

**UNSTEADY MHD FREE CONVECTION
BOUNDARY-LAYER-FLOW OF ALONG A STRETCHING SLIP
WITH THERMAL HEAT AND VISCOUS INDULGENCE
PROPERTY**

P. SREEDIVYA¹ AND K. SUNITHA²

Abstract

In the present paper we study the unsteady MHD free convection boundary layer flow along a stretching slip with thermal heat and viscous indulgence property depend on the magnetic parameter ,radiation parameter ,prandti number, Eckert number, lewis number, brownain motion etc, the non linear governing partial differential equation obtained are converted in ordinary differential equation by means of suitable similarly transform runge-kutta method with various method and graphically. We opine that findings of this present study will be found useful for environment system in pollution control and ventilation and serve as complementary for researches.

1. Introduction

Magnetohydrodynamics (MHD) boundary-layer flow and heat transfer over a linearly stretched exterior have recognized a set of awareness in the field of numerous industrial, scientific, and engineering applications in recent years. Fluids have numerous applications in the industries since materials of nano meter size have exclusive chemical and

physical properties. various applications of fluids, the cooling applications of fluids include silicon mirror cooling, electronics cooling, vehicle cooling, transformer cooling, etc. the MHD heat and mass transfer stagnation point flow of an incompressible, viscous horizontal plate in the attendance of a oblique magnetic field. This study is more important in industries such as hot rolling; melt spinning, extrusion, glass fiber production, wire drawing, and manufacture of plastic and rubber sheets, polymer sheet and filaments, etc.

2. Review Literature

- Sakiadis [1] examine the boundary layer flow on a uncontrollably surface is first author.
- Vajravelu and Hadjinalaou [6] examine the heat transfer individuality over a stretching exterior with viscous dissipation in the occurrence of internal heat construction or absorption.
- Crane [2] obtained an exact solution of the Newtonian fluid caused by the stretching of an elastic sheet moving in its own plane linearly. Indulgence is the process of converting automatic energy of downward-flowing water into thermal and acoustical energy.
- ([7,8]) considered the radiation individual possessions on the MHD free convection flow of a gas past a semi-infinite upright plate. And measured the thermal radiation effect on a steady flow.
- Sattar and Alam [11] presented unsteady free convection and mass transfer flow of a viscous, incompressible, and electrically conducting fluid past a moving unlimited vertical porous plate by thermal diffusion result.
- Singh et al. [13] investigate the thermal radiation and magnetic field effects on an unsteady stretching permeable sheet in the presence of free stream velocity. The natural convective boundary-layer flows of a fluid past a vertical plate.
- Kuznetsov and Nield [16,17,18]. The representation, the Brownian motion and thermophoresis are accounted by the simplest possible boundary conditions. The steady boundary-layer flow of a fluid past a moving semi-infinite flat plate in a

uniform free stream. It was assumed that the plate is moving in the same or opposite directions to the free stream to define.

- Hamad and Pop [21] discussed the boundary-layer flow near the stagnation-point flow on a permeable stretching sheet in a porous medium saturated with a fluid.
- Hamad et al. [22] examine free convection flow of a fluid past a semi-infinite vertical flat plate with the influence of magnetic field.
- Hady et al. [23] investigated the effects of thermal radiation on the viscous flow of a fluid and heat transfer over a non-linearly make longer sheet. However,
- the aim of the current effort is to study the unsteady free convection boundary-layer fluid flows the length of a stretching surface by the power of magnetic field and radiation result. An finite difference procedure [24] has been taken to solve the obtained nonsimilar equations with stability and convergence analysis.

3. Methods Presentation of the Hypothesis

- An unsteady two-dimensional MHD free convection laminar boundary-layer flow of a viscous incompressible and electrically conduct fluid the length of a vertical stretch sheet under the authority of thermal radiation and viscous dissipation.
- The physical collection and organize scheme is shown in Figure 1. Introducing the Cartesian coordinate scheme, the x -axis is taken along the stretching piece in the upright increasing direction, and the y -axis is taken as normal to the sheet.
- Two the same and overturn forces are introduce along the x -axis so that the sheet is stretched, charge the origin fixed. immediately at time $t > 0$, the temperature of the plate and the order attention are raised to $T_w(> T_\infty)$ and $C_w(> C_\infty)$, correspondingly, which be present following maintain stable, where T_w and C_w are the temperature and order attention at the partition correspondingly, and T_∞ and C_∞ are the temperature and species attention distant away from the plate.
- A strong magnetic field is useful in the y direction. The uniform magnetic field strength (magnetic induction) B_0 can be taken as $B = (0, B_0, 0)$. The Rosse-

land approximation is used to describe the radioactive heat flux q_r in the energy equation.

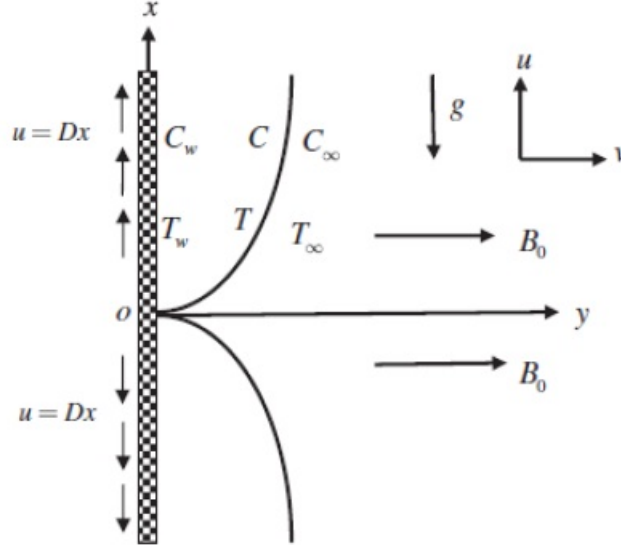


Figure 1 Physical model and coordinate system.

- the usual boundary layer approximation, the MHD free convection unsteady fluid flow and heat and mass transfer with the radiation effect are governed by the following equations:

Continuity equation :

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

Momentum equation :

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\partial u_e}{\partial t} + u_e \frac{\partial u_e}{\partial x} + \gamma \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho} (u - u_e) - \frac{\lambda}{K_P} (u - u_e) = 0 \quad (2)$$

Energy equation :

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + V \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{1}{C_P} u_e \frac{\partial u_e}{\partial x} (u_e - u) = 0. \quad (3)$$

Concentration equation :

$$\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + V \frac{\partial C}{\partial y} = D_m \frac{\partial^2 C}{\partial y^2} = D_m \frac{\partial^2 C}{\partial y^2} + S(C) \quad (4)$$

where u and v are the velocity components in the x and y directions, correspondingly, ν is the kinematic viscosity, k is the thermal conductivity, DB is the Brownian diffusion coefficient, DT is the thermophoresis diffusion coefficient, $D(> 0)$ is the stretching stable, g is the increase of rate appropriate to gravity, ρ is the density of the fluid, and c_p is the exact heat at stable pressure.

The Rosseland approximation [25] is expressed for radiative heat flux and leads to the form

$$\begin{aligned} u(t, x, 0) &= \beta \frac{\partial u}{\partial y}(t, x, 0), V(t, x, 0) = v_W, -k \frac{\partial T}{\partial y}(t, x, 0) = h_f(T_f - T(t, x, 0)) \\ u(t, x, \infty) &= u_e = ax, T(t, x, \infty) = T_\infty, C(t, x, \infty) = C_\infty \\ \eta &= y \sqrt{\frac{a}{\gamma(1-\lambda t)}}, \quad \psi(x, y) = x \sqrt{\frac{a\gamma}{\gamma(1-\lambda t)}} f(\eta) \\ \theta(\eta) &= \frac{T - T_\infty}{T_f - T_\infty}, \quad \varphi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}, \end{aligned} \quad (6)$$

where

σ is the Stefan-Boltzmann constant

κ^* is the mean absorption coefficient.

The temperature variation inside the flow is suitably small such that T_4 may be expressed as a linear function of the temperature, then Taylor's series for T_4 is about T_∞ after neglecting higher order terms:

$$\text{And} \quad F_W = -V_W \sqrt{\frac{1-\lambda t}{a\gamma}} \quad (7)$$

the subsequent non-dimensional variables,

$$f''' + ff'' - (f') - A \left(f' + \frac{1}{2} \eta f'' - 1 \right) - [Ha + Da](f' - 1) + 1 = 0. \quad (8)$$

Then, Equations 1 to 5 become

$$\frac{1}{Pr} \theta'' + Ec(-f') + f\theta' - \frac{1}{2} A\mu\theta' = 0 \quad (9)$$

$$\frac{1}{Sc} \phi'' + f\phi' + \varsigma e^{n\phi} - \frac{1}{2} A\eta\theta' = 0 \quad (10)$$

$$f'(0) = \delta f''(0, f) = F_W,$$

$$-\theta'(0) = B_i(1 - \theta(0)), \phi(0) = 1,$$

$$f(\infty) = 1, \phi(\infty) = \theta(\infty) = 0. \quad (11)$$

The non-dimensional boundary situation are

$$Sc = \frac{\gamma}{D_m}, \quad Ha = \frac{\sigma B_0^2}{\rho a}(1 - \lambda t), \quad Pr = \frac{\gamma}{\alpha},$$

$$Bi = \frac{h_f}{k} \sqrt{\frac{\gamma(1 - \lambda t)}{\alpha}}, \quad Da = \frac{\gamma}{k_p}(1 - \lambda t),$$

$$Ec = \frac{u_e^2}{C_P(T_f - T_\infty)}, \quad A = \frac{\lambda}{a}, \quad \varsigma = \frac{Q(1 - \lambda t)}{a(C_W - C_\infty)},$$

$$n = b(C_w - C_\infty), \quad \delta = \beta \sqrt{\frac{a}{\gamma(1 - \lambda t)}} \quad (12)$$

where the magnetic parameter $2M = \frac{\sigma B_0^2 v}{\rho u_0^2}$, Grashof number $Gr = \frac{g\beta(T_w - T_\infty)}{U_0^3}$, radiation $r = \frac{16\sigma T_\infty^3}{3kk^*}$, Prandtl number $Pr = \frac{v}{\alpha}$, Eckert number $Ec = \frac{U_0^2}{c_p(T_w - T_\infty)}$, Lewis number $Le = \frac{v}{D_B}$, Brownian parameter $N_b = \frac{\tau D_B (C_w - C_\infty)}{v}$, and thermophoresis parameter $N_t = \frac{D_T}{T_\infty} \frac{\tau}{v} (T_w - T_\infty)$.

Results and Discussion

To obtain the steady state solution of the calculation, the calculation have been approved out up to non-dimensional time $\tau = 5$ to 80. The velocity, temperature, and attention profiles do not show any change after non-dimensional time $\tau = 50$. Therefore, the solution for $\tau \geq 50$ is the steady-state solution. The graphical representation of the problem has been shown in Figures 2, 3, 4, 5, 6, 7, 8, and 9. Dimensionless velocity, temperature, and attention distributions are plotted against and corresponding values of Grashof number Gr, magnetic parameter M, radiation parameter R, Eckert number Ec, Prandtl number Pr, Brownian motion parameter Nb, thermophoresis parameter Nt, and Lewis number Le, respectively.

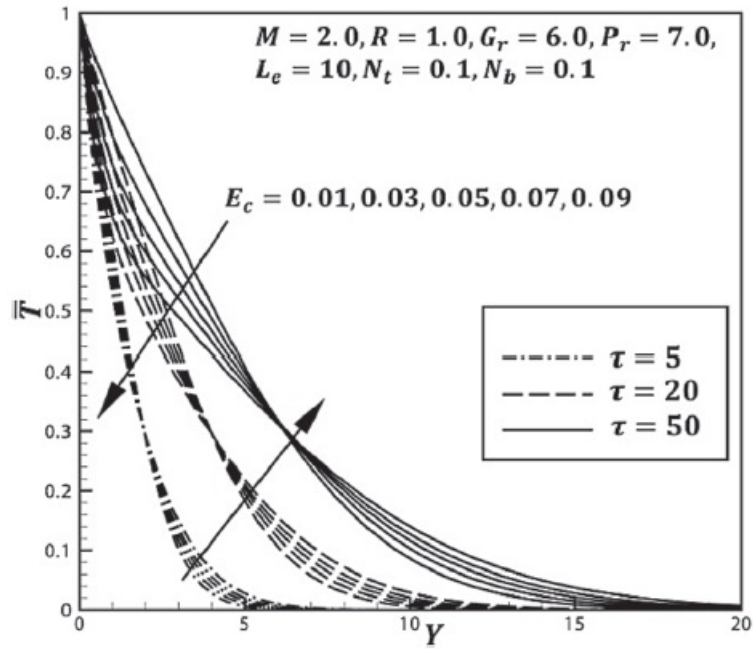


Figure 2. Eckert Number (E_c) effect on temperature profiles.

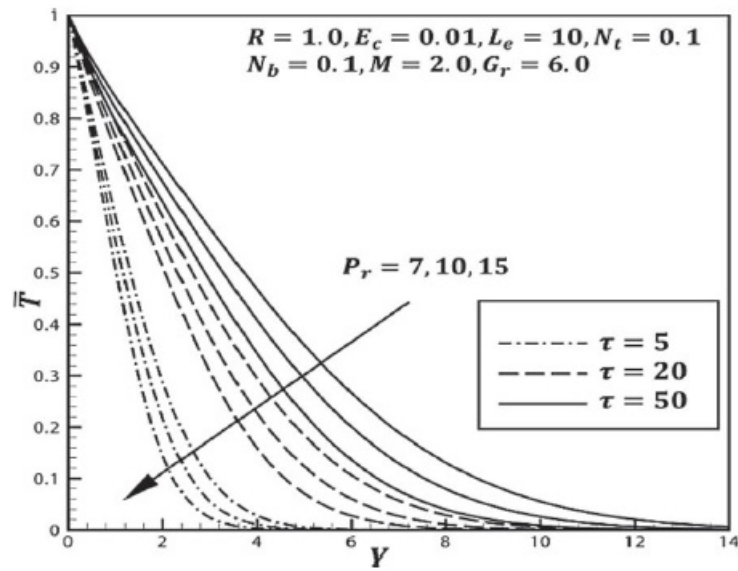
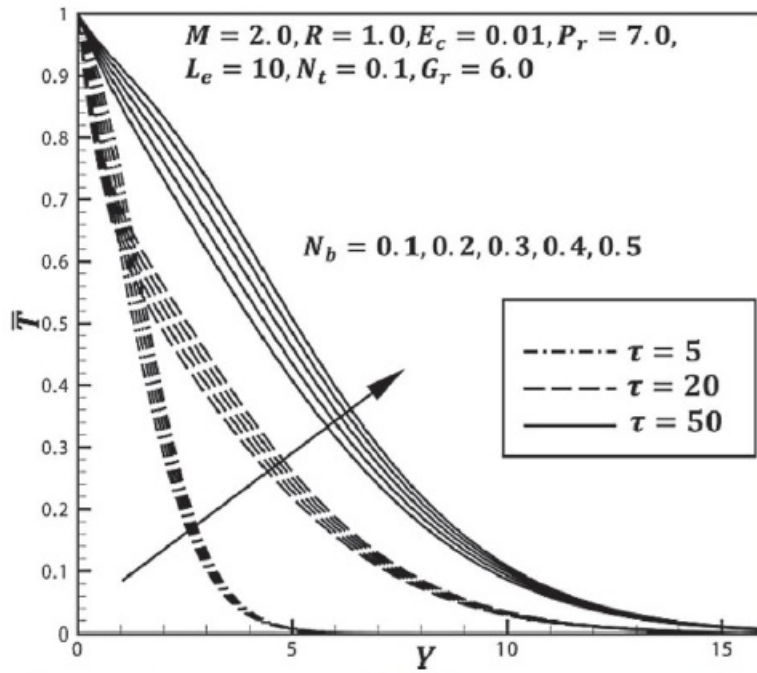
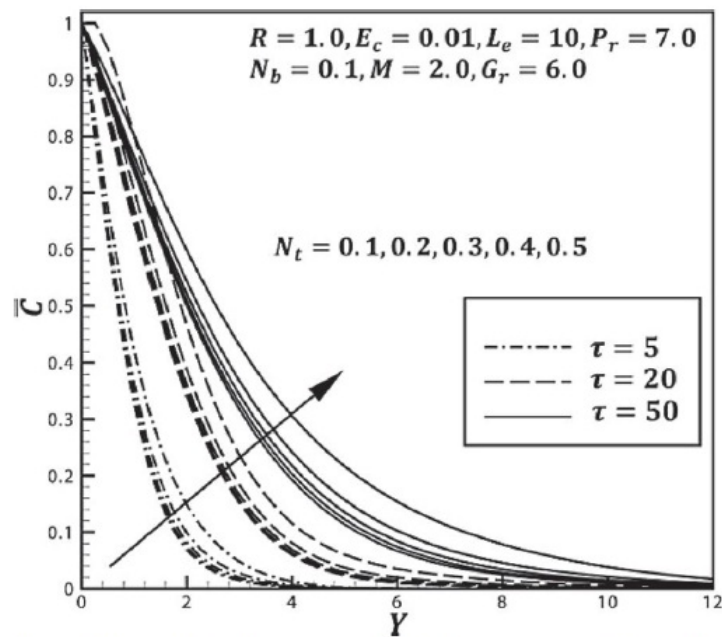


Figure 3 Prandtl number (P_r) effect on temperature profiles

Figure 4 Brownian parameter (N_b) effect on temperature profiles.Figure 5 Thermophoresis parameter (N_t) effect on concentration profiles

- ([19]) (similar solution) and the value of magnetic parameter M , radiation parameter R , Eckert number Ec , and Grashof number Gr are considered zero.
- In Figures 2, 3, 4, and 5, the temperature distribution is plotted respectively for different values of R , Ec , Pr , and Nb . Here, it is observed that when the values of R increase, then the temperature profiles increase; when the values of Ec increase, then the temperature profiles also increase; when the values of Pr increase, then the temperature profiles decrease; and when the values of Nb increase, then the temperature profiles increase.

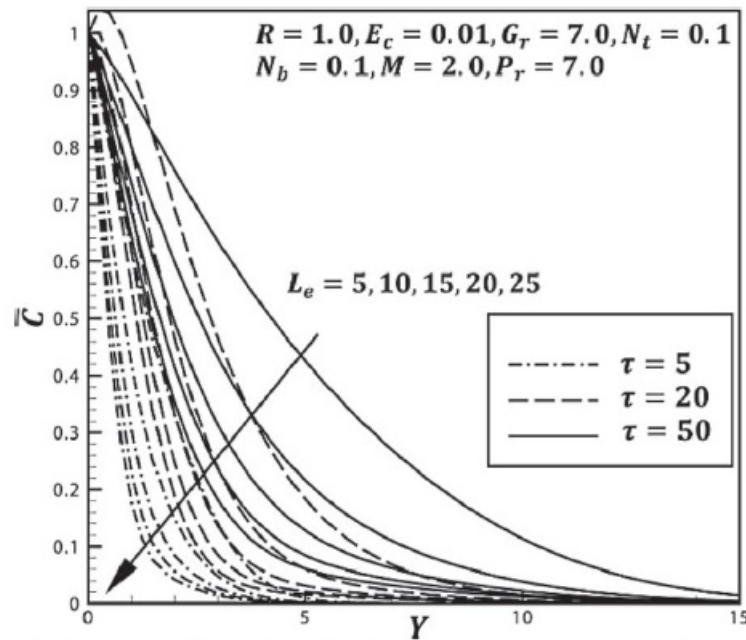


Figure 6 Lewis number (L_e) effect on concentration profiles.

- Here, it is observed that when the values of Nt increase, then the concentration profiles increase, but when the values of Le increase, then the concentration profiles decrease.
- ([19]) study when the values of magnetic parameter M , radiation parameter R , Eckert number Ec , and Grashof number Gr are considered zero.

4. Conclusions

An unsteady free convection boundary-layer flow of a fluid due to a stretch piece is calculated with the manipulate of magnetic field and thermal energy. finite difference [24] technique with stability and convergence analysis.

For the unsteady case (time-dependent), The results are presented for the effect of various parameters. Of velocity, temperature, and attention property on the sheet are

1. superior values of the Grashof number showed a significant effect on momentum boundary layer. Brownian motion and thermophoresis stabilizes the boundary layer growth.
2. The boundary layers are highly influenced by the Prandtl number. The thermal boundary layer thickness increases as a result of increasing radiation.
3. The presence of heavier species (large Lewis number) decreases the concentration in the boundary layer.

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