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SOME ASPECTS OF A KIND OF PAIRWISE SEMI GENERALIZED CLOSED SETS

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Abstract

The aim of this paper is to introduce a new class of sets, called ij - semi (δ, θ)-open sets and study some of it's properties. Using these sets, we shall also define the notion of generalized $ij - s\wedge_\delta^\theta$ sets. The major properties of this new concept will be studied. Finally, we introduce, a new form of generalized closed sets called $ij - (\delta, \theta)$ semi generalized closed sets by utilizing ij - semi (δ, θ)-closure operator. Also we introduce $ij - (\delta, \theta)$-sg continuity, $ij - (\delta, \theta)$-sg irresolute maps and investigates some of their fundamental properties.

1. Introduction

Throughout the present paper, (X, τ_1, τ_2) (or briefly X) always mean a bitopological space. Also $i, j = 1, 2$ and $i \neq j$. Let A be a subset of (X, τ_1, τ_2). By $i - int(A)$ and

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Key Words : *ij - semi (δ, θ)-open sets, $g - ij - s\wedge_\delta^\theta$ sets, $ij - s\wedge_\delta^\theta$ closed sets, $ij - (\delta, \theta)$-sg closed sets, $ij - (\delta, \theta)$-sg continuity, $ij - (\delta, \theta)$-sg irresoluteness.*

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$i-cl(A)$, we mean respectively the interior and the closure of A in the topological space (X,τ_i) for $i=1,2$. A subset A of X is called[8] ij - regular open (resp. ij - semi open) if $A=i-int[j-cl(A)]$ (resp. $A\subseteq j-cl[i-int(A)]$). A point x of X is called an $ij-\delta$ - cluster point of A if $i-int(j-cl(U))\cap A\neq\phi$ for every τ_i - open set U containing x. The set of all $ij-\delta$ - cluster points of A is called the $ij-\delta$ - closure of A and is denoted by $ij-\delta cl(A)$.

In this paper, we introduce a new class of sets, called ij - semi (δ,θ)-open sets and study some of it's properties. Using these sets, we shall also define the notion of generalized $ij-s\wedge_\delta^\theta$ set. The major properties of this new concept will be studied. Finally, we introduce a new form of generalized closed sets called $ij-(\delta,\theta)$ semi generalized closed sets by utilizing ij - semi (δ,θ)-closure operator. Also we introduce $ij-(\delta,\theta)$-sg continuity, $ij-(\delta,\theta)$-sg irresolute maps and studies some of their fundamental properties.

Recall the following definitions.

Definition 1.1 [9] : A subset A is said to be $ij-\delta$ closed if $ij-\delta Cl(A)=A$. The complement of an $ij-\delta$ closed set is said to be $ij-\delta$ open. The set of all $ij-\delta$ open (resp. $ij-\delta$ closed) sets of X will be denoted by $ij-\delta$O(X)(resp. $ij-\delta$C(X)).

Definition 1.2 [4] : A subset A of a bitopological spaces (X,τ_1,τ_2) is called $ij-\delta$ semi open if there exists a $ij-\delta$ open set U of X such that $U\subseteq A\subseteq j-cl(U)$. The set of all $ij-\delta$ semi open (resp. $ij-\delta$ semi closed) sets of X will be denoted by $ij-\delta SO(X)$(resp.$ij-\delta SC(X)$).

Definition 1.3 [11] : A subset A of a bitopological spaces (X,τ_1,τ_2) is called ij - semi-generalized closed set (briefly ij - sg-closed) if $ji-scl(A)\subseteq U$ whenever $A\subseteq U$ and U is ij - semi open set in X.

Definition 1.4 [4] : A bitopological space (X,τ_1,τ_2) is called $ij-\delta s-T_1$ if for each distinct points $x,y\in X$, there exist two $ij-\delta$ semi open sets U and V such that $x\in U\backslash V$ and $y\in V\backslash U$. If X is $12-\delta s-T_1$ and $21-\delta s-T_1$, then it is called pairwise $\delta s-T_1$ $(P-\delta s-T_1)$. A bitopological space (X,τ_1,τ_2) is pairwise $\delta s-T_1$ if and only of every singleton is pairwise δ - semi closed.

Definition 1.5 [4] : A function $f:(X,\tau_1,\tau_2)\longrightarrow(Y,\sigma_1,\sigma_2)$ is said to be $ij-\delta s$ continuous, if $f^{-1}(V)$ is $ij-\delta$ semi open set in X for every σ_i - open set V in Y.

Definition 1.6 [4] : A function $f:(X,\tau_1,\tau_2)\longrightarrow(Y,\sigma_1,\sigma_2)$ is said to be $ij-\delta s$ irresolute, if $f^{-1}(V)$ is $ij-\delta$ semi open set in X for every $ij-\delta$ semi open set V in Y.

Definition 1.7 [11] : A function $f:(X,\tau_1,\tau_2)\longrightarrow(Y,\sigma_1,\sigma_2)$ is said to be ij - sg continuous, if $f^{-1}(V)$ is ij - sg closed set in X for every σ_j - open set V in Y.

2. ij - Semi (δ,θ)-Open Sets.

Definition 2.1 : A point $x\in X$ is said to be an ij - semi (δ,θ)-cluster point of A if $ji-\delta scl(U)\cap A\neq\phi$ for every $ij-\delta$ semi open set U containing x. The set of all ij - semi (δ,θ)-cluster point of A is called the ij - semi (δ,θ)-closure of A and is denoted by $ij-\delta scl_\theta(A)$. A subset A is called ij - semi (δ,θ)-closed if $ij-\delta scl_\theta(A)=A$.
The set $\{x\in X: ji-\delta scl(U)\subseteq A$, for some $ij-\delta$ semi open $U\}$ is called the ij - semi (δ,θ)-interior of A and is denoted by $ij-\delta sint_\theta(A)$. A subset A is called ij - semi (δ,θ)-open if $A=ij-\delta sint_\theta(A)$.

Theorem 2.2 : Let A be a subset of a bitopological space (X,τ_1,τ_2). Then
(i) $X\backslash ij-\delta sint_\theta(A)=ij-\delta scl_\theta(X\backslash A)$.
(ii) $X\backslash ij-\delta scl_\theta(A)=ij-\delta sint_\theta(X\backslash A)$.

Proof : (i) Let $x\notin ij-\delta scl_\theta(X\backslash A)$. Then there exists $U\in ij-\delta SO(X)$ containing x such that $ji-\delta scl(U)\cap(X\backslash A)=\phi$. Thus $x\in U\subseteq ji-\delta scl(U)\subseteq A$ and $x\in ij-\delta sint_\theta(A)$. Hence $x\notin X\backslash ij-\delta sint_\theta(A)$. Now let $x\notin X\backslash ij-\delta sint_\theta(A)$. Thus $x\in ij-\delta sint_\theta(A)$ and there exists $U\in ij-\delta SO(X)$ such that $x\in U\subseteq ji-\delta scl(U)\subseteq A$. Hence $ji-\delta scl(U)\cap(X\backslash A)=\phi$ and $x\notin ij-\delta scl_\theta(X\backslash A)$.
(ii) The proof is similar to that of (i).

Theorem 2.3 : Let A be a subset of a bitopological space (X,τ_1,τ_2). If A is $ij-\delta$ semi open, then $ji-\delta scl(A)=ji-\delta scl_\theta(A)$.

Definition 2.4 : A subset A of a bitopological space (X,τ_1,τ_2) is called $ij-\delta$ semi regular if A is both $ji-\delta$ semi open and $ij-\delta$ semi closed. The set of all $ij-\delta$ semi regular subsets of a space X is denoted by $ij-\delta SR(X)$.

Theorem 2.5 : If $U\in ij-\delta SO(X)$, then $ji-\delta scl(U)\in ji-\delta SR(X)$.

Theorem 2.6 : Let A be a subset of a bitopological space (X,τ_1,τ_2), then
(i) $ij-\delta scl_\theta(A)$ is ij - semi (δ,θ)-closed.
(ii) $ij-\delta scl_\theta(A)=\bigcap\{ij-\delta scl(V): A\subseteq V\in ji-\delta SO(X)\}$.

Proof : (i) We show that $ij-\delta scl_\theta(A)=ij-\delta scl_\theta[ij-\delta scl_\theta(A)]$, for a subset A of X. Since $A\subseteq ij-\delta scl_\theta(A)$, then $ij-\delta scl_\theta(A)\subseteq ij-\delta scl_\theta[ij-\delta scl_\theta(A)]$.
Conversely, let $x\notin ij-\delta scl_\theta(A)$. This means there exists $U_0\in ij-\delta SO(X)$ containing

x such that $ji - \delta scl_\theta(U_0) \cap A = \phi$. Suppose that $ji - \delta scl_\theta(U) \cap ij - \delta scl_\theta(A) \neq \phi$, for every $U \in ij - \delta SO(X)$. Therefore, there exists a point $y \in X$ such that $y \in ji - \delta scl(U) \in ij - \delta scl_\theta(A)$. On the other hand, $y \in ij - \delta scl_\theta(A)$ for any $V \in ij - \delta SO(X)$ containing y, we have $ji - \delta scl(V) \cap A \neq \phi$. Since $y \in ji - \delta scl(U)$ and by theorem 2.5, $ji - \delta scl(U)$ is $ij - \delta$ semi open, then $ji - \delta scl(U) \cap A \neq \phi$. But this is a contradiction, since $x \notin ij - \delta scl_\theta(A)$. Thus $x \notin ij - \delta scl_\theta[ij - \delta scl_\theta(A)]$. This implies $ij - \delta scl_\theta(A) = ij - \delta scl_\theta[ij - \delta scl_\theta(A)]$.
(ii) By theorem 2.3, $ij - \delta scl_\theta(A) \subseteq \bigcap\{ij - \delta scl(V) : A \subseteq V \in ji - \delta SO(X)\}$.
Conversely, let $x \notin ij - \delta scl_\theta(A)$ and $A \subseteq V \in ji - \delta SO(X)$. Then there exists $U \in ji - \delta SO(X)$ containing x such that $ji - \delta scl(U) \cap A \neq \phi$. Thus for each $y \in A$, we have $y \in V$, $y \notin ji - \delta scl(U)$ and so, $U \cap V = \phi$. This implies that $U \cap ij - \delta scl(V) = \phi$ and $x \notin ij - \delta scl(V)$. Hence $x \notin \bigcap\{ij - \delta scl(V) : A \subseteq V \in ji - \delta SO(X)\}$ and $\bigcap\{ij - \delta scl(V) : A \subseteq V \in ji - \delta SO(X)\} \subseteq ij - \delta scl_\theta(A)$. From this, we note that $ij - \delta scl_\theta(A)$ is $ij - \delta$ semi closed set in X.

Theorem 2.7 : Let A be a subset of a bitopological space (X, τ_1, τ_2). If $A \in ji - \delta SR(X)$, then A is both ij - semi (δ, θ)-closed and ji - semi (δ, θ)-open.

Proof : Let $A \in ij - \delta SR(X)$, then $A \in ji - \delta SO(X)$ and $A \in ij - \delta SC(X)$. If A is $ji - \delta$ semi open set of X, then by theorem 2.5, $ij - \delta scl(A) \in ij - \delta SR(X)$. Thus $ij - \delta scl(A)$ is $ji - \delta$ semi open set of X and hence by theorem 2.3, $A = ij - \delta scl(A) = ij - \delta scl_\theta(A)$. This implies that A is ij - semi (δ, θ)-closed. Therefore, if A is $ij - \delta$ semi closed, then $X \backslash A$ is $ij - \delta$ semi open set of X. Similar $X \backslash A$ is ji - semi (δ, θ)-closed set and A is ji - semi (δ, θ)-open set of X.

Theorem 2.8 : In a bitopological space (X, τ_1, τ_2), a singleton is an ij - semi (δ, θ)-open if and only if it is $ji - \delta$ semi regular.

Proof : Let $x \in X$ and $\{x\}$ be an ij - semi (δ, θ)-open set of X. Then by the fact that a set A is ij - semi (δ, θ)-open if and only if there exists $U \in ij - \delta SO(X)$ containing x such that $ji - \delta scl(U) \subseteq A$. Since the superset $ji - \delta$ semi closed contained in $\{x\}$ is $\{x\}$, hence $\{x\}$ is $ji - \delta$ semi regular. The converse part of the proof is follows directly from theorem 2.7.

3. Generalized $ij - s\wedge_\delta^\theta$ Sets

Definition 3.1 : For a subset A of a bitopological space (X, τ_1, τ_2), we define $A^{\delta s \wedge_{ij}}$

and $A^{\delta s\vee_{ij}}$ as follows, $A^{\delta s\wedge_{ij}} = \bigcap\{U : A \subseteq U, U \in ij - \delta SO(X)\}$ and $A^{\delta s\vee_{ij}} = \bigcup\{U : U \subseteq A, U^C \in ij - \delta SO(X)\}$.

Definition 3.2 : Let A be a subset of a bitopological space (X, τ_1, τ_2), then we define
(a) $A_\delta^{\theta s\wedge_{ij}} = \{x \in X : ij - \delta scl_\theta(\{x\}) \cap A \neq \phi\}$.
(b) A is called $ij - s\wedge_\delta^\theta$ set if $A = A_\delta^{\theta s\wedge_{ij}}$.
(c) A is called generalized $ij - s\wedge_\delta^\theta$ set (briefly $g - ij - s\wedge_\delta^\theta$ set) if $A_\delta^{\theta s\wedge_{ij}} \subseteq U$ whenever $A \subseteq U$ and U is a $ji - \delta$ semi closed set of (X, τ_1, τ_2).

Theorem 3.3 : For any subset A of a bitopological space (X, τ_1, τ_2), $A_\delta^{\theta s\wedge_{ij}} = \bigcap\{U : A \subseteq U, U is ij - semi(\delta, \theta) - open\}$.

Proof : Let $H = \bigcap\{U : A \subseteq U, U is ij - semi(\delta, \theta) - open\}$ and $x \in H$. Suppose that $x \notin A_\delta^{\theta s\wedge_{ij}}$. That is $ij - \delta scl_\theta(\{x\}) \cap A = \phi$. Hence $x \notin X\backslash ij - \delta scl_\theta(\{x\})$, where $X\backslash ij - \delta scl_\theta(\{x\})$ is ij - semi (δ, θ)-open set containing A, by theorem 2.6. But $x \in H$. Therefore $x \in A_\delta^{\theta s\wedge_{ij}}$.
Conversely, Let $x \in A_\delta^{\theta s\wedge_{ij}}$. If $x \notin H$, then there exists an ij - semi (δ, θ)-open set U such that $A \subseteq U$ and $x \notin U$. Assume that $y \in ij - \delta scl_\theta(\{x\}) \cap A$. Thus $y \in U$ and $x \notin U$. This is contradiction. Therefore $x \in H$ and hence $x \in A_\delta^{\theta s\wedge_{ij}}$.

Theorem 3.4 : Let (X, τ_1, τ_2) be a bitopological space, then
(a) For any set $A \subset X$, $A \subseteq A^{\delta s\wedge_{ij}} \subseteq A_\delta^{\theta s\wedge_{ij}} \subseteq ji - \delta scl_\theta(A)$.
(b) Every ij - semi (δ, θ)-closed is an $ji - \wedge_\delta^s$ set.
(c) Every $g - ij - s\wedge_\delta^\theta$ is $g - ij - \wedge_\delta^s$ set.

Proof : (a) Obviously, $A \subseteq A^{\delta s\wedge_{ij}}$. Now we prove that $A^{\delta s\wedge_{ij}} \subseteq A_\delta^{\theta s\wedge_{ij}}$. Suppose that $x \notin A_\delta^{\theta s\wedge_{ij}}$. It follows that $A \subseteq X\backslash ij - \delta scl_\theta(\{x\}) = U$. Since $ij - \delta scl_\theta(\{x\})$ is ij - semi (δ, θ)-closed by theorem 2.6, so U is ij - semi (δ, θ)-open. Hence there exists an $ij - \delta$ semi open set U containing A but not x, then $x \notin A_\delta^{\theta s\wedge_{ij}}$. Thus $A^{\delta s\wedge_{ij}} \subseteq A_\delta^{\theta s\wedge_{ij}}$. To prove that $A_\delta^{\theta s\wedge_{ij}} \subseteq ji - \delta scl_\theta(A)$, let $x \notin ji - \delta scl_\theta(A)$. Then there exists $U \in ij - \delta SO(X)$ containing x such that $ij - \delta scl(U) \cap A = \phi$. Since $U \in ji - \delta SO(X)$ it follows by theorem 2.3, $ij - \delta scl_\theta(U) \cap A = \phi$. Then $ij - \delta scl_\theta(\{x\}) \cap A = \phi$. Therefore $x \notin A_\delta^{\theta s\wedge_{ij}}$. This shows that $A_\delta^{\theta s\wedge_{ij}} \subseteq ji - \delta scl_\theta(A)$.
(b) Let A be an ij - semi (δ, θ)-closed set. Then $ij - \delta scl_\theta(A) = A$. Thus by $A^{\delta s\wedge_{ij}} \subseteq ji - \delta scl_\theta(A)$, $A^{\delta s\wedge_{ji}} \subseteq ij - \delta scl_\theta(A)$ and hence $A^{\delta s\wedge_{ji}} = A$. Thus A is a $ji - \wedge_\delta^s$.
(c) Let $A \subseteq U$ where U is $ji - \delta$ semi closed. By (i), we have $A \subseteq A^{\delta s\wedge_{ij}} \subseteq A_\delta^{\theta s\wedge_{ij}}$. This implies that A is a $g - ij - \wedge_\delta^s$ set.

Definition 3.5 : For a subset A of a bitopological space (X,τ_1,τ_2), we define $A_\delta^{\theta s\vee_{ij}}$ as follows, $A_\delta^{\theta s\vee_{ij}} = \bigcup\{U : U \subseteq A, U^C isij - semi(\delta,\theta) - open\}$.

Theorem 3.6 : For any subsets A and B of a bitopological space (X,τ_1,τ_2), the following are hold:

(a) $(A^C)_\delta^{\theta s\wedge_{ij}} = (A_\delta^{\theta s\wedge_{ij}})^C$.

(b) $A_\delta^{\theta s\vee_{ij}} \subseteq A$.

(c) If A is ij - semi (δ,θ)-open, then $A = A_\delta^{\theta s\vee_{ij}}$.

Proof. (a) By the definition,$(A_\delta^{\theta s\wedge_{ij}})^C = \bigcap\{U^C : U^C \supseteq A^C, U^C isij - semi(\delta,\theta) - open\} = (A^C)_\delta^{\theta s\wedge_{ij}}$.

(b) Obviously. Clear by the definition.

(c) If A is ij - semi (δ,θ)-closed subset of (X,τ_1,τ_2), then A^C is ij - semi (δ,θ)-open. By (a) and (b), we have $A^C = (A^C)_\delta^{\theta s\wedge_{ij}} = (A_\delta^{\theta s\wedge_{ij}})^C$. Therefore $A = A_\delta^{\theta s\wedge_{ij}}$.

Theorem 3.7 : Let A and $\{A_\alpha, \alpha \in J\}$ be the subsets of a bitopological space (X,τ_1,τ_2). Then the following are valid:

(a) $[\bigcup_{\alpha\in J} A_\alpha]_\delta^{\theta s\wedge_{ij}} = \bigcup_{\alpha\in J}(A_\alpha)_\delta^{\theta s\wedge_{ij}}$.

(b) $[\bigcap_{\alpha\in J} A_\alpha]_\delta^{\theta s\wedge_{ij}} \subseteq \bigcap_{\alpha\in J}(A_\alpha)_\delta^{\theta s\wedge_{ij}}$.

(c) $[\bigcup_{\alpha\in J} A_\alpha]_\delta^{\theta s\vee_{ij}} \supseteq \bigcup_{\alpha\in J}(A_\alpha)_\delta^{\theta s\vee_{ij}}$.

Proof : (a) Suppose there exists a point x such that $x \notin [\bigcup_{\alpha\in J} A_\alpha]_\delta^{\theta s\wedge_{ij}}$. Then there exists a ij - semi (δ,θ)-open subset U, such that $\bigcup_{\alpha\in J} A_\alpha \subseteq U$ and $x \notin U$. Thus for each $\alpha \in J$, we have $x \notin (A_\alpha)_\delta^{\theta s\wedge_{ij}}$. This implies that $x \notin \bigcup_{\alpha\in J}(A_\alpha)_\delta^{\theta s\wedge_{ij}}$.

Conversely, Suppose there exists a point x such that $x \notin \bigcup_{\alpha\in J}(A_\alpha)_\delta^{\theta s\wedge_{ij}}$. Then by the definition, there exist ij - semi (δ,θ)-open subsets U_α, for each $\alpha \in J$ such that $x \notin U_\alpha$ and $A_\alpha \subseteq U_\alpha$. Let $U = \bigcup_{\alpha\in J} U_\alpha$. Then $x \notin \bigcup_{\alpha\in J} U_\alpha$, $\bigcup_{\alpha\in J} A_\alpha \subseteq U$ and U is a ij - semi (δ,θ)-open set. Thus $x \notin [\bigcup_{\alpha\in J} A_\alpha]_\delta^{\theta s\wedge_{ij}}$.

(b) Suppose there exists a point x such that $x \notin \bigcap_{\alpha\in J}(A_\alpha)_\delta^{\theta s\wedge_{ij}}$, then there exists $\alpha \in J$ such that $x \notin (A_\alpha)_\delta^{\theta s\wedge_{ij}}$. Hence there exists a ij - semi (δ,θ)-open set U such that $U \supseteq A_\alpha$ and $x \notin U$. Thus $x \notin [\bigcap_{\alpha\in J} A_\alpha]_\delta^{\theta s\wedge_{ij}}$.

(c) $[\bigcup_{\alpha\in J} A_\alpha]_\delta^{\theta s\vee_{ij}} = [[(\bigcup_{\alpha\in J} A_\alpha)^C]_\delta^{\theta s\vee_{ij}}]^C = [[(\bigcap_{\alpha\in J} A_\alpha^C)]_\delta^{\theta s\wedge_{ij}}]^C \supseteq [\bigcap_{\alpha\in J}(A_\alpha^C)_\delta^{\theta s\wedge_{ij}})]^C = [\bigcap_{\alpha\in J}[(A_\alpha)_\delta^{\theta s\wedge_{ij}}]^C]^C$, by theorem 3.5(a). By (b), we have $[\bigcup_{\alpha\in J} A_\alpha]_\delta^{\theta s\vee_{ij}} \supseteq \bigcup_{\alpha\in J}(A_\alpha)_\delta^{\theta s\vee_{ij}}$.

Theorem 3.8 : Let A be a subset of a bitopological space (X,τ_1,τ_2). Then $A^{\delta s\wedge_{ij}} = (A_\delta^{\theta s\wedge_{ij}})_\delta^{\theta s\wedge_{ij}}$.

Proof : From the definition of $A_\delta^{\theta s\wedge_{ij}}$, we have $(A_\delta^{\theta s\wedge_{ij}})_\delta^{\theta s\wedge_{ij}} = \bigcap\{U : A_\delta^{\theta s\wedge_{ij}} \subseteq U, U is ij - semi(\delta,\theta) - open\} \subseteq \bigcap\{U : (\bigcap\{V : A \subseteq V, V is ij - semi(\delta,\theta) - open\}) \subseteq U, U is ij - semi(\delta,\theta) - open\} \subseteq \bigcap\{U : A \subseteq U, U is ij - semi(\delta,\theta) - open\} = A_\delta^{\theta s\wedge_{ij}}$. That is $(A_\delta^{\theta s\wedge_{ij}})_\delta^{\theta s\wedge_{ij}} \subseteq A_\delta^{\theta s\wedge_{ij}}$. On the other hand, since $A \subseteq A_\delta^{\theta s\wedge_{ij}}$, for any subset A, $A_\delta^{\theta s\wedge_{ij}} \subseteq (A_\delta^{\theta s\wedge_{ij}})_\delta^{\theta s\wedge_{ij}}$. Therefore $A^{\delta s\wedge_{ij}} = (A_\delta^{\theta s\wedge_{ij}})_\delta^{\theta s\wedge_{ij}}$.

Theorem 3.9 : A bitopological space (X, τ_1, τ_2) is a pairwise $\delta s - T_1$ space if and only if every subset is a $ij - \wedge_\delta^s$ set.

Proof : Let A be a subset of a pairwise $\delta s - T_1$ space. Suppose that there exists a point $x \in X$ such that $x \notin A$. By definition 1.4, $X\backslash\{x\}$ is an $ij - \delta$ semi open set containing A. This implies $x \notin A^{\delta s\wedge_{ij}}$. Hence $A^{\delta s\wedge_{ij}} \subseteq A$ and by theorem 3.4 (a), $A \subseteq A^{\delta s\wedge_{ij}}$. Thus $A = A^{\delta s\wedge_{ij}}$.

Conversely if $x \in X$, then by hypothesis $X\backslash\{x\}$ is $ij - \wedge_\delta^s$ set. Hence $\{x\}$ is the union of $ij - \delta$ semi closed sets and thus $ij - \delta$ semi closed. This shows that X is a pairwise $\delta s - T_1$ space, by definition 1.4.

Definition 3.10 : A subset A of a bitopological space (X, τ_1, τ_2) is called $ij - s\wedge_\delta^\theta$ closed if $A = G \cap F$, where G is a $ij - s\wedge_\delta^\theta$ set and F is a ji - semi $(\delta, \theta$-closed.

Theorem 3.11 : For a subset A of a bitopological space (X, τ_1, τ_2) the following conditions are equivalent:

(i) A is $ij - s\wedge_\delta^\theta$ closed.

(ii) $A = G \cap ji - \delta scl_\theta(A)$, where G is a $ij - s\wedge_\delta^\theta$ set.

(iii) $A = A_\delta^{\theta s\wedge_{ij}}$.

Proof : (i)$\Longrightarrow$(ii) Suppose that $A = G \cap F$ where G is a $ij - s\wedge_\delta^\theta$ set and F is ji - semi $(\delta, \theta$-closed. Then $A \subseteq G$ and $A \subseteq ji - \delta scl_\theta(A) \subseteq F$. Now, we have $A \subseteq G \cap ji - \delta scl_\theta(A) \subseteq G \cap F = A$. This means that $A = G \cap ji - \delta scl_\theta(A)$.

(ii)$\Longrightarrow$(iii) Suppose that $A = G \cap ji - \delta scl_\theta(A)$ where G is a $ij - s\wedge_\delta^\theta$ set. We have $A \subseteq A_\delta^{\theta s\wedge_{ij}} \subseteq G$ and $A \subseteq ji - \delta scl_\theta(A)$. So $A = A_\delta^{\theta s\wedge_{ij}} \cap ji - \delta scl_\theta(A)$ and hence $A = A_\delta^{\theta s\wedge_{ij}}$ by theorem 3.4.

(iii)$\Longrightarrow$(i) By theorem 3.8, $A_\delta^{\theta s\wedge_{ij}} = (A_\delta^{\theta s\wedge_{ij}})_\delta^{\theta s\wedge_{ij}}$ for any set A. Therefore $A_\delta^{\theta s\wedge_{ij}}$ is a $ij - s\wedge_\delta^\theta$ set. Suppose that $A = A_\delta^{\theta s\wedge_{ij}}$. By theorem 3.4, we have $A = A_\delta^{\theta s\wedge_{ij}} \cap ji - \delta scl_\theta(A)$. Clearly A is the intersection of a $ij - s\wedge_\delta^\theta$ set and a ji - semi $(\delta, \theta$-closed set and hence A is $ij - s\wedge_\delta^\theta$ closed.

Definition 3.12 : A subset A of a bitopological space (X, τ_1, τ_2) is called ij - quasi semi (δ,θ)-closed (briefly ij-qs(δ,θ)-closed) if $ji-\delta scl_\theta(A)\subseteq U$ wherever $A\subseteq U$ and U is ij - semi (δ,θ)-open in (X,τ_1,τ_2).

Theorem 3.13 : A subset A of a bitopological space (X,τ_1,τ_2) is ij-qs(δ,θ)-closed if and only if $ji-\delta scl_\theta(A)\subseteq A_\delta^{\theta s\wedge_{ij}}$.

Proof : Let $x\in X$ such that $x\notin A_\delta^{\theta s\wedge_{ij}}$. So there exists an ij - semi (δ,θ)-open subset U such that $A\subseteq U$ with $x\notin U$. This means that $x\notin ji-\delta scl_\theta(A)$, since A is ij-qs(δ,θ)-closed. Therefore $ji-\delta scl_\theta(A)\subseteq A_\delta^{\theta s\wedge_{ij}}$.

Conversely, let U be an ij - semi (δ,θ)-open set such that $A\subseteq U$. By assumption $A_\delta^{\theta s\wedge_{ij}}$ and by theorem 3.3, then $ji-\delta scl_\theta(A)\subseteq U$. Thus A is ij-qs(δ,θ)-closed.

Theorem 3.14 : For a subset A of a bitopological space (X,τ_1,τ_2) the following are equivalent:

(a) A is ji - semi (δ,θ)-closed.

(b) A is ij-qs(δ,θ)-closed and $ij-s\wedge_\delta^\theta$ closed.

Proof : (a)$\Longrightarrow$(b) Let A be a subset of X such that , $A\subseteq U$ where U is ij - semi (δ,θ)-open set. Since A is ji - semi (δ,θ)-closed, $A=ji-\delta scl_\theta(A)$. Thus $ji-\delta scl_\theta(A)\subseteq U$. Hence A is ij-qs(δ,θ)-closed. Therefore, by theorem 3.4(a) and theorem 3.13, $ji-\delta scl_\theta(A)=A_\delta^{\theta s\wedge_{ij}}=A$. Thus $A=A_\delta^{\theta s\wedge_{ij}}\cap ji-\delta scl_\theta(A)$ and hence A is a $ij-s\wedge_\delta^\theta$ closed.

(b)$\Longrightarrow$(a) Since A is ij-qs(δ,θ)-closed, then by theorem 3.13, $ji-\delta scl_\theta(A)\subseteq A_\delta^{\theta s\wedge_{ij}}$. Thus $A=A_\delta^{\theta s\wedge_{ij}}\cap ji-\delta scl_\theta(A)=ji-\delta scl_\theta(A)$. Hence A is ji- semi (δ,θ)-closed.

Theorem 3.15 : The following statements are equivalent for any points x and y in a bitopological space (X,τ_1,τ_2). (i) $\{x\}_\delta^{\theta s\wedge_{ij}}\neq\{y\}_\delta^{\theta s\wedge_{ij}}$.

(ii) $ij-\delta scl_\theta(\{x\})\neq ij-\delta scl_\theta(\{y\})$.

Proof : (i)$\Longrightarrow$(ii) Let $\{x\}_\delta^{\theta s\wedge_{ij}}\neq\{y\}_\delta^{\theta s\wedge_{ij}}$. Then there exists a point z in X such that $z\in\{x\}_\delta^{\theta s\wedge_{ij}}$ and $z\notin\{y\}_\delta^{\theta s\wedge_{ij}}$. By $z\in\{x\}_\delta^{\theta s\wedge_{ij}}$ it follows that $\{x\}\cap ij-\delta scl_\theta(\{z\})\neq\phi$. This implies $x\in ij-\delta scl_\theta(\{z\})$. By $z\notin\{y\}_\delta^{\theta s\wedge_{ij}}$ we obtain $\{y\}\cap ij-\delta scl_\theta(\{z\})=\phi$. Since $x\in ij-\delta scl_\theta(\{z\})$, $x\in ij-\delta scl_\theta(\{x\})\subseteq ij-\delta scl_\theta(\{z\})$ and $\{y\}\cap ij-\delta scl_\theta(\{x\})=\phi$. Hence it follows that $ij-\delta scl_\theta(\{x\})\neq ij-\delta scl_\theta(\{y\})$.

(ii)$\Longrightarrow$(i) Let $ij-\delta scl_\theta(\{x\})\neq ij-\delta scl_\theta(\{y\})$. Then there exists a point z in X such that $z\in\{x\}_\delta^{\theta s\wedge_{ij}}$ and $z\notin\{y\}_\delta^{\theta s\wedge_{ij}}$. This means that there exists an ij- semi (δ,θ)-open set containing z and therefore x but not y. Thus $y\notin\{x\}_\delta^{\theta s\wedge_{ij}}$. Hence $ij-\delta scl_\theta(\{x\})\neq ij-\delta scl_\theta(\{y\})$.

Definition 3.16 : A bitopological space (X, τ_1, τ_2) is a pairwise semi $(\delta, \theta) - R_0$ space if for every ij - semi (δ, θ)-open set U, $x \in U$ implies $ji - \delta scl_\theta(\{x\}) \subseteq U$.

Theorem 3.17 : In a bitopological space (X, τ_1, τ_2), the following statements are equivalent:

(a) (X, τ_1, τ_2) is pairwise semi $(\delta, \theta) - R_0$.
(b) For any $x \in X$, $ij - \delta scl_\theta(\{x\}) \subseteq \{x\}_\delta^{\theta s \wedge_{ij}}$.
(c) For any $x, y \in X$, $y \in \{x\}_\delta^{\theta s \wedge_{ij}}$ if and only if $x \in \{y\}_\delta^{\theta s \wedge_{ji}}$.
(d) For any $x, y \in X$, $y \in ij - \delta scl_\theta(\{x\})$ if and only if $x \in ji - \delta scl_\theta(\{y\})$.
(e) For any ij- semi (δ, θ)-closed set F and a point $x \notin F$, there exists a ji - semi (δ, θ)-open set U such that $x \notin U$, $F \subset U$.
(f) Each ij- semi (δ, θ)-closed , $F = \bigcap\{U : F \subseteq U$,and U is ji - semi (δ, θ)-open$\}$.
(g) Each ij - semi (δ, θ)-open set , $U = \bigcup\{F : U \subseteq F$,and F is ji - semi (δ, θ)-closed$\}$.
(h) For each ij - semi (δ, θ)-closed set F, $x \notin F$ implies $ji - \delta scl_\theta(\{x\}) \cap F = \phi$.

Proof : (a)$\Longrightarrow$(b) For any $x \in X$, we have by theorem 3.3, $\{x\}_\delta^{\theta s \wedge_{ij}} = \bigcap\{U : \{x\} \subseteq U, U$ is ji-semi (δ, θ)-open$\}$. Since (X, τ_1, τ_2) is pairwise semi $(\delta, \theta) - R_0$, each ji - semi (δ, θ)-open U containing x contains $ij - \delta scl_\theta(\{x\})$. Hence $ij - \delta scl_\theta(\{x\}) \subseteq \{x\}_\delta^{\theta s \wedge_{ij}}$.

(b)$\Longrightarrow$(c) For any $x, y \in X$, if $y \in \{x\}_\delta^{\theta s \wedge_{ij}}$, then $x \in ij - \delta scl_\theta(\{y\})$. By (b), we have $x \in \{y\}_\delta^{\theta s \wedge_{ji}}$. Similarly we can prove the other hand.

(c)$\Longrightarrow$(d) For any $x, y \in X$, if $y \in ij - \delta scl_\theta(\{x\})$, then $x \in \{y\}_\delta^{\theta s \wedge_{ij}}$. By (c), $y \in \{x\}_\delta^{\theta s \wedge_{ji}}$ and so $x \in ji - \delta scl_\theta(\{y\})$. Similarly we can prove the other hand.

(d)$\Longrightarrow$(e) Let F be an ij - semi (δ, θ)-closed set and a point $x \notin F$. Then for any point $y \in F$, $ij - \delta scl_\theta(\{y\}) \subseteq F$ and $x \notin ij - \delta scl_\theta(\{y\})$. By (d), $x \notin ij - \delta scl_\theta(\{y\})$ implies $y \notin ji - \delta scl_\theta(\{x\})$. Hence there exists a ji - semi (δ, θ)-open set U_y such that $y \in U_y$ and $x \notin U_y$. Let $U = \bigcup_{y \in F}\{U_y : y \in U_y and x \notin U_y, U_y is ji - semi(\delta, \theta) - open\}$. Then U is a ji - semi (δ, θ)-open set such that $x \notin U$ and $F \subseteq U$.

(e)$\Longrightarrow$(f) Let F be an ij - semi (δ, θ)-closed set and suppose that $H = \bigcap\{U : F \subseteq U, is ji - semi(\delta, \theta) - open\}$. Then $F \subseteq H$ and we show that $H \subseteq F$. Let $x \notin F$. Then by (e), there exists a ji - semi (δ, θ)-open set U such that $x \notin U$ and $F \subseteq U$ and hence $x \notin H$. Therefore $H \subseteq F$ and so, $H = F$.

(f)$\Longrightarrow$(g) Obvious.

(g)$\Longrightarrow$(h) Let F be an ij - semi (δ, θ)-closed set and $x \notin F$. Then $X \backslash F = U$ is an ij - semi (δ, θ)-open set containing x. Then by (g), there exists a ji - semi (δ, θ)-closed set

H such that $x \in H \subseteq U$ and hence $ji - \delta scl_\theta(\{x\}) \subseteq U$. Thus $ji - \delta scl_\theta(\{x\}) \cap F = \phi$.
(h)$\Longrightarrow$(a) Let U be an ij - semi (δ, θ)-open set and $x \in U$. Then $x \notin X \backslash U$ which is ij - semi (δ, θ)-closed set and by (h), $ji - \delta scl_\theta(\{x\}) \cap X \backslash U \subseteq U$. Thus $ji - \delta scl_\theta(\{x\}) \subseteq U$. Hence (X, τ_1, τ_2) is pairwise semi $(\delta, \theta) - R_0$ space.

Definition 3.18 : In a bitopological space (X, τ_1, τ_2) for any x,
(a) $P - \delta scl_\theta(\{x\}) = 12 - \delta scl_\theta(\{x\}) \cap 21 - \delta scl_\theta(\{x\})$.
(b) $\{x\}_\delta^{\theta s \wedge_P} = \{x\}_\delta^{\theta s \wedge_{12}} \cap \{x\}_\delta^{\delta s \wedge_{21}}$.

Theorem 3.19 : In a pairwise semi $(\delta, \theta) - R_0$ space, for any x and y we have either $P - \delta scl_\theta(\{x\}) = P - \delta scl_\theta(\{y\})$ or $P - \delta scl_\theta(\{x\}) \cap P - \delta scl_\theta(\{y\}) = \phi$. **Proof** : Suppose that $P - \delta scl_\theta(\{x\}) \cap P - \delta scl_\theta(\{y\}) \neq \phi$ and let $z \in [[12 - \delta scl_\theta(\{x\}) \cap 21 - \delta scl_\theta(\{x\})] \cap [12 - \delta scl_\theta(\{y\}) \cap 21 - \delta scl_\theta(\{y\})]], then 12 - \delta scl_\theta(\{z\}) \subseteq [12 - \delta scl_\theta(\{x\}) \cap 12 - \delta scl_\theta(\{y\})] and 21 - \delta scl_\theta(\{z\}) \subseteq [21 - \delta scl_\theta(\{x\}) \cap 21 - \delta scl_\theta(\{y\})]$. Also since $z \in 12 - \delta scl_\theta(\{x\})$, then $21 - \delta scl_\theta(\{x\}) \subseteq 21 - \delta scl_\theta(\{y\})$. This is so because by theorem 3.17 (d), if $z \in 12 - \delta scl_\theta(\{x\})$, then $x \in 21 - \delta scl_\theta(\{z\})$. This implies $21 - \delta scl_\theta(\{x\}) \subseteq 21 - \delta scl_\theta(\{z\}) \subseteq 21 - \delta scl_\theta(\{y\})$. Similarly, $z \in 21 - \delta scl_\theta(\{x\})$ implies $12 - \delta scl_\theta(\{x\}) \subseteq 12 - \delta scl_\theta(\{y\})$ and $z \in 12 - \delta scl_\theta(\{y\})$ implies $21 - \delta scl_\theta(\{x\}) \subseteq 21 - \delta scl_\theta(\{y\})$ and $z \in 21 - \delta scl_\theta(\{y\})$ implies $12 - \delta scl_\theta(\{y\}) \subseteq 12 - \delta scl_\theta(\{x\})$. Therefore, $12 - \delta scl_\theta(\{x\}) \cap 21 - \delta scl_\theta(\{x\}) \subseteq 12 - \delta scl_\theta(\{y\}) \cap 21 - \delta scl_\theta(\{y\})$ and $12 - \delta scl_\theta(\{y\}) \cap 21 - \delta scl_\theta(\{y\}) \subseteq 12 - \delta scl_\theta(\{x\}) \cap 21 - \delta scl_\theta(\{x\})$. Hence $12 - \delta scl_\theta(\{y\}) \subseteq 21 - \delta scl_\theta(\{y\}) = 12 - \delta scl_\theta(\{x\}) \cap 21 - \delta scl_\theta(\{x\})$.

Theorem 3.20 : In a pairwise semi $(\delta, \theta) - R_0$ space, for any x and y we have either $\{x\}_\delta^{\theta s \wedge_P} = \{y\}_\delta^{\theta s \wedge_P}$ or $\{x\}_\delta^{\delta s \wedge_P} \cap \{y\}_\delta^{\theta s \wedge_P} \neq \phi$.
Proof : Obvious. Similar to theorem 3.19.

4. Pairwise (δ, θ)-Semi Generalized Closed Sets

Definition 4.1 : A subset A of a bitopological space (X, τ_1, τ_2) is called $ij - (\delta, \theta)$ semi generalized closed sets (briefly $ij - (\delta, \theta)$-sg closed sets) if $ji - \delta scl_\theta(A) \subseteq U$ wherever $A \subseteq U$ and U is $ij - \delta$ semi open in (X, τ_1, τ_2). The complement of an $ij - (\delta, \theta)$ semi generalized closed set is called $ij - (\delta, \theta)$ semi generalized open (briefly $ij - (\delta, \theta)$-sg open).

If $A \subset X$ is $12 - (\delta, \theta)$-sg closed and $21 - (\delta, \theta)$-sg closed, then A is called pairwise (δ, θ)-sg closed.

Example 4.2 : Let (X, τ_1, τ_2) be bitopological space with $X = \{a, b, c, d, e\}$ and $\tau_1 = \{\phi, X, \{a\}, \{b\}, \{e\}, \{a, b\}, \{a, e\}, \{b, e\}, \{a, b, e\}\}$, $\tau_2 = \{\phi, X, \{c\}, \{d\}, \{e\}, \{c, d\}, \{c, e\}, \{d, e\}, \{c, d, e\}\}$. Then $12-(\delta,\theta)$-sg closed sets are $\phi, X, \{c, d\}, \{c, d, e\}$.

Theorem 4.3 : Every ji - semi (δ,θ)-closed set is $ij-(\delta,\theta)$-sg closed.

Proof : Let $A \subset X$ be a ji - semi (δ,θ)-closed set. Then $ji-\delta scl_\theta(A) = A$. Let $A \subseteq U$ where U is an $ij-\delta$ semi open in (X, τ_1, τ_2). It follows that $ji-\delta scl_\theta(A) \subseteq U$. Therefore A is $ij-(\delta,\theta)$-sg closed.

Theorem 4.4 Every $ij-(\delta,\theta)$-sg closed set is ij - sg closed.

Proof : Let A is $ij-(\delta,\theta)$-sg closed. Since $ji-\delta scl(A) \subseteq ji-scl(A) \subseteq ji-\delta scl_\theta(A)$ for every subset A of X, then A is ij - sg closed.

The following example shows that the converse of above theorem needs not true in general.

Example 4.5 : Let (X, τ_1, τ_2) be bitopological space with $X = \{a, b, c, d, e\}$ and $\tau_1 = \{\phi, X, \{a\}, \{b\}, \{e\}, \{a, b\}, \{a, e\}, \{b, e\}, \{a, b, e\}\}$, $\tau_2 = \{\phi, X, \{c\}, \{d\}, \{e\}, \{c, d\}, \{c, e\}, \{d, e\}, \{c, d, e\}\}$. Then $\{a, b\}$ is 12 - sg closed but not $12-(\delta,\theta)$-sg closed.

Remark 4.6 : From above discussion, we conclude the following diagram:

$ji-\delta$ Closed

↓

$ji-\delta$ Semi closed

↓

ij - Semi (δ,θ)-closed

↓

$ij-(\delta,\theta)$-sg Closed

↓

ij - sg Closed

Theorem 4.7 : A subset A of a bitopological space (X, τ_1, τ_2) is $ij-(\delta,\theta)$-sg open if and only if $U \subseteq ji-\delta sint_\theta(A)$ whenever U is $ij-\delta$ semi closed in X and $U \subseteq A$.

Proof : Let A be $ij-(\delta,\theta)$-sg open and $U \subseteq A$ where U is $ij-\delta$ semi closed. Then $X\backslash A \subseteq X\backslash U$. This implies that $ji-\delta scl_\theta(X\backslash A) \subseteq X\backslash U$. Then by theorem 2.2, $ji-\delta scl_\theta(X\backslash A) = X\backslash ji-\delta sint_\theta(A) \subseteq X\backslash U$. Thus $U \subseteq ji-\delta sint_\theta(A)$.

Conversely, if U is an $ij-\delta$ semi closed set with $U \subseteq ji-\delta sint_\theta(A)$ whenever $U \subseteq A$ then it follows that $X\backslash A \subseteq X\backslash U$ and $X\backslash ji-\delta sint_\theta(A) \subseteq X\backslash U$. Thus $ji-\delta scl_\theta(X\backslash A) \subseteq X\backslash U$. Therefore $X\backslash A$ is $ij-(\delta,\theta)$-sg closed and therefore A is $ij-(\delta,\theta)$-sg open.

Theorem 4.8 : Let A be an $ij-(\delta,\theta)$-sg closed subset of (X,τ_1,τ_2). Then

(a) $ji-\delta sint_\theta(A)\backslash A$ does not contain a nonempty $ij-\delta$ semi closed set.

(b) $ji-\delta sint_\theta(A)\backslash A$ is $ij-(\delta,\theta)$-sg open.

Proof : (a) Let U be an $ij-\delta$ semi closed set such that $U \subseteq ji-\delta scl_\theta(A)\backslash A$. Since $X\backslash U$ is $ij-\delta$ semi open and $A \subseteq X\backslash U$, it follows that $ji-\delta scl_\theta(A) \subseteq X\backslash U$ and $U \subseteq X\backslash ji-\delta scl_\theta(A)$. This implies that $U \subseteq X\backslash ji-\delta scl_\theta(A) \cap ji-\delta scl_\theta(A) = \phi$. Hence $U = \phi$.

(b) If A is $ij-(\delta,\theta)$-sg closed and U is an $ij-\delta$ semi closed set such that $U \subseteq ji-\delta scl_\theta(A)\backslash A$ then by (a), U is empty and therefore $U \subseteq ji-\delta sint_\theta[ji-\delta sint_\theta(A)\backslash A]$. By above theorem, $ji-\delta scl_\theta(A)\backslash A$ is $ij-(\delta,\theta)$-sg open.

Theorem 4.9 : If A is an $ij-(\delta,\theta)$-sg closed set of a space (X,τ_1,τ_2) such that $A \subseteq B \subseteq ji-\delta scl_\theta(A)$ then B is also an $ij-(\delta,\theta)$-sg closed set.

Proof : Let U be an $ij-\delta$ semi open set of (X,τ_1,τ_2) such that $B \subseteq U$. Then $A \subseteq U$. Since A is $ij-(\delta,\theta)$-sg closed, then $ji-\delta scl_\theta(A) \subseteq U$. Also since $ij-\delta scl_\theta(A) = \bigcap\{ij-\delta scl(V) : A \subseteq V \in ji-\delta SO(X)\}$. Hence $ji-\delta scl_\theta(B) \subseteq ji-\delta scl_\theta[ji-\delta scl_\theta(A)] = ji-\delta scl_\theta(A) \subseteq U$. Therefore B is also an $ij-(\delta,\theta)$-sg closed set of (X,τ_1,τ_2).

Theorem 4.10 : A bitopological space (X,τ_1,τ_2) is $ij-\delta s-T_1$ space if and only if every $ij-(\delta,\theta)$-sg closed set is ij - semi (δ,θ)-closed.

Proof : Let $A \subset X$ be $ij-(\delta,\theta)$-sg closed and $x \in ji-\delta scl_\theta(A)$. Since X is $ij-\delta s-T_1$ space, $\{x\}$ is $ij-\delta$ semi closed by definition 1.3. Thus $x \notin ji-\delta scl_\theta(A)\backslash A$ by theorem 4.9. Since $x \in ji-\delta scl_\theta(A)$, then $x \in A$. Hence $ji-\delta scl_\theta(A) \subseteq A$ and A is ij - semi (δ,θ)-closed.

Conversely, let $x \in X$. Assume that $\{x\}$ is not $ij-\delta$ semi closed. Then $X\backslash\{x\}$ is not $ij-\delta$ semi open. Since the only $ij-\delta$ semi open superset of $X\backslash\{x\}$ is X, then $X\backslash\{x\}$ is $ij-(\delta,\theta)$-sg closed. By hypothesis $X\backslash\{x\}$ is ij - semi (δ,θ)-closed and thus $\{x\}$ is ji - semi (δ,θ)-open. By theorem 2.8, $\{x\}$ is $ij-\delta$ semi regular. Thus $\{x\}$ is $ij-\delta$ semi closed. Hence by definition 1.4, (X,τ_1,τ_2) is $ij-\delta s-T_1$ space.

5. $ij-(\delta,\theta)$-sg Continuous and $ij-(\delta,\theta)$-sg Irresolute Functions

Definition 5.1 : A function $f:(X,\tau_1,\tau_2)\longrightarrow(Y,\sigma_1,\sigma_2)$ is called an $ij-(\delta,\theta)$ semi generalized continuous (briefly $ij-(\delta,\theta)$-sg continuous) if $f^{-1}(V)$ is $ij-(\delta,\theta)$-sg closed in X for every $ji-\delta$ semi closed V of Y.

Theorem 5.2 : If a function $f:(X,\tau_1,\tau_2)\longrightarrow(Y,\sigma_1,\sigma_2)$ is $ij-(\delta,\theta)$-sg continuous, then f is ij-sg continuous.

Proof : Let V be σ_j - closed set of Y. Since f is $ij-(\delta,\theta)$-sg continuous, then $f^{-1}(V)$ is $ij-(\delta,\theta)$-sg closed. Since every $ij-(\delta,\theta)$-sg closed set is ij - sg closed, then $f^{-1}(V)$ is ij-sg closed set of X and hence f is ij-sg continuous.

Definition 5.3 : A function $f:(X,\tau_1,\tau_2)\longrightarrow(Y,\sigma_1,\sigma_2)$ is called an $ij-(\delta,\theta)$ semi generalized irresolute (briefly $ij-(\delta,\theta)$-sg irresolute) if $f^{-1}(V)$ is $ij-(\delta,\theta)$-sg closed in X for every $ij-(\delta,\theta)$-sg closed set V of Y.

Theorem 5.4 : If $f:(X,\tau_1,\tau_2)\longrightarrow(Y,\sigma_1,\sigma_2)$ is an $ij-(\delta,\theta)$-sg irresolute and $g:(Y,\sigma_1,\sigma_2)\longrightarrow(Z,\rho_1,\rho_2)$ is $ij-(\delta,\theta)$-sg continuous, then gof is $ij-(\delta,\theta)$-sg continuous.

Proof. Let W be a $ji-\delta$ semi closed set of Z. Since g is an $ij-(\delta,\theta)$-sg continuous, $g^{-1}(W)$ is $ij-(\delta,\theta)$-sg closed set of Y. Since f is $ij-(\delta,\theta)$-sg irresolute, $(gof)^{-1}(W)=f^{-1}[g^{-1}(W)]$ is an $ij-(\delta,\theta)$-sg closed set of X. Hence gof is an $ij-(\delta,\theta)$-sg continuous.

Theorem 5.5 : If $f:(X,\tau_1,\tau_2)\longrightarrow(Y,\sigma_1,\sigma_2)$ and $g:(Y,\sigma_1,\sigma_2)\longrightarrow(Z,\rho_1,\rho_2)$ are $ij-(\delta,\theta)$-sg irresolute functions, then gof is $ij-(\delta,\theta)$-sg irresolute.

Proof : Let W be a $ij-(\delta,\theta)$-sg closed set of Z. Since g is an $ij-(\delta,\theta)$-sg irresolute, $g^{-1}(W)$ is $ij-(\delta,\theta)$-sg closed set of Y. Since f is $ij-(\delta,\theta)$-sg irresolute, $(gof)^{-1}(W)=f^{-1}[g^{-1}(W)]$ is an $ij-(\delta,\theta)$-sg closed set of X. Hence gof is an $ij-(\delta,\theta)$-sg irresolute.

Definition 5.6 : A function $f:(X,\tau_1,\tau_2)\longrightarrow(Y,\sigma_1,\sigma_2)$ is said to be $ij-(\delta,\theta)$ semi generalized closed (briefly $ij-(\delta,\theta)$-sg closed) if $f(U)$ is an $ij-(\delta,\theta)$-sg closed set of Y for every $ji-\delta$ semi closed set U of X.

Definition 5.7 : A function $f:(X,\tau_1,\tau_2)\longrightarrow(Y,\sigma_1,\sigma_2)$ is said to be ij - pre-semi (δ,θ)-open (resp. ij - pre-semi (δ,θ)-closed) if $f(U)$ is an ij - semi (δ,θ)-open (resp. ij - semi (δ,θ)-closed) set of Y for every ij - semi (δ,θ)-open (resp. ij - semi (δ,θ)-closed) set U of X.

Theorem 5.8 : If a function $f:(X,\tau_1,\tau_2)\longrightarrow(Y,\sigma_1,\sigma_2)$ is a bijective ij - pre-semi (δ,θ)-open $ij-(\delta,\theta)$-sg continuous, then f is $ij-(\delta,\theta)$-sg irresolte.

Proof : Let V be an $ij-(\delta,\theta)$-sg closed set of Y. Assume that $f^{-1}(V)\subseteq U$, where U

is an $ij-\delta$ semi open in X. Then $V \subseteq f(U)$. Since $f(U) \in ij-\delta SO(X)$ and V is an $ij-(\delta,\theta)$ closed set, we have $ji-\delta scl_\theta(V) \subseteq f(U)$ and therefore $f^{-1}[ji-\delta scl_\theta(V)] \subseteq U$. Since f is $ij-(\delta,\theta)$ continuous and $ji-\delta scl_\theta(V)$ is $ji-\delta$ semi closed, it follows that $ji-\delta scl_\theta[ji-\delta scl_\theta(V)] \subseteq U$ and thus $ji-\delta scl_\theta[f^{-1}(V)] \subseteq U$. This means that $f^{-1}(V)$ is $ij-(\delta,\theta)$ closed in X. Therefore f is $ij-(\delta,\theta)$ irresolute.

Theorem 5.9 : If a function $f:(X,\tau_1,\tau_2) \longrightarrow (Y,\sigma_1,\sigma_2))$ is $ij-\delta s$ irresolute and $ij-(\delta,\theta)$ closed, then $f(U)$ is $ij-(\delta,\theta)$ closed in Y for an $ij-(\delta,\theta)$ closed set U of X.
Proof : Let U be $ij-(\delta,\theta)$ closed in X. Assume that $f(U) \subseteq V$, where V is an $ij-\delta$ semi open set in Y. Thus $U \subseteq f^{-1}(V)$. Since f is $ij-\delta s$ irresolute, then $f^{-1}(V)$ is $ij-\delta$ semi open and U is $ij-(\delta,\theta)$ closed, hence $ji-\delta scl_\theta(U) \subseteq V$. Therefore $f[ji-\delta scl_\theta(U)] \subseteq V$. Thus $ji-\delta scl_\theta[f[ji-\delta scl_\theta(U)]] \subseteq V$, since f is $ij-(\delta,\theta)$ closed and $ji-\delta scl_\theta(U)$ is $ij-\delta$ semi closed. Hence $ji-\delta scl_\theta[f(U)] \subseteq V$. Thus $f(U)$ is $ij-(\delta,\theta)$ closed in Y.

Theorem 5.10 : For a function $f:(X,\tau_1,\tau_2) \longrightarrow (Y,\sigma_1,\sigma_2))$, the following are equivalent: (a) f is ij- pre-semi (δ,θ)-closed.
(b) For each $U \subset X$, $ij-\delta scl_\theta[f(U)] \subseteq f[ij-\delta scl_\theta(U)]$.
(c) If $f^{-1}(V) \subseteq U$, where $V \subset Y$ and U is ij- semi (δ,θ)-open in X, then there exists an ij- semi (δ,θ)-open set $W \subset Y$ such that $V \subset W$ and $f^{-1}(W) \subset U$.
(d) If $f^{-1}(y) \subseteq U$ where $y \in Y$ and U is ij - semi (δ,θ)-open in X, then there exists an ij - semi (δ,θ)-open set $W \subset Y$ such that $y \in W$ and $f^{-1}(W) \subseteq U$.
Proof : (a)$\Longrightarrow$(b) Let $U \subset X$. Then $ij-\delta scl_\theta(U)$ is ij - semi (δ,θ)-closed. Since f is ij - pre-semi (δ,θ)-closed, then $f[ij-\delta scl_\theta(U)]$ is ij - semi (δ,θ)-closed. Moreover, $f(U) \subseteq f[ij-\delta scl_\theta(U)]$ and $ij-\delta scl_\theta[f(U)] \subseteq ij-\delta scl_\theta[f[ij-\delta scl_\theta(U)]]$. Thus $ij-\delta scl_\theta[f(U)] \subseteq f[ij-\delta scl_\theta(U)]$.
(b)$\Longrightarrow$(a) Let U be an ij - semi (δ,θ)-closed set of X. By (b), $ij-\delta scl_\theta[f(U)] \subseteq f[ij-\delta scl_\theta(U)] = f(U)$. Hence f is ij - pre-semi (δ,θ)-closed.
(a)$\Longrightarrow$(c) Let $U \subseteq X$, $W = Y\backslash f(X\backslash U)$. Since $f^{-1}(V) \subseteq U$, where U is an ij - semi (δ,θ)-open set of X and $V \subseteq W$, then $X\backslash U$ is an ij - semi (δ,θ)-closed. By (a), $f(X\backslash U)$ is ij - semi (δ,θ)-closed set in Y. Hence W is ij - semi (δ,θ)-open in Y and $f^{-1}(W) = X\backslash f^{-1}[f(X\backslash U)] \subseteq U$.
(c)$\Longrightarrow$(d) Obvious.
(d)$\Longrightarrow$(a) Let F be an ij - semi (δ,θ)-closed set of X. Suppose that $y \in Y\backslash f(F)$. Then

$f^{-1}(y) \subseteq X \backslash F$, where $X \backslash F$ is ij - semi (δ, θ)-open set of X. By (d), there exists an ij - semi (δ, θ)-open set $W \subset Y$ such that $f^{-1}(W) \subseteq X \backslash F$. Thus $F \subseteq f^{-1}(Y \backslash W)$, this implies $f(F) \subseteq Y \backslash W$. Hence $y \in W \subseteq Y \backslash f(F)$ and $Y \backslash f(F)$ is ij - semi (δ, θ)open set of Y. It follows that $f(F)$ is ij - semi (δ, θ)-closed set in Y and hence f is an ij-pre-semi (δ, θ)-closed.

Theorem 5.11 : If a function $f : (X, \tau_1, \tau_2) \longrightarrow (Y, \sigma_1, \sigma_2))$ is $ij - \delta s$ irresolute and ji - pre-semi (δ, θ)-closed, then for every $ij - (\delta, \theta)$-sg closed set F of X, $f(F)$ is an $ij - (\delta, \theta)$-sg closed set of Y.

Proof. Suppose that F is an $ij - (\delta, \theta)$-sg closed set of X. Assume $f(F) \subseteq U$, where $U \in ij - \delta SO(Y)$. Now $F \subseteq f^{-1}(U)$ and since f is $ij - \delta s$ irresolute $f^{-1}(U) \in ij - \delta SO(X)$. But F is an $ij - (\delta, \theta)$-sg closed. Therefore $ij - \delta scl_\theta(F) \subseteq f^{-1}(U)$. Thus $f[ij - \delta scl_\theta(F)] \subseteq U$. Now we have $ij - \delta scl_\theta[f(F)] \subseteq f[ij - \delta scl_\theta(F)] \subseteq U$. This shows that $f(F)$ is an $ij - (\delta, \theta)$-sg closed set of Y.

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