International J. of Pure & Engg. Mathematics (IJPEM)
ISSN 2348-3881, Vol. 4 No. I (April, 2016), pp. 1-13

TWO RING SHAPED CRACKS ARE OPENED BY A WEDGE IN AN INFINITE SOLID

ANJANA SINGH
Department of Mathematics,
Govt. Girls (PG) College, Rewa (M.P.), India
E-mail: dranjanasingh@yahoo.in

Abstract

The closed form expressions of stress-intensity factors and of crackshape are obtained by using Fourier transform method for two similar ring shaped cracks in an isotropic infinite solidand cracks are opened by wedge.

1. Introduction

Wedge is a hard particle or body in the medium. Iron rod are hard in comparison to matrix made of concrete and cement. Iron bar, in circular shape opens two ring shaped cracks. Crack faces are stress-free.

These are used by civil engineers in constructing the pillars for bridges etc. he continuous use of bridges, the iron frame which is circular in shape leaves the matrix. This causes discontinuity in the medium. The discontinuities are in ring shape. The height and

---

Key Words : *Stress Intensity Factors (S.I.F.), Crack Opening Displacement (C.O.D.), Fourier Transform (F.T.), Modified Bessels Function (MBF), Wedge.*

radius of the pillar are large in comparison to radius or width of ring shaped discontinuity. Therefore it is considered as infinite three dimensional isotropic solid with ring shaped discontinuities whose axes coincide with z-axis.

Linear fracture mechanics has established itself as highly satisfory working tool in studying the phenomenon of brittle fracture and crack propagation in solid structures. The technique appear to be more effective when plane-strain conditions prevail. The crack problems in shell's type solid structure or crack discontinuity in shell shape solids poses limitation.

Two major limitations arise from geometry and material behavior. The geometrical factors include the relative size of the crack with respect to radius of curvature of shellsand orientation of crack. So far as material properties are concerned we take up isotropic homogeneous solid having shell-type discontinuity.

Estekanchi et. al. [1] and Ferreira et. al. [2] discussed buckling of composite shells by using numerical methods. The research work in [3-12] discussed about buckling of cylindrical shells under different loading conditions and different material properties with or without cracks.

Erdogan and Ratwani [13] calculated the stresses causing fatigue and fracture of isotropic cylindrical shell containing circumferential crack by using numerical method. Erdogan [14] extended above method to orthotropic cylindrical shell having axial crack.

Ma et. al [15] obtained stress-intensity factors for axial cracks in hollow isotropic cylindrical shell by using finite-element technique.Liu et. al [16] analysed the crack closure effect on stress-intensity factors for circumferentially cracked cylindrical shell. Lal et.al [17] and Lal [18] has discussed thermo-elastic problem with penny-shaped crack reducing the problem to Abel integral equation.Jaunky et.al [19] discussed the mechanical response of laminated composite cylindrical panel in axial compression by using shell theories.

The problem in present research endeavour is of two ring shaped cracks having axis parallel to z-axis and opened by an interior wedge. The infinite 3-D isotropic body is now considered as cylinder of infinite radius and axis as z-axis. The ring shaped cracks occupy the space $r = d, b < |z| < c, 0 \leq \theta \leq 2\pi$ (see figure-1).

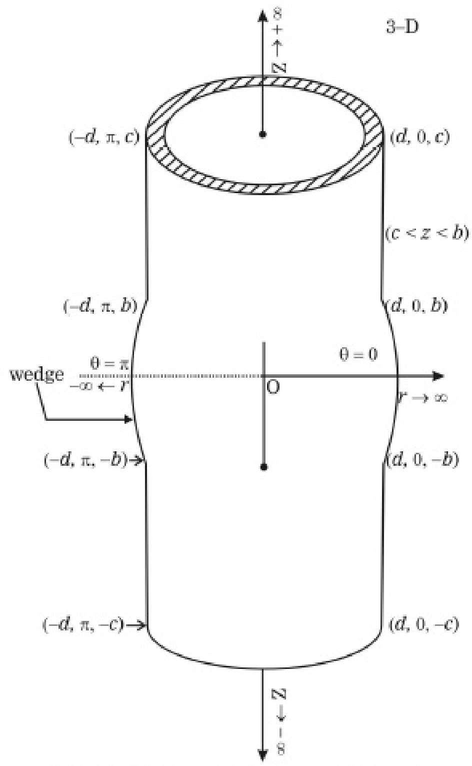

FIGURE - 1. Two Cylinderical Shell Type Discontinuities opened by wedge.

The cracks are formed by an wedge and crack's faces are stress-free. The medium is such that the cross-sections obtained by any $\theta = \alpha$ are same. It reduces the 3-dimensional problem to 2-dimensional ie. r and z only two variables. We take cross-section by $\theta = 0$ and $\theta = \pi$, (see figure 2a). It is being assumed that $\sigma_{\theta\theta} = 0$ and the operator $\frac{\partial}{\partial\theta}$ is null operator. Thus the co-ordinates of any point will be r and z when cylindrical co-ordinate system is taken.

Thus the physical problem is reduced to the following mixed-boundary value problem.

$$\sigma_{rr}(d,z) = 0, \quad b < |z| < c, \quad \sigma_{rz}(d,z) = 0, \quad 0 \leq |z| < \infty \qquad (1.1)-(1.2)$$

$$u_r(d,z) = \begin{cases} u(z) & 0 \leq |z| \leq b \\ \\ 0, & c \leq |z| < \infty \end{cases} \qquad (1.3)$$

and all physical quantities, i.e., the components of stress and of displacement are zero as $r, z \Rightarrow \infty$. $u(z)$ is wedge shape function. We checked throughout that

$$u_r(d,z) > 0, \quad b < |z| < c \qquad (1.4)$$

which means that cracks really open out and the faces of crack do not meet each other, other than at crack tips, see Burniston [20]. The symmetry of geometry and of loading reduce the boundary and mixed-boundary conditions (1)-(3) to, (see figure 2b).

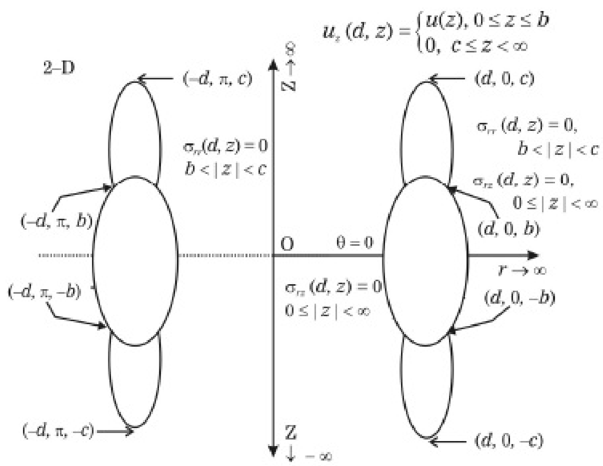

FIGURE - 2a. Four Griffith Cracks in 2-D.

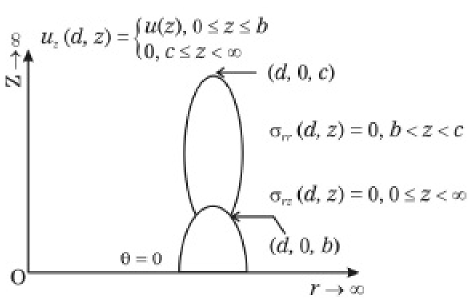

FIGURE - 2b. A Griffith crack in Ist quadrant opened by wedge.

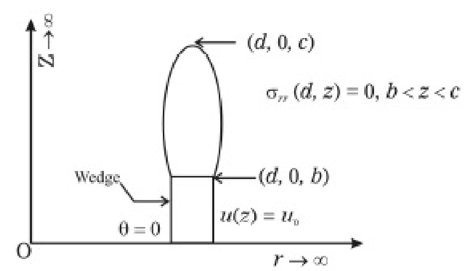

FIGURE - 3. A Crack Opened by rectangular wedge Crack Faces are stress free.

$$\sigma_{rr}(d,z) = 0, \quad b < z < c, \quad \sigma_{rz}(d,z) = 0, \quad 0 \le z < \infty \qquad (1.5)-(1.6)$$

$$u_r(d,z) = \begin{cases} u(z) & 0 \le z \le b \\ \\ 0, & c \le z < \infty \end{cases} \qquad (1.7)$$

The plan of the paper is as follows : Section 1 introduces the problem and reduces to mixed-boundary value problem. Section 2 formulates the mixed-boundary value problem and reduces to triple integral equation. Section 3 solves the triple integral equation and reduces to Fredholm integral equation of second kind. Section 4 solves the Fredholm integral equation. Physical quantities are given in Section 5. This section takes one special case of wedge shape.

2. Formulation and Reduction to Triple Integral Equation

The equations of equilibrium, after using stress-strain relations are reduced to fourth order partial differential equation in u_r as :

$$\Delta^2(\Delta^2 u,(r,z)) = 0, \quad \Delta^2 = \frac{1}{r}\frac{\partial}{\partial r} r\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}. \tag{2.1}$$

And the other displacement component u_z is related with u_r in the following manner

$$u_z(r,z) = \frac{1}{P}\left[(\lambda+2\mu)\int\left\langle\frac{u_r}{r}+\frac{\partial u_r}{\partial r}\right\rangle dz + \int\frac{\partial u_r}{dz}dr\right] \tag{2.2}$$

where λ and μ are Lame's constants. We assume the solution of (2.1) as

$$u_r(r,z) = \int_0^\infty \cos(sz)[A(s)I_1(sr) + rB(s)I_0(sr)]ds. \tag{2.3}$$

Then

$$u_z(r,z) = -\frac{1}{P}\int_0^\infty \sin(sz)\left[QA(s)I_0(sr) + B(s)\left\{(2+rs)I_1(sr) + \frac{(2-rs)}{rs}I_0(sr)\right\}\right]ds \tag{2.4}$$

where $P = \lambda + \mu$ $Q = 1 + \mu + P$ and $I_0(sr), I_1(sr)$ are modified Bessel's functions of kind first with order zero and one. The use of stress-strain relations and (2.3)-(2.4) we get

$$\begin{aligned}\sigma_{rr}(r,z) &= \int_0^\infty s\cos(sz)\left[-A(s)\left\{I_0(rs)\alpha_0 + \frac{I_1(sr)}{sr}\right\}\right.\\ &\qquad \left. + B(s)\{\alpha_1 I_0(sr) + (sr(1+Q)+1)I_1(sr)\}\right]ds\end{aligned} \tag{2.5}$$

$$\begin{aligned}\sigma_{rz}(r,z) &= -\frac{\mu}{P}\int_0^\infty s\sin(sz)\left[(P+Q)A(s)I_1(sr)\right.\\ &\qquad \left. + \frac{B(s)}{s}\{I_0(rs)(2r^2sp + r^2s^2) + (rs+2)I_1(sr)\}\right]ds\end{aligned} \tag{2.6}$$

where $A(s)$ and $B(s)$ are two arbitrary constants to be determined. The quantities in (2.3)-(2.6) vanishes $(z,r)\to\infty$. The boundary condition (1.6), with (2.6), gives

$$b_1 A(s) = -\frac{b_2 B(s)}{2} \tag{2.7}$$

with

$$b_1 = I_0(ds)\left[2d^2 + \frac{d}{s}\right] + \frac{s+2}{s}I_1(ds), \quad b_2 = (p+Q)I_1(ds). \tag{2.8}$$

Now, the substitution of u_r and σ_{rr} from (2.3) and (2.5), respectively, and using (2.7) -(2.8), in boundary conditions (1.7) and (1.5), respectively, give

$$\int_0^\infty \psi(s)\cos(sz)dx = \begin{cases} u(z), & 0 \le z \le b \\ 0, & c \le z < \infty \end{cases} \tag{2.9}$$

$$\int_0^\infty \psi(s)\cos(sz)ds = -P_1(z), \quad b < z < c \tag{2.10}$$

$$b_1\psi(s) = B(s)[db_1 I_0(ds) - b_2 I_1(sd)] \tag{2.11}$$

$$P_1(z) = \int_0^\infty s\cos(sz)\psi(s)M(ds)ds \tag{2.12}$$

$$M(sd) = (b_2 b_4 - b_1 b_3 - b_5)/b_3, \; b_3 = (3-Q)I_0(sd) + I_1(sd)[1 + s(1+Q)] \tag{2.13}$$

$$b_4 = I_0(sd)\left[\frac{Q-P}{P} + \frac{I_1(sd)}{s}\right], \quad b_5 = b_1 d I_0(sd) - b_2 I_1(sd). \tag{2.14}$$

Thus the physical problem is reduced to triple integral equation given by (2.9)-(2.10).

3. Solution of Triple Integral Equation and Expansion of Some Function

Solution of Triple Integral Equation

The solution of triple integral equation (2.9) - (2.10) is obtained through the method of Srivastava and Lowengrub [21]. The solution is assumed as,

$$s\pi\psi(s) = 2\int_b^c g(t)\frac{\sin st}{}dt - 2\int_0^b u'(t)\frac{\sin(st)}{}dt. \tag{3.1}$$

Then the use of (3.1) in (2.9) along with the following integral

$$\int_0^\infty \frac{\sin st\cos st}{t}dt = \begin{cases} \pi/2, & s > x \\ \pi/4, & s = x \\ 0, & s < x \end{cases}$$

will satisfy (2.9), if

$$\int_b^c g(t)dt = u(b). \tag{3.2}$$

Then using (3.1) in (2.10) and the value of integral

$$\int_0^\infty \frac{\sin st \sin xt}{s} ds = \frac{1}{2} \log \left| \frac{t+x}{t-x} \right|$$

will give

$$g(t) = -\frac{2}{\pi^2} \frac{1}{\psi(t)} \left[\Delta_0(t) + \left\langle \int_b^c g(\alpha) - \int_0^b u'(\alpha) \right\rangle M_1(\alpha, t) dx \right], \ b < t < c \tag{3.3}$$

$$M_1(\alpha, t) = \int_b^c \frac{\psi(z)}{z^2 - r^2} K_1(\alpha, z) dz, \quad \Delta_0(t) = \int_b^c \frac{z\psi(z)}{z^2 - r^2} p_2(z) dz + D \qquad (3.4) - (3.5)$$

$$K_1(\alpha, z) = \int_0^\infty M(sd) \cos(sz) \sin(s\alpha) ds, \quad p_2(z) = \int_0^b \frac{yu'(y)dy}{y^2 - z^2} \tag{3.6}$$

$$\psi(t) = \left\{ \left| (t^2 - b^2)(c^2 - t^2) \right| \right\}^{1/2} \tag{3.7}$$

$M(sd)$ is defined in (2.13). D is an arbitrary constant to be determined through (3.2). (’) over function represents differentiation with respect to argument. (3.3) is Fredholm integral equation of second kind.

Expansion of Some Functions

We make use of expansion of modified Bessel’s function $I_\nu(z)$ of order ν. In this case $\nu = 0$ and $\nu = 1$.

$$I_\nu(z) = e^{-z} \sum_{m=0}^{d-i} (\nu, m)(-1)^m \left(\frac{z}{2}\right)^m, \quad (\nu, m) = \left|\overline{v + m + \frac{1}{2}}\right. /m! \left|\overline{\nu + \frac{m}{2}}\right. \tag{3.8}$$

see [22]. It is real part of $I_\nu(z)$. To get the approximate expansion of $M(sd)$ the following is needed

$$b_2 b_4 - b_1 b_3 = e^{-sd} \sum_{m=0}^{n-1} \sum_{r=0}^{n-1} (-1)^{m+r} \left(\frac{sd}{2}\right)^{m+r} e_1(m, r, d)$$

with

$$
\begin{aligned}
e_1(m,r,s) &= \left|\overline{m+\frac{1}{2}}\right.\left|\overline{r+\frac{1}{2}e_{11}}\right.(m,r,s)\\
e_{11}(m,r,s) &= \left(r+\frac{1}{2}\right)d_4 + d_5\left(r+\frac{1}{2}\right)\left(m+\frac{1}{2}\right)d_6\\
b_5 &= \sum_{r=0}^{n-1}\sum_{m=0}^{n-1}(-1)^{m+r}\left(\frac{sd}{2}\right)^{m+r}\left[d_7\left|\overline{m+\frac{1}{2}}\right.\left|\overline{r+\frac{3}{2}}\right. + d_6\left|\overline{m+\frac{3}{2}}\right.\left|\overline{r+\frac{3}{2}}\right.\right]\\
b_5^{-1} &= \frac{\pi}{4}\sum_{c=0}^{\infty}(2d_7-d_8)^{-1}\left[\frac{sd}{4}\left(\frac{2d_7-3d_8}{2d_7-d_8}\right)^e\right]\\
M(sd) &= \pi\sum_{R=0}^{\infty}\sum_{l=0}^{\infty}\sum_{p=0}^{l}\sum_{m=0}^{n-1}\sum_{r=0}^{n-1}(-1)^{m+r+p+l}e_2(m,r,d)\\
&\quad\left(\frac{\alpha d}{2}\right)^{m+r+l-2p-2k}\ {}^{l}C_p\begin{pmatrix}p\\k\end{pmatrix}(d_1 s-2)^k \qquad (3.9)
\end{aligned}
$$

where $d_1 \sim d_8$ alongwith other variables are given in appendix-I.

4. Solution of Fredholm Integral Equation

To solve Fredholm integral equation given in (3.3), we expand the function $g(t)$ in terms of 'd' i.e. distance of ring shaped crack from z-axis.

$$g(t) = \sum_{r=0}^{\infty} g_r(t)d^{-r}. \qquad (4.1)$$

And then substitute (4.1) in (3.3) and compare the coefficients of $\{d^{-m}\}$ from both sides. Before we proceed for above analysis we take appropriate values of k, l, p, m, r so that in the expansion of $M(sd)$ we retain upto d^{-5} only. Then from (3.9)

$$
\begin{aligned}
M(sd) &= \frac{2}{3P}\left[\frac{t_6}{d^2 s} + \frac{1}{d^4 s^2}\left\langle t_1 + \frac{2\sqrt{\pi}}{3}\right\rangle + \frac{1}{d^6 s^3}\left\langle \frac{2\sqrt{p}}{3}t_7 + \frac{4\pi}{9}t_6 - t_2\right\rangle\right]\\
&\quad\left[1 + \frac{P+Q}{2Pd} + \frac{\sqrt{\pi}}{Pd^2}\right]
\end{aligned}
$$

This $M(sd)$ gives $K_1(\alpha, z)$ from (3.6), after evaluating integrals, as

$$K_1(\alpha,t) = \frac{1}{d^2}\left[t_8 + \frac{t_9}{d} + \frac{t_{10}}{d^3}\right]T(\alpha,t) \qquad (4.2)$$

$$T(\alpha,t) = \frac{\alpha}{2}\left[\log\left|\frac{\alpha+t}{\alpha-t}\right| + t^2\log|\alpha^2-t^2| - |\alpha^2-t^2|\right],\quad b<\alpha,\ t<c \qquad (4.3)$$

where $t_i, i = 1, 2, \cdots, 10$ are given in Appendix-II. Evaluate $M_1(\alpha, t)$ from (3.4) after using (4.2) and evaluating integrals which is given as,

$$M_1(\alpha, t) = \left(\frac{t_8}{d^2} + \frac{t_9}{d^3} + \frac{t_{10}}{d^5} \right) T_1(\alpha, t) \tag{4.4}$$

$$T_1(\alpha, t) = \frac{\pi\alpha}{2}[e_1 + e_2\alpha^2 + e_3 t^2 + 2\alpha^2 t^2 - 2t^2 - 2t^4], \quad b < \alpha, \ t < c \tag{4.5}$$

$$e_1 = \frac{(b^2 - c^2)^2}{16}, \text{‘} \ e_2 = \frac{16}{2} - 2(b^2 + c^2), \quad e_3 = b^3 + c^2 - 4.$$

Now we use (4.1) in (3.3) and relevant function there in and compare coefficient of $\{d^{-m}\}, m = 0, 1, 2, 3, 4, 5$ only. Then we get,

$$\left.\begin{array}{lll} g_0(t) = \frac{2}{\pi^2}\frac{\Delta_0(t)}{\psi(t)}, & g_1(t) = 0, & g_2(t) = \frac{t_8 \Delta_1(t)}{4\pi^2 \psi(t)}, \\ g_3(t) = \frac{t_9 \Delta_1(t)}{\pi^4 \psi(t)}, & g_4(t) = \frac{4 t_8 \Delta_2(t)}{\pi^4 \psi(t)}, & g_5(t) = \frac{8}{\pi^6 \psi(t)} \Delta_3(t) \end{array}\right\} \tag{4.6}$$

where $\Delta_0(t)$ is defined in (3.5). And

$$\Delta_0(t) = \frac{\pi}{2}[a_0 - a_1(t)] + D \tag{4.7}$$

$$a_0 = \int_0^b y u'(y) dy, \ a_1(t) = \int_1^b y \frac{\psi(y) u'(y) dy}{y^2 - t^2} \tag{4.7a}$$

$$\Delta_1(t) = \frac{\pi^2}{4} \left[a_1 \langle L_1(t) k_1 + L_2(t) k_2 \rangle + \frac{\pi}{2} D \left\langle L_1(t) + L_2(t) \frac{2}{\pi} P_1 \right\rangle - L_1(t) a_{01} - L_2(t) a_{02} \right] \tag{4.8}$$

$$\left.\begin{array}{l} k_1 = \int_b^c \frac{\alpha^2 d\alpha}{\psi(\alpha)}, k_2 = \int_b^c \frac{\alpha^4 d\alpha}{\psi(\alpha)} \\ L_1(t) = e_1 + t^2(e_3 - 2) - 2t^4, L_2(t) = e_2 + 2t^2 \\ P_n = \sum\limits_{r=0}^{n} {}^nC_r (-1)^r C^{2n-2r} (c^2 - b^2)^r \frac{\left|\overline{\frac{2r+1}{2}}\right| \overline{\frac{1}{2}}}{|\overline{n^1}} \\ a_{01} = u(b) - u(0), \quad a_{02} = \int_0^b y^2 u'(y) dy \end{array}\right\} \tag{4.9}$$

$$\Delta_2(t) = t_8 M_7(t) - M_5(t), \Delta_3(t) = t_{10} \langle M_6(t) - M_5(t) \rangle t t_9 t_9 M_7(t) \tag{4.10}$$

$$M_7(t) = \int_b^c \frac{\Delta_1(\alpha}{\psi(\alpha)} \cdot T_1(\alpha, t) d\alpha, \ M_5(t) = \int_0^b u'(y) T_1(y, t) dy \tag{4.10a}$$

$$M_6(t) = \int_b^c \frac{\Delta_0(\alpha)}{\psi(\alpha)} T_1(\alpha, t) d\alpha. \tag{4.10b}$$

Thus,

$$g(t) = \frac{2\Delta_4(t)}{\pi^2 \psi(t)} \tag{4.11}$$

$$\Delta_4 = \left[\Delta_0(t) + d^{-2} t_8 \Delta_1(t) \frac{t^2 d^{-3} t_9}{\pi^2} \Delta_1(t) + \frac{2}{\pi^2} d^{-4} t_8 \Delta_2(t) + \frac{2}{\pi^4} d^{-5} \Delta_3(t)\right], b < t < c \tag{4.11a}$$

The integrals involved can easily be obtained by any numerical integration method. Thus the solution of Fredholm integral equation, given by (3.3), is obtained, which includes one unknown constant D. This is obtained with the help of (3.2) and taking $g(t)\square g_0(t)$

$$D = -\frac{u(b) + \int_b^c z\psi(z) P_2(z) dz \int_b^c \frac{dt}{\psi(t)(z^2 - t^2)}}{F\left(\frac{\pi}{2}, \mu_0\right)} c$$

5. Physical Quantities in General and a Special Wedge

The crack opening displacement and normal stress-component are quantities which are important in fracture design parameters.

Crack Opening Displacement

The crack opening displacement is the value of integral in (2.9) for z in (b, c). Now using (3.1) in (2.9) and evaluating the integral we get

$$u_r(d, z) = \int_z^c g(t) dt, \quad b < z < c, \tag{5.1}$$

where $g(t)$ is to be taken from (4.11). Displacement is smooth at crack-tips (d, b) and (d, c).

Stress Components

Shear Stress

The component of shear stress at $r = d$ is assumed to be zero for all z.

Normal Stress

The normal stress component is obtained from (2.10) for z in $[0, b) \cup (c, \infty)$ after taking second term on left hand side and is given as

$$\sigma_{rr}(d, z) = \pm \frac{1}{\pi \psi(z)} [\Delta_4(z)] - \int_0^\infty g(t) m_2(t, z) dt, \quad 0 \le z < b, \; c < z < \infty \tag{5.2}$$

$$m_2(p, z) = \int_0^\infty M(ds) \cos(ps) \sin(zs) ds \tag{5.3}$$

where ($\pm$) signs are to be taken for $0 \leq z < b$ as $(+)$ and $c < z < \infty$ as $(-)$ in (5.2). Stress-component σ_{rr} possesses Cauchy type singularities at crack tips, (d, b) and (d, c).

Stress Intensity Factors

The stress-intensity factors at crack tips are defined as

$$(K_c, N_c) = \lim_{z \to e^-} \sqrt{z - c}(\sigma_{rr}(d, z), \sigma_{rz}(d, z))$$

$$(K_b, N_b) = \lim_{z \to b^+} \sqrt{b - z}[\sigma_{rr}(d, z), \sigma_{rz}(d, z)]. \tag{5.3}$$

But $N_c, N_b = 0$ using (5.2) in (5.3) and evaluating the limits K_c and K_b are given as :

$$K_c = \frac{-\Delta_4(c)}{\pi\sqrt{2c(c^2 - b^2)}}, \quad K_b = \frac{\Delta_4(b)}{\pi\sqrt{2b(c^2 - b^2)}}. \tag{5.4}$$

Special Case of Wedge

We consider that the cracks were opened by wedge, therefore, we take, wedge shape $u(z)$ as

$$u(z) = u_0 = \text{ constant.} \tag{5.5}$$

Now evaluating integral in (3.5) by taking (5.6), we get

$$P_2(z) = 0 \ \text{ and then} \tag{5.6}$$

$$a_0 = 0, \ a_1(t) = 0, \ a_{01} = 0, \ a_{02} = 0, \ M_5(t) = 0$$

$$\Delta_0(t) = D = \text{ constant.} \tag{5.7}$$

Now we evaluate the constant D using (t) by taking $g(t) = g_0(t)$, we get

$$D = \frac{\psi_0 \pi^2}{2}[F(\pi/2, \mu_0)], \quad D = \frac{e u_0 \pi^2}{2F\left(\frac{\pi}{2}, \mu_0\right)} \tag{5.8}$$

where F is complete elliptic integral of 1^{st} kind [22]. Thus knowing $\Delta_0(z)$ from (5.7) we can easily get $\Delta_4(c)$ and $\Delta_4(b)$, then K_c and K_b from (5.4), respectively.

Appendix - I

$$d_1 = d^2(2p+s), d_2 = \frac{s+2}{s}, d_3 = 14s(1+Q), d_4 = \frac{d_1}{s} + \frac{Q-P}{P}d_2 - (P+Q)(3-Q)$$
$$d_5 = \frac{d_1(Q-P)}{P}, d_6 = \frac{d_2}{s} - (P+Q)d_3, d_7 = P+Q+d_1, d_8 = d_2$$
$$e_2(m,r,s) = e_{21}(m,r,s)\left|\overline{m+\frac{1}{2}}\right.\left|\overline{r+\frac{1}{2}}\right.$$
$$e_{21}(m,r,s) = e_1(m,r,s) - \left|\overline{r+\frac{1}{2}}\right.(d_7) + d_8\left(m+\frac{1}{2}\right)\left(r+\frac{1}{2}\right).$$

Appendix - II

$$t_1 = [2(1+Q)\sqrt{\pi} - 3]/4,\ 2t_2 = \sqrt{\pi} - 6Q, t_3 = t_2,$$
$$t_4 = 2(P + \sqrt{\pi} - 2/P), t_5 = (4Q - 7P)/4P$$
$$t_6 = t_4 t_5 + 3(1+Q)P, t_7 = \sqrt{\pi}(t_4 + 8t_5)/2 - t_1,$$
$$t_8 = 2[t_7 + 2\sqrt{\pi}/3]/3P, t_9 = (P+Q)t_{10}/\sqrt{\pi}$$
$$t_{10} = \sqrt{\pi}t_8/p.$$

References

[1] Estekanchi H. E., Vafai A., On the buckling of cylindrical shells with through cracks under axial load. Thin Wall Struct, 35 (1999), 255-274.

[2] Ferreira A. J. M., Barbosa J. T., Buckling behavior of composite shells. Compos. Struct., 50 (2000), 93-98.

[3] Sai Ram K. S., Sreedhar Babu T. Buckling of laminated composite shells under transverse load. Compos Struct, 55 (2002), 157-68.

[4] Tafreshi A., Buckling and post-buckling analysis of composite cylindrical shells with cutouts subjected to internal pressure and axial compression loads. Int J Pres Ves Pip, 79:35 (2002), 1-9.

[5] Diaconu C. G., Masaki S., Sekine H., Buckling characteristics and lay-up optimization of long laminated composite cylindrical shells subjected to combined loads using lamination parameters. Compos Struct. 58 (2002), 423-33.

[6] Geier B., Meyer-Peiening H. R., Zimmermann R., On the influence of laminated stacking on buckling of composite cylindrical shells subjected to axial compression. Compos. Struct. 55 (2002), 467-74.

[7] Weaver P. M., Driesen J. R., Roberts P., The effects of flexural/twist anisotropy on compression buckling of laminated cylindrical shells. Compos. Struct. 55 (2002), 195-204.

[8] Hilburger M. W., Starnes Jr. J. H., Effects of imperfections on the buckling response of compression-loaded composite shells. Int. J. Non-Linear Mech, 37 (2002), 623-643.

[9] Weaver P. M., Dickenson R., Interactive local/Euler buckling of composite cylindrical shells. Comput Struct, 81 (2003), 2767-73.

[10] Hilburger M. W., Starnes Jr. J. H., Buckling behavior of compression-loaded composite cylindrical shells with reinforced cutouts. Int. J. Non-Linear Mech., 7 (2005), 1005-21.

[11] Vaziri A., Estekanchi H. E., Buckling of cracked cylindrical thin shells under combined internal pressure and axial compression. Thin Wall Struct 44 (2006), 141-51.

[12] Vaziri Ashkan, On the buckling of cracked composite cylindrical shells under uniaxial compression, Composite Structure, 80 (2007), 152-158.

[13] Erdogan F. and Ratwani M., Fatigue and fractures of cylindrical shells containing a circumferential Crack, Int. J. Fracture Mech., 6 (1970), 371-392.

[14] Erdogan F., An axial crack in specially orthotropic cylindrical shells, Mechanics of Fracture of Plates and Shells. Edited by. G. C. Shih, Noordoff International Publishing Leyden, (1977).

[15] Ma C. C., Huang J. I. and Tsai C. H., Weight functions and stress-intensity factors for axial cracks in hollow cylinders, Journal of Pressure Vessel Technology, 116 (Nov. 1994), 423-430.

[16] Liu, Rong, Zhang, Tie, Wu X. J. and Wang C. H., Crack closure effect on stress intensity factors of an axially and circumferenteally cracked cylindrical shell. Int. J. of Fracture, 125(3-4) (2004), 227-248.

[17] Lal M. and Pandey D., The axisymmetic thermo-elastic stress distribution in an infinite solid containing Penny shaped crack, Proc. Nat. Acad. Sci., 45(a) (1975), 175-180.

[18] Lal M., Some axisymmetric thermal stress distribution in an elastic solid containing an annular crack. Lett. Appl. Eng. Sci., 20(11) (1982), 1261-1273.

[19] Jaunky N., Knight Jr. N. F., An assessment of shell theories for buckling of circular cylindrical laminated composite panels loaded in axial compression. Int J Solids Struct, 36 (1999), 799-820.

[20] Burniston E. E., An example of partially closed Griffith crack, Int. J. of Fracture Mech., (1969), 17-24.

[21] Srivastava K. N. and Lowengrub M., Finite Hilbert transform technique for triple integral equation with trigonometric kernels. Proc. Roy. Soc. Edinb., A68 (1970), 309-321.

[22] Gradstyen I. S. and Rizhik I. N., Tables of Integrals, Series and Products, Academic press London, (1965).