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# SOME NEW RESULTS OF PRIME CORDIAL LABELING 

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#### Abstract

A prime cordial labeling of a graph $G=(V(G), E(G))$ is a bijection f from $\mathrm{V}(\mathrm{G})$ to $\{1,2, \ldots|V(G)|\}$ such that for each edge uv is assigned the label 1 if $\operatorname{gcd}(f(u), f(v))=$ 1 and 0 if $\operatorname{gcd}(f(u), f(v))>1$; then the number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1 . If a graph has a prime cordial labeling, then it is called a prime cordial graph. In this paper, we prove that the subdivision of each edge of bistar $B_{n, n}$, the graph $P_{n} \bigodot k_{1, n-1}$ and the disconnected graph $P_{n} \bigcup P_{m}$ admit prime cordial labeling.


## 1. Introduction

In this paper we consider only finite, simple undirected graphs. We consider a graph $G=(V(G), E(G))$ and we let $|V(G)|=p$ and $|E(G)|=q$. For graph theoretic notations and terminology we follow Harary [4] and for number theory we follow Burton [1]. A labeling of a graph $G$ is a mapping that carries vertices and/or edges into a set of

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numbers, usually integers. An excellent survey of graph labeling and various types of graph labeling can be found in Gallian [3]. We provide a brief summary of results which will be useful for the present investigations.
Definition 1.1: Let $G=(V(G), E(G))$ be a graph. A mapping $f: V(G) \rightarrow\{0,1\}$ is called a binary vertex labeling of $G$ and $f(v)$ is called the label of the vertex $v \in V(G)$ under $f$. The induced edge labeling $f^{*}: E(G) \rightarrow\{0,1\}$ is defined by $f^{*}(e=u v)=$ $|f(u)-f(v)|$.
We denote $v_{f}(i)$ is the number of vertices of $G$ having label i and $e_{f}(i)$ is the number of edges of $G$ having label $i$ under $f^{*}$, where $i=0,1$.
Definition 1.2: A binary vertex labeling $f$ of a graph $G$ is called a cordial labeling if $\left|v_{f}(0)-v_{f}(1)\right| \leq 1$ and $\left|e_{f}(0)-e_{f}(1)\right| \leq 1$. A graph $G$ is called cordial if it admits cordial labeling.
The concept of cordial labeling was introduced by Cahit [2].
Definition 1.3: A prime cordial labeling of a graph $G$ with vertex set $V(G)$ is a bijection $f: V(G) \rightarrow\{1,2,3, \ldots,|V(G)|\}$ defined by
$f(e=u v)=\left\{\begin{array}{ll}1 & \text { if } \operatorname{gcd}(f(u), f(v))=1 ; \\ 0 & \text { otherwise. }\end{array} \quad\right.$ further $\quad\left|e_{f}(0)-e_{f}(1)\right| \leq 1$.
A graph which admits prime cordial labeling is called a prime cordial graph. The concept of prime cordial labeling was introduced by Sundaram et al. [6].
Now let us recall the definitions of the following graphs.

Definition 1.4: If each edge of the bistar $B_{n, n}$ is subdivided then the resulting graph is called the subdivision of the bistar graph and the graph is denoted by $S\left(B_{n, n}\right)$.
Definition 1.5: $P_{n} \bigodot k_{1, n-1}$ is a graph obtained from a path $P_{n}$ by attaching a star graph $k_{1, n-1}$ at each vertex of the path $P_{n}$.
Definition 1.6: $P_{n} \bigcup P_{m}$ is a graph which consists of the two disconnected paths $P_{n}$ and $P_{m}$.
In this paper we prove that the subdivision each of bistar graph $S\left(B_{n, n}\right)$, the graph $P_{n} \bigodot k_{1, n-1}$ and the graph $P_{n} \cup P_{m}$ are prime cordial graphs.

Theorem 1.7: The subdivision of the bistar graph $S\left(B_{n, n}\right)$ admits prime cordial labeling.
Proof: Let $G$ be the graph $S\left(B_{n, n}\right)$ with vertex set
$V(G)=\left\{u, v, w, u_{i}^{j}, v_{i}^{j}, 1 \leq i \leq 2,1 \leq j \leq n\right\}$.
Then $|V(G)|=4 n+3$ and $|E(G)|=4 n+2$.
We define the vertex labeling $f: V(G) \rightarrow\{1,2,3, \ldots 4 n+3\}$ as follows:
$f(u)=3, f(w)=9, f(v)=2$
$f\left(u_{1}^{1}\right)=1, f\left(u_{1}^{2}\right)=5$ and $f\left(u_{1}^{j}\right)=4 j+1, j=3,4, \ldots n$
$f\left(u_{2}^{1}\right)=11, f\left(u_{2}^{2}\right)=7$ and $f\left(u_{2}^{j}\right)=4 j+3, j=3,4, \ldots n$
$f\left(v_{1}^{j}\right)=4 j, 1 \leq j \leq n$ and $f\left(v_{2}^{j}\right)=4 j+2,1 \leq j \leq n$
Suppose $u / u_{1}^{j}$ then replace $u_{1}^{j}$ by $u_{2}^{j}$ for any $j=3,4, \ldots n$.
From the above labeling pattern, we have
$e_{f}(0)=2 n+1=e_{f}(1)$.
We conclude that $\left|e_{f}(0)-e_{f}(1)\right| \leq 1$.
Hence $G$ is a prime cordial graph.

Example 1.8: Prime cordial labeling of graph $S\left(B_{3,3}\right)$ is shown in Figure 1.


Figure 1 : Prime Cordial Labeling for $S\left(B_{3,3}\right)$

Theorem 1.9 : Thegraph $P_{n} \bigodot k_{1, n-1}$ is a prime cordial graph, where $n$ is odd.
Proof : Let $G$ be the graph $P_{n} \bigodot k_{1, n-1}$ and let $\left\{u_{i}: 1 \leq i \leq n\right\}$ be the vertices of the path $P_{n}$.
Let $\left\{u_{i}^{j}: 1 \leq i, j \leq n\right\}$ be the pendent vertices of $G$.
Then $|V(G)|=n^{2}$ and $|E(G)|=n^{2}-1$.
We define $f: V(G) \rightarrow\left\{1,2,3, \ldots n^{2}\right\}$ as follows.
$f\left(u_{i}\right)=2 i ; 1 \leqslant i \leqslant n$.
$f\left(u_{j}^{i}\right)=(n-1) j+2 i-1 ; j=2,4, \ldots n-1,1 \leqslant i \leqslant n-1$.
$f\left(u_{1}^{i}\right)=2 i-1 ; 1 \leqslant i \leqslant n-1$.
$f\left(u_{j}^{i}\right)=n(j-1)+2 i-j+3 ; j=3,5, \ldots n, 1 \leqslant i \leqslant n-1$. (except for $i=n-1$ and $j=n$ )
and $f\left(u_{n}^{n-1}\right)=n(j-1)+2 i-j+2$.
In view of the labeling pattern defined above we have $e_{f}(0)=\frac{n^{2}-1}{2}=e_{f}(1)$.
Thus we have $\left|e_{f}(0)-e_{f}(1)\right| \leq 1$.
Hence $P_{n} \bigodot k_{1, n-1}$ is a prime cordial graph.
Example 1.10 : Prime cordial labeling of the graph $P_{5} \odot k_{1,4}$ is shown in Figure 2.


Figure 2 : Prime cordial labeling for $P_{5} \odot k_{1,4}$
Theorem 1.11: The disconnected graph $P_{n} \bigcup P_{m}$ is a prime cordial graph, where $n, m \geq 2$.
Proof: Consider $P_{n}$ with $V\left(P_{n}\right)=\left\{u_{i} ; 1 \leq i \leq n\right\}$ and $P_{m}$ with $V\left(P_{m}\right)=\left\{v_{i} ; 1 \leq i \leq\right.$ $m\}$.
Let $G$ be the disconnected graph $P_{n} \bigcup P_{m}$.
Then $|V(G)|=n+m$ and $|E(G)|=n+m-2$.
To define $f: V(G) \rightarrow\{1,2,3, \ldots n+m\}$, we consider following three cases.
Case 1: $m=n$
$f\left(u_{i}\right)=2 i-1 ; 1 \leqslant i \leqslant m$.
$f\left(v_{i}\right)=2 i ; 1 \leqslant i \leqslant n$.
In view of the labeling pattern defined above we have $e_{f}(0)=n-1=e_{f}(1)$.
Case 2: $n>m$
Subcase $(i): m+n$ is even.
$f\left(u_{i}\right)=2 i ; i=1,2, \ldots\left(\frac{m+n}{2}\right)$
$f\left(u_{i}\right)=2 i-(k+1) ;\left(\frac{m+n}{2}\right)+1 \leq i \leq n$, where $k=|n-m|$
$f\left(v_{i}\right)=2 i-1 ; 1 \leqslant i \leqslant n$.
In view of the labeling pattern defined above we have $e_{f}(0)=\left(\frac{m+n-2}{2}\right)=e_{f}(1)$.
Subcase ( $i i$ ): $m+n$ is odd.
$f\left(u_{i}\right)=2 i ; i=1,2, \ldots\left(\frac{m+n-1}{2}\right)$
$f\left(u_{i}\right)=2 i-k ;\left(\frac{m+n+1}{2}\right) \leq i \leq n$, where $k=|n-m|$
$f\left(v_{i}\right)=2 i-1 ; 1 \leqslant i \leqslant n$.
In view of the labeling pattern defined above we have $e_{f}(0)=\left(\frac{m+n-3}{2}\right)$ and $e_{f}(1)=$ ( $\frac{m+n-1}{2}$ ).
Case 3: $n<m$
Subcase ( $i$ ): $m+n$ is even.
$f\left(u_{i}\right)=2 i-1 ; 1 \leqslant i \leqslant m$,
$f\left(v_{i}\right)=2 i ; i=1,2, \ldots\left(\frac{m+n}{2}\right)$
$f\left(v_{i}\right)=2 i-(k+1) ;\left(\frac{m+n+2}{2}\right)+1 \leq i \leq n$, where $k=|n-m|$
In view of the labeling pattern defined above we have $e_{f}(0)=\left(\frac{m+n-2}{2}\right)=e_{f}(1)$.
Subcase (ii): $m+n$ is odd.
$f\left(u_{i}\right)=2 i-1 ; 1 \leqslant i \leqslant m$
$f\left(v_{i}\right)=2 i ; i=1,2, \ldots\left(\frac{m+n-1}{2}\right)$
$f\left(v_{i}\right)=2 i-k ;\left(\frac{m+n+1}{2}\right) \leq i \leq n$, where $k=|n-m|$
In view of the labeling pattern defined above we have $e_{f}(0)=\left(\frac{m+n-3}{2}\right)$ and $e_{f}(1)=$ $\left(\frac{m+n-1}{2}\right)$.
Thus from the above cases we have $\left|e_{f}(0)-e_{f}(1)\right| \leq 1$.
Hence the disconnected graph $P_{n} \bigcup P_{m}$ is a prime cordial graph.

Example 1.12: The Prime cordial labeling of the graphs $P_{5} \bigcup P_{5}$ and $P_{4} \bigcup P_{2}$ are shown in Figure 3 and Figure 4 respectively.

Figure 3: Prime cordial labeling for $P_{4} \bigcup P_{2}$

| 2 |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathbf{u}$. | $u_{2}$ | $u_{3}$ | $u_{4}$ | $u_{4}$ | 1 |

Figure 4: Prime cordial labeling for $P_{4} \bigcup P_{2}$

## 2. Concluding Remarks

In this paper, we have proved that the graphs $S\left(B_{n, n}\right), P_{n} \odot k_{1, n-1}$ when $n$ is odd and the disconnected graph $P_{n} \bigcup P_{m}$ admit prime cordial labeling and hence these graphs are prime cordial graphs. It is an interesting open area of research to find out some new class of graphs which admit prime cordial labeling.

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