analysis of the System 1 $[A_{T1}(\infty)]$

For the availability Equation (1) case in Figure 1, the initial conditions for this problem are the same as for the reliability case

$$P(0) = [P_{2,0}(0),\ P_{1,1}(0),\ P_{1,2}(0),\ P_{1,3}(0),\ P_{1,4}(0),\ P_{0,5}(0)] = (1,0,0,0,0).$$

The differential equation form can be expressed as $\dot{P} = QP$

Or

$$\begin{pmatrix} \dot{P}_{2,0} \\ \dot{P}_{1,1} \\ \dot{P}_{1,2} \\ \dot{P}_{1,3} \\ \dot{P}_{1,4} \\ \dot{P}_{0.5} \end{pmatrix} = \begin{pmatrix} -(\lambda+\lambda'+\beta+\alpha) & \eta & \eta' & \beta & \delta' & \mu \\ \lambda & -(\eta+\alpha') & 0 & 0 & 0 & 0 \\ \lambda' & 0 & -(\eta+v') & 0 & 0 & 0 \\ \alpha & \alpha' & 0 & -(\theta+\beta') & 0 & 0 \\ \beta & 0 & v' & 0 & -(\delta'+\beta') & 0 \\ 0 & 0 & 0 & \theta & \theta' & -\mu \end{pmatrix} \begin{pmatrix} P_{2,0} \\ P_{1,1} \\ P_{1,2} \\ P_{1,3} \\ P_{1,4} \\ P_{0.5} \end{pmatrix} \tag{14}$$

The steady state availabilities can be obtained using the following procedure. In the steady-state, the derivatives of the state probabilities become zero. That allows us to calculate the steady-state probabilities with

$$A_{T_1}(\infty) = 1 - P_{0,5}(\infty)$$

and

$$QP(\infty) = 0$$

or in the matrix form

$$\begin{pmatrix} -(\lambda+\lambda'+\beta+\alpha) & \eta & \eta' & \beta & \delta' & \mu \\ \lambda & -(\eta+\alpha') & 0 & 0 & 0 & 0 \\ \lambda' & 0 & -(\eta+v') & 0 & 0 & 0 \\ \alpha & \alpha' & 0 & -(\theta+\beta') & 0 & 0 \\ \beta & 0 & v' & 0 & -(\delta'+\beta') & 0 \\ 0 & 0 & 0 & \theta & \theta' & -\mu \end{pmatrix} \begin{pmatrix} P_{2,0}(\infty) \\ P_{1,1}(\infty) \\ P_{1,2}(\infty) \\ P_{1,3}(\infty) \\ P_{1,4}(\infty) \\ P_{0.5}(\infty) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{15}$$

To obtain $P_{0,5}(\infty)$ we solve above and the following normalization condition:

$$P_{2,0}(\infty) + P_{1,1}(\infty) + P_{1,2}(\infty) + P_{1,3}(\infty) + P_{1,4}(\infty) + P_{1,5}(\infty) = 1 \tag{16}$$

We substitute (16) in any one of the redundant rows in (15) to yield

$$\begin{pmatrix} -(\lambda+\lambda'+\beta+\alpha) & \eta & \eta' & \beta & \delta' & \mu \\ \lambda & -(\eta+\alpha') & 0 & 0 & 0 & 0 \\ \lambda' & 0 & -(\eta+v') & 0 & 0 & 0 \\ \alpha & \alpha' & 0 & -(\theta+\beta') & 0 & 0 \\ \beta & 0 & v' & 0 & -(\delta'+\beta') & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} P_{2,0}(\infty) \\ P_{1,1}(\infty) \\ P_{1,2}(\infty) \\ P_{1,3}(\infty) \\ P_{1,4}(\infty) \\ P_{0.5}(\infty) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \tag{17}$$

The solution above of (17) provides the steady state probabilities in the availability case. For Figure 1, the explicit expression for $A_{T_1}(\infty)$ is given by,

$$A_{T_1}(\infty) = \frac{\mu\cdot(\theta'+\delta')(\theta+\beta')(\eta+\alpha')(\gamma'+\eta')}{\begin{array}{l}(\mu\cdot\alpha\cdot(\lambda+\eta)\cdot(\eta'+\gamma')(\beta'+\theta)+(\lambda+\beta)(\delta'+\theta')(\gamma'+\eta')\beta'\cdot\alpha' \\ +\lambda'\cdot\beta'\cdot\theta'\cdot\gamma'\cdot\alpha'+\eta\cdot\eta'\cdot\alpha\cdot\beta'\cdot\mu+(\eta'+\gamma')\theta\cdot\mu\{\eta\cdot\alpha+\eta+\lambda\} \\ +\eta\cdot\gamma'\cdot\theta\cdot\lambda'\cdot\mu+(\alpha+\lambda')\eta\cdot\gamma'\cdot\beta'\cdot\mu+\{(\eta\cdot\gamma'+\eta'\cdot\alpha'+\eta\cdot\eta') \\ (\beta+\beta')\cdot\mu+(\alpha'+\eta)(\theta+\beta')\lambda'\cdot\mu\cdot+\eta'\cdot\lambda\cdot\mu\cdot(\alpha'+\beta') \\ +\eta\gamma'\alpha'\cdot\beta'\cdot\mu+(\gamma'+\theta)\eta'\cdot\alpha'\mu+\eta\cdot\beta'\cdot\gamma'+\eta\cdot\mu\eta'\cdot\beta\}(\delta'+\theta') \\ +\{\alpha\cdot\mu\cdot\alpha'(\gamma'+\eta')+\eta\cdot\lambda'\cdot\theta'\cdot\gamma'\}(\theta+\beta')+(\eta'\cdot\alpha'+\gamma'\cdot\theta) \\ \theta'\cdot\lambda\cdot\mu\cdot\alpha'+\theta\cdot\lambda'\cdot\gamma'\cdot\alpha'(\theta'+\mu)+\gamma'\cdot\theta'\cdot\lambda\cdot\mu(\alpha'+\beta') \\ +\gamma'\cdot\alpha'\cdot\theta'\cdot\beta\cdot\mu+(\lambda'\cdot\alpha'+\delta'\cdot\lambda)\gamma'\cdot\beta'\cdot\mu+\gamma'\cdot\delta'\cdot\alpha'\cdot\mu\cdot(\beta+\lambda))\end{array}} \tag{18}$$

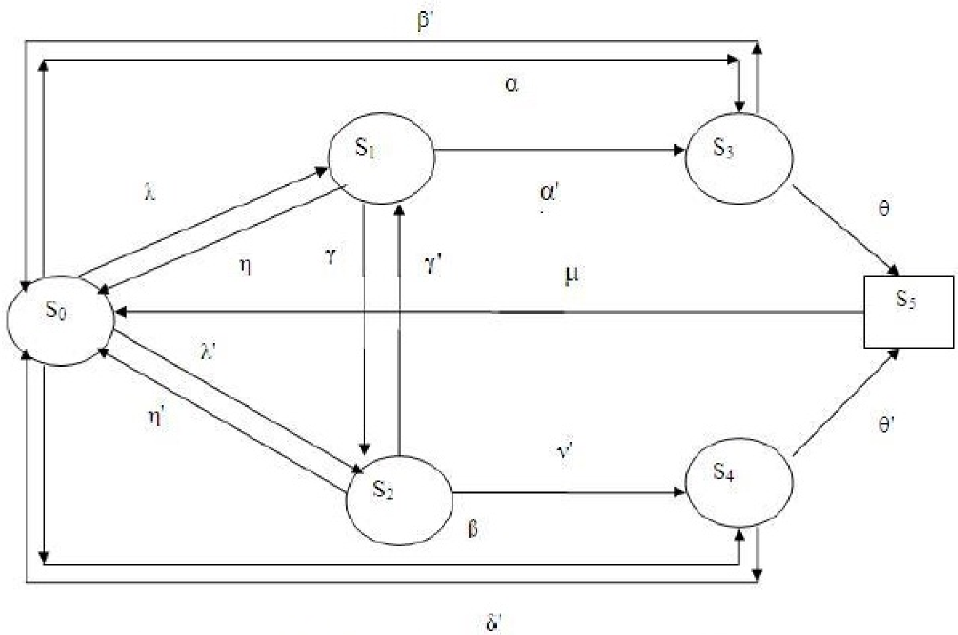
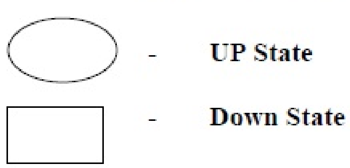


Figure 2. State Transition Diagram for System 2



4. Case (ii) Switching of State 1 and 2 for Preventive Maintenance

(a) **Up states** : $S_0 = (A_N, B_N), S_1 = (A_{Np}, B_N), S_2 = (A_N, B_{Np}), S_3 = (A_N, B_F), S_4 = (A_F, B_N)$

(b) **Down state** : $S_5 = (A_F, B_F)$.

4.1 Mean Time to System Failure ($MTSF_2$)

For figure (2), we obtain the following differential equation for this problem are :

$$\frac{dP'_{1,1}}{dt} = \lambda P_{2,0} - (\alpha' + \eta + \gamma)P_{1,1} + \gamma' P_{1,2} + 0P_{1,3} + 0P_{1,4} + 0P_{0,5} \tag{19}$$

$$\frac{dP'_{1,2}}{dt} = \lambda' P_{2,0} + \gamma P'_{1,1} - (\eta + v' + \gamma')P_{1,2} + 0P_{1,3} + 0P_{1,4} + 0P_{0,5}. \tag{20}$$

Equation (19) and (20) with equation in case (4)-(7) yields in to the form

$$\dot{P} = QP$$

where,

$$Q' = \begin{bmatrix} -(\lambda+\lambda'+\beta+\alpha) & \eta & \eta' & \beta & \delta' & \mu \\ \lambda & -(\eta+\alpha'+\gamma') & \gamma & 0 & 0 & 0 \\ \lambda' & \gamma' & -(\eta+\alpha'+\gamma) & 0 & 0 & 0 \\ \alpha & \alpha' & 0 & -(\theta+\beta) & 0 & 0 \\ \beta & 0 & v' & 0 & -(\delta'+\beta') & 0 \\ 0 & 0 & 0 & \theta & \theta' & -\mu \end{bmatrix} \tag{21}$$

To evaluate the transient solution is too complex. Therefore, we will restrict ourselves in calculating the MTSF2; we take the transpose matrix of Q and delete the rows and columns for the absorbing states. The new matrix is called B, the expected time to reach an absorbing states is calculated from

$$E_1[T_{p(0)\to p(\text{absorbing})}] = P(0)(-B^{-1}) \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

where,

$$B=\begin{bmatrix} -(\lambda+\lambda'+\beta+\alpha) & \lambda & \lambda' & \alpha & \beta \\ \eta & -(\eta+\alpha'+\gamma') & \gamma' & \alpha' & 0 \\ \eta' & \gamma & -(\eta+v'+\gamma) & 0 & v' \\ \beta & 0 & 0 & -(\theta+\beta') & 0 \\ \delta' & 0 & 0 & 0 & -(\delta'+\beta') \end{bmatrix} \tag{22}$$

and

$$\int_0^\infty e^{At}dt=-B^{-1}.$$

We obtain the following explicit expression for the $MTSF_2$

$$E_1[T_{p(0)\to p(\text{absorbing})}]=P(0)(-B^{-1})\begin{pmatrix}1\\1\\1\\1\\1\\1\end{pmatrix}$$

$$MTTF_2=\frac{\begin{array}{l}\{(\theta+\beta')((\delta'+\theta')\eta\cdot\gamma+(v'+\eta')(\eta'\cdot\alpha'+\eta\cdot\delta')+v'\cdot\gamma'\cdot\theta')\\ +\{\alpha'\cdot\gamma\cdot\beta'\cdot+\theta\cdot\eta'\cdot\gamma\}(\theta'+\delta')+(v'+\eta)\cdot(\theta+\beta')((\alpha'+\eta)\cdot\theta'+\delta'\cdot\gamma')\\ +\theta'\cdot\beta'\cdot\eta'\cdot\gamma'+\theta\cdot\delta\cdot\alpha'\gamma\}+(\theta+\beta')\cdot(\delta+\theta')\cdot((\lambda+\lambda')\cdot(\gamma+v'\cdot a)+\eta\cdot\lambda)\\ +(\delta+\theta')\cdot\{(\theta+\beta')\cdot(\eta\cdot b+\lambda\cdot p+\lambda'\cdot(\alpha'+\gamma'))+\theta\cdot\lambda'\gamma'\}+\{\eta\cdot\beta\gamma(\delta'+\theta')\}\\ +\{\eta\cdot(\delta'+\theta')+\delta'\cdot\gamma'\}\beta\cdot(v'+\eta')+((\eta'+\gamma)\delta'\alpha'+(\delta'+\theta')v'\cdot f)(\lambda+\beta)\\ +(\lambda+\lambda')\cdot\theta'\cdot\alpha'\cdot\gamma+\theta'\cdot\eta'\cdot\beta\cdot(\alpha'+\gamma')+\theta'\cdot\beta\cdot(v'\cdot\gamma'+\gamma\cdot\alpha')\\ +(\theta'\cdot\eta'\cdot\lambda\cdot\alpha'+\delta'\cdot\lambda'\cdot\alpha'\cdot\gamma)+(\eta+\gamma'+\alpha')\alpha\cdot\theta(v'+\eta')\\ +\{(\eta+\alpha')\gamma\cdot\alpha+\eta\cdot\lambda'\cdot v'+\lambda'\cdot\alpha'\beta'\}(\theta+\beta')+\theta\cdot v'\cdot\gamma'(\lambda+\lambda')\\ +(\lambda'+\alpha)v'\cdot\gamma\cdot\beta'+(v'\cdot\lambda+\eta'\cdot\alpha)\cdot\gamma'\beta'+\alpha\cdot v'\cdot\beta'(\eta+\alpha)\end{array}}{\begin{array}{l}\theta'\{((\theta+\beta')(\alpha'+\eta)+\gamma'\cdot\beta')\cdot\alpha(v'+\eta')+\lambda'\cdot\alpha'\cdot\beta'(v'+\gamma)\\ +((\lambda+\lambda')\cdot v'\cdot\gamma'+\gamma\cdot\alpha(\eta+\alpha'))\beta'+((\theta+\beta')\lambda'\cdot v'+(\theta+\alpha')\gamma\cdot\alpha)\eta\\ +(\lambda'+\alpha)(\lambda'\cdot\alpha'+\lambda\gamma')(\alpha'+\gamma')\theta\cdot v'\}+(\delta'+\theta')\{((\gamma+v'+\eta')\cdot\eta\\ +(v'+\eta')\cdot\gamma)\cdot\beta\cdot\beta'+((\lambda+\beta)(v'+\lambda)+h\cdot\lambda)\cdot\alpha'\beta'\}+\lambda'\cdot\alpha'\cdot\gamma\cdot\beta'\cdot\delta'\\ +\theta\cdot\eta\cdot\alpha\cdot\gamma'\cdot\theta'\end{array}} \tag{23}$$

4.2 Availability Analysis of the System $2[A_{T2}(\infty)]$

For the availability of figure 2, the initial conditions are the same as for the reliability case

$$P(0) = [P_{2,0}(0), P'_{1,1}(0), P'_{1,2}(0), P_{1,3}(0), P_{1,4}(0), P_{0,5}(0)] = (1, 0, 0, 0, 0, 0).$$

The differential equation can be expressed as

$$\begin{pmatrix} \dot{P}_{2,0} \\ \dot{P}_{1,1} \\ \dot{P}_{1,2} \\ \dot{P}_{1,3} \\ \dot{P}_{1,4} \\ \dot{P}_{0.5} \end{pmatrix} = \begin{pmatrix} -(\lambda+\lambda'+\beta+\alpha) & \eta & \eta' & \beta & \delta' & \mu \\ \lambda & -(\eta+\alpha'+\gamma) & \gamma & 0 & 0 & 0 \\ \lambda' & \gamma' & -(\eta+v+\gamma) & 0 & 0 & 0 \\ \alpha & \alpha' & 0 & -(\theta+\beta') & 0 & 0 \\ \beta & 0 & v' & 0 & -(\delta'+\beta') & 0 \\ 0 & 0 & 0 & \theta & \theta' & -\mu \end{pmatrix} \begin{pmatrix} P_{2,0} \\ P_{1,1} \\ P_{1,2} \\ P_{1,3} \\ P_{1,4} \\ P_{0.5} \end{pmatrix} \tag{24}$$

The steady state availabilities can be obtained using the following procedure in the steady state, the derivatives of the state probabilities becomes zero. That allows us to calculate the steady state probabilities with

$$A_{T_2}(\infty) = 1 - P_{0,5}(\infty)$$

and

$$QP(\infty) = 0$$

or in the matrix form

$$\begin{pmatrix} -(\lambda+\lambda'+\beta+\alpha) & \eta & \eta' & \beta & \delta' & \mu \\ \lambda & -(\eta+\alpha'+\gamma') & \gamma & 0 & 0 & 0 \\ \lambda' & \gamma' & -(\eta+v'+\gamma) & 0 & 0 & 0 \\ \alpha & \alpha' & 0 & -(\theta+\beta') & 0 & 0 \\ \beta & 0 & v' & 0 & -(\delta'+\beta') & 0 \\ 0 & 0 & 0 & \theta & \theta' & -\mu \end{pmatrix} \begin{pmatrix} P_{2,0}(\infty) \\ P_{1,1}(\infty) \\ P_{1,2}(\infty) \\ P_{1,3}(\infty) \\ P_{1,4}(\infty) \\ P_{0.5}(\infty) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \tag{25}$$

To obtain $P_{0,3}(\infty)$ we solve above and the following normalization condition:

$$P_{2,0}(\infty) + P_{1,1}(\infty) + P_{1,2}(\infty) + P_{1,3}(\infty) + P_{1,4}(\infty) + P_{1,5}(\infty) = 1$$

We substitute (10) in any one of the redundant rows in (9) to yield

$$\begin{pmatrix} -(\lambda+\lambda'+\beta+\alpha) & \eta & \eta' & \beta & \delta' & \mu \\ \lambda & -(\eta+\alpha'+\gamma') & \gamma & 0 & 0 & 0 \\ \lambda' & \gamma' & -(\eta+v'+\gamma) & 0 & 0 & 0 \\ \alpha & \alpha' & 0 & -(\theta+\beta') & 0 & 0 \\ \beta & 0 & v' & 0 & -(\delta'+\beta') & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} P_{2,0}(\infty) \\ P_{1,1}(\infty) \\ P_{1,2}(\infty) \\ P_{1,3}(\infty) \\ P_{1,4}(\infty) \\ P_{0.5}(\infty) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \tag{26}$$

The solution above of (26) provides the steady state probabilities in the availability case. For Figure 2, the explicit expression for $A_{T2}(\infty)$ is given by the following equations

$$A_{T_2}(\infty) = \frac{\begin{array}{l} \mu\cdot(\theta'+\beta')\,[\eta\cdot\gamma(\theta'+\beta')+\delta(v'+\eta')(\alpha'+\eta)+v'\cdot\gamma'\cdot(\theta'+\delta') \\ +2\theta'\cdot(v'+\eta')\cdot(\alpha'+\theta')+(\theta'+\delta')\cdot(\eta'+\alpha')\cdot(\gamma'+\gamma)] \end{array}}{\begin{array}{l} [\{\lambda'\cdot v'\cdot\theta'(\theta+\beta')+(\delta'+\theta')\cdot v'\cdot\beta'\cdot\beta+\theta'\cdot\eta'\cdot\beta\cdot\beta'+\mu\cdot\delta'\cdot v'\cdot\beta\} \\ (\alpha'+\gamma')+\delta'\cdot\eta\cdot\beta\cdot\beta'\cdot(v'+\gamma)+\theta'\cdot\eta\cdot\beta\cdot v'\cdot\beta'+\theta'\cdot\eta\cdot\theta\cdot\gamma+\alpha \\ +\eta'\cdot\delta'\cdot\alpha'\cdot\beta'(\beta+\lambda)+\{(\mu+\beta)\lambda'\cdot\gamma+\eta'\cdot\eta\cdot\alpha'\beta\cdot\beta'+\mu\cdot\eta\cdot\gamma\cdot\beta \\ +(\mu+\beta')v'\cdot\lambda\cdot\alpha'+\mu\cdot\eta'\cdot\lambda\cdot\alpha'\}(\delta'+\theta')+\theta'\cdot\eta'\cdot\lambda\cdot\alpha'\cdot\beta' \\ +\theta'\cdot\eta\cdot\gamma\cdot\beta'(\beta+\alpha)+4\gamma\cdot\alpha'\cdot(\mu+\beta')(\theta'+\delta')(\lambda+\beta)\delta'\cdot\eta'\cdot\beta\cdot\beta'\cdot\gamma' \\ +\mu\cdot\delta'\cdot v'(\eta\cdot\beta+\alpha'\cdot\beta')+\theta'\cdot\alpha\cdot\gamma'(\theta+\beta')(\gamma+'+\eta') \\ +\mu\cdot\beta(\delta'\cdot\eta'+\theta'\cdot\eta'+\theta'\cdot v')(\eta+\alpha'+\gamma')+v'\cdot\lambda\cdot\gamma'\cdot\theta' \\ +\mu\cdot\gamma\cdot\alpha\cdot\alpha'\cdot\beta'+\{\mu\cdot(\theta'\cdot\lambda'+\eta'\alpha)\cdot(\gamma+\alpha'+\eta)+\mu\cdot\eta'(\delta'+\theta') \\ (\mu\cdot\eta'(\gamma'+\eta+\alpha'+\lambda)+\gamma'v'+\mu\cdot\alpha'\cdot\gamma)+\mu\cdot\lambda'\cdot(\delta+v')(\gamma'+\alpha'+\eta) \\ +\alpha\cdot\theta'\cdot\alpha'\gamma+\mu\cdot\theta'\cdot v'\cdot(\alpha'+\lambda)+\mu\cdot\lambda'\cdot\beta'\cdot\gamma+\mu\cdot\theta'\cdot\lambda\cdot(\gamma+\gamma') \\ +\mu\cdot\theta'\cdot\lambda'\cdot\gamma'+\mu\cdot\delta'\cdot\eta\cdot\alpha'+\lambda'\cdot\theta'\cdot\eta\cdot v'+\mu\cdot\eta\cdot(v'+\gamma)\cdot\alpha \\ +\mu\cdot v'\cdot\gamma'(\lambda+\alpha)+\mu\cdot\alpha\cdot v'\cdot\alpha'+v'\cdot\lambda\cdot\gamma'\cdot\theta'+\mu\cdot\delta'\cdot\lambda\cdot(v'+\gamma') \\ +\theta'\cdot(\eta+\alpha')\cdot(v'+\eta')+\mu\cdot\delta'\cdot\lambda\cdot\gamma\}(\theta+\beta')] \end{array}} \tag{27}$$

5. Calculations

To observe the effect of switching for preventive maintenance in the systems, we assume following parameters; fixing γ and γ' with 0.5 and $\beta = 1.5, \alpha = 1.5, \eta = 3$, $acute\alpha = 2.8, \beta' = 2.5, \acute{\eta} = 2.75, v' = 2, \theta = 3.5$, $\delta' = 2.8, \theta' = 2.0$ while the frequent change in some of the parameters is done, and shown in following tables.

5.1 Results of MTSF for Figure 1 and Figure 2

(a) When failure rates are common:

Table (1)

Failure rate (common)	$\lambda = \lambda' = 0.25$	$\lambda = \lambda' = 0.3$	$\lambda = \lambda' = 0.5$
System 1	1.0533	1.0554	10634
System 2	1.0243	1.0212	1.0098

(b) When failure rate are different

Table (2)

Failure rates	$\lambda = 0.3, \lambda' = 0.25$	$\lambda = 0.5, \lambda' = 0.75$
Figure 1	1.2568	1.0791
Figure 2	1.1952	1.0116

5.2 Results of the Availability of Systems Shown in Figure 1 and Figure 2

(a) When failure rate are common

Table (3)

Failure rate	$\lambda = \lambda' = 0.25$	$\lambda = \lambda' = 0.30$	$\lambda = \lambda' = 0.5$
Figure 1	0.4753	0.4679	0.4405
Figure 2	0.4761	0.4724	0.4419

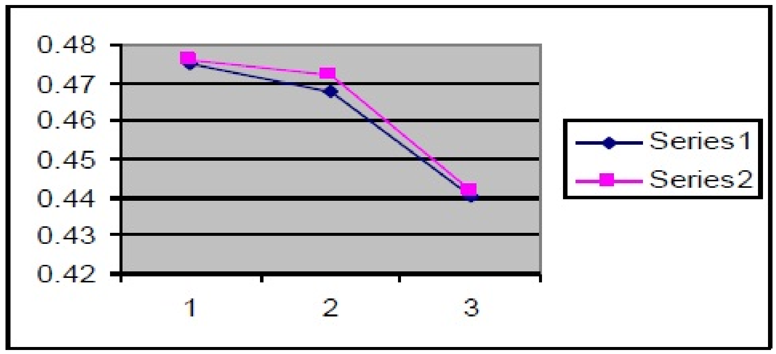
(b) When failure rate are different

Table (4)

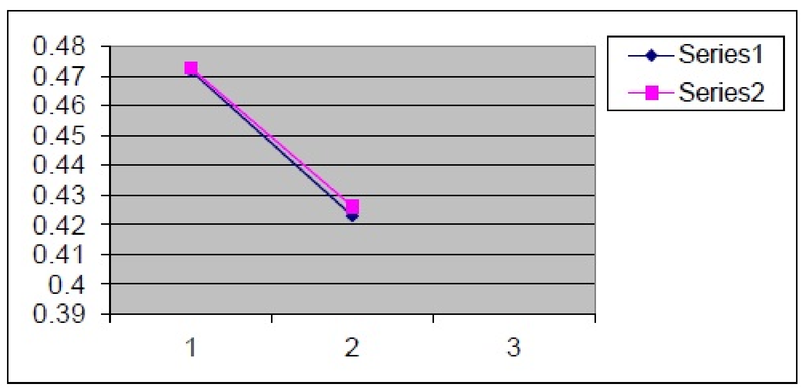
Failure rate	$\lambda = 0.3, \lambda' = 0.25$	$\lambda = 0.5, \lambda' = 0.75$
Figure 1	0.4717	0.4230
Figure 2	0.4726	0.4262

Graphs showing improvement in availabilities corresponding to table (3) and (4)

Graph(A)



Graph (B)



Series 1: SHOWS PERFOMANCE OF SYSTEM DISCRIBED IN FIGURE 2

Series 2: SHOWS PERFOMANCE OF SYSTEM DISCRIBED IN FIGURE 1 .

6. Conclusions

This is very clear from graph (A) and graph (B) that on application of switching for switching for preventive maintenance between states (1)and (2)results in increasing availability of the system in both the cases for common failure rates as well with different failure rates. While mean time to failure is significantly reduced in both the cases after applying switching in between states S_1 and S_2.

References

[1] Said K. M. and EL Sherbney M.S, On the evaluation of reliability and availability characteristics of two different systems, Information and Management Science, 1(1) (2007), 81-90.

[2] Said K. M. and EL Sherbney M. S., Profit analysis of a two unit cold stand by with preventive maintenance and random change in units. Department of Mathematics, Faculty of science, Helwan University, Cairo, Egypt.

[3] Goel L. R. and Gupta P., Stochastic analysis of a two-unit parallel system with partial and catastrophic failures and preventive maintenance, Microelectron Reliability, 24(55) (1984), 881.

[4] Goel L. R., Sharma G. C. and Gupta P., Reliability analysis of a system with preventive maintenance and inspection and two types of repair, Microelectron Reliability, 26(3) (1986), 429-433.

[5] Goel L. R. and Shrivastava P., Comaprison of reliability characterstics of two (double component) systems with bivariate exponential lifetimes, Journal Systems Sci., 23(1) (1992), 135-144.

[6] Gopalan M. N. and Nagarwalla H. E., Cost benefit analysis of a one-server two-unbit cold stand by system with repair and preventive maintenance, Microelectron. Reliab., 25(2) (1985), 27-269.

[7] Emara S. A. A. and El-Said Kh. M., Repairable system with different failure modes, Math. Congress proceeding of non-linear analysis, Tempa, Florida, U.S.A., (Aug 1992), 19-26.

[8] Rander M. C., Suresh K. and Ashok K., Cost analysis of two dissimilar cold standby system with preventive maintenance and replacement of standby, Microelectron. Reliab., 34 (1994), 171-174.

[9] El-Said Kh. M. and EL-Sherbeny M.S., Evaluation of reliability and availability characteristics of two different systems by using linear first order differential equations. J. Math. Stat., 1 (2005), 119-123.

[10] Goel L. R., Gupta R. and Singh S. K., Profit analysis of a cold standby system with two repair distribution. Microelectron. Reliab., 25 (1986), 467-472.

[11] Haggag M. Y., Cost Analysis of two-dissimilar-unit cold standby system with three states and preventive maintenance using linear first order differential equations. Journal of Mathematics and Statistics, 5(4) (2006), 395-400.

[12] Sridharan V. and Mohanavadivu P., Stochastic behavior of a two-unit standby with two types of repairmen and patience time, Math. Compute. Modeling, 28(9) (1998), 63-71.

[13] Lai C. D., Xie M., Poh K. L., Dai Y. S. and Yang P., A model for availability analysis of distributed software/hardware systems, Information and Software Technology, 44 (2002), 343-350.

[14] Wang K. H., Liou Y. C. and Pearn W. L., Cost benefit analysis of series systems with warm standby components and general repair time, Mathematical Methods of Operations Research, 61 (2005), 329-343.

[15] Wang K. H., Hsieh C. H. and Liou C. H., Cost benefit analysis of series systems with cold standby components and repairable service station, Quality Technology and Quantitative Management, 3(1) (2006), 77-92.