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RELIABILITY IN MAXIMUM LIKEHOOD ESTIMATOR OF $R_{s,k}$ EXPONENTIAL DISTRIBUTION

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Abstract

In this study, we consider ,A multicomponent system of k-components having strengths following k-independently and identically distributed random variables $x_1, x_2, \cdots, x_k$, and each component experiencing a random stress Y is considered. The system is regarded as alive only if at least s out of k $(s < k)$ strengths exceed the stress. The reliability of such a system is obtained when strength and stress variates are given by generalized exponential distribution with different shape parameters. The reliability is estimated using MLE method of estimation in samples drawn from strength and stress distributions. The reliability estimators are compared asymptotically confidence of R and its asymptotic distribution are obtained made through monte carlo simulation.

1. Introduction

The two-parameter exponential distribution (GE) has been introduced and studied quite extensively by Gupta & Kundu (1999, 2001, 2002). The two-parameter E distribution is an alternative to the well known two-parameter gamma, two-parameter Weibull or two parameter log-normal distributions. The two-parameter Exponential distribution has the following probability density function and the distribution function, respectively

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Key Words : *Asymptotic confidence interval, Maximum likelihood estimation, Reliability, Stress-strength model.*

$$f(x;\sigma,\theta)=\sigma\theta x^{\sigma-1}e^{-}\theta x^{\sigma} \quad X>0,\ \ \theta,\ \sigma>0 \tag{1}$$

$$F(x;\sigma,\theta)=1-e^{-}\theta x^{\sigma}, \qquad \theta,\ \sigma>0. \tag{2}$$

Here σ and θ are the shape and scale parameters, respectively. Now onwards exponential distribution with the shape parameter a and scale parameter θ will be denoted by $E(\sigma,\theta)$.

The purpose of this paper is to study the reliability in a multicomponent stress-strength based on X, Y being two independent random variables, where $X \sim E(\theta,\sigma)$ and $Y \sim E(\sigma,\theta)$.

Let the random samples Y, $x_1, x_2, \cdots, x_k$, being independent, $G(y)$ be the continuous distribution function of Y and $F(x)$ be the common continuous distribution function of $X_1, X_2, \cdots, X_k$. The reliability in a multicomponent stress-strength model developed by Bhattacharyya & Johnson (1974) is given by $R_{s,k} = p$ [at least S of the where $x_1, x_2, \cdots, x_k$ exceeds Y are independently identically distributed (iid) with common distribution function $F(x)$, this system is subjected to common random stress Y.

Studied estimation of reliability in multicomponent stress-strength for the log-logistic distribution. Suppose that a system, with k identical components, functions if s $(1 \leq s \leq k)$ or more of the components simultaneously operate. In this operating environment, the system is subjected to a stress Y which is a random variable with distribution function $G(\cdot)$. The strengths of the components, that is the minimum stress to cause failure, are independent and identically distributed random variables with distribution function $F(\cdot)$. Then, the system reliability, which is the probability that the system does not fail, is the function $R_{s,k}$ given in (3). The estimation of the survival probability in a multicomponent stress-strength system when the stress follows a two-parameter GE distribution has not received much attention in the literature. Therefore, an attempt is made here to study the estimation of reliability in multicomponent stress-strength model with reference to the twoparameter GE probability distribution. In Section 2, we derive the expression for $R_{s,k}$ and develop a procedure for estimating it. More specifically, we obtain the maximum likelihood estimates of the parameters. The Maximum Likelihood Estimators (MLEs) are employed to obtain the asymptotic distribution and confi- dence intervals for $R_{s,k}$. The small sample comparisons are made through Monte

Carlo simulations in Section 3. Also, using real data, we illustrate the estimation process. Finally, some conclusion and comments are provided in Section 4.

2. Review Literature

1. Bhattacharyya, G. K. & Johnson, R. A. (1974), Estimation of reliability in multicomponent stress-strength model

2. Jae, J. K. & Eun, M. K. (1981), Estimation of reliability in a multicomponent stress-strength model in Weibull case,

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5. Bhattacharyya & Johnson 1974). The survival probability of a single component stress-strength version has been considered by several authors assuming various lifetime distributions for the stress-strength random variates,

6. Enis & Geisser (1971), Downtown (1973), Awad & Gharraf (1986), McCool (1991), Nandi & Aich (1994), Surles & Padgett (1998), Raqab & Kundu (2005), Kundu & Gupta (2005), Kundu & Gupta (2006), Raqab, Modi & Kundu (2008), Kundu & Raqab (2009). The reliability in a multicomponent stress-strength was developed Revista Colombiana de Estadstica 35 (2012) 6776 Estimation of Reliability in Multicomponent Stress-strength based on GED 69 by Bhattacharyya & Johnson (1974),

7. Pandey & Uddin (1985), and the references therein cover the study of estimating in many standard distributions assigned to one or both stress, strength variates. Recently, Rao & Kantam (2010).

Maxiimum likelihoos estimator of $R_{s,k}$

Let $X \sim E(\theta, \sigma)$ and $Y \sim E(\emptyset, \sigma)$ wit hunkown shape parameters θ and $\emptyset$ and common scale parameter σ where X and Y are independently distributed. The reliability in multicomponents stress-strength for two - parameter exponential distribution using (1) - (2)

$$\begin{aligned} R_{s,k} &= \sum_{i=s}^{k} \sum_{k} \left(\begin{array}{c} k \\ i \end{array} \right) \emptyset, \sigma \int_{0}^{\infty} Y^{\sigma-1} e^{-y^{\sigma(\sigma+\theta)}} (1 - e^{-\theta y^{\sigma}}) k - 1 dy \\ &= \sum_{i=s}^{k} \sum_{k} \left(\begin{array}{c} k \\ i \end{array} \right) \sum_{k} \left(\begin{array}{c} k-i \\ j \end{array} \right) \frac{(-1)^{j}}{i}. \end{aligned}$$

The estimator of $R_{s,k}$. If that n systems are put on like testing experiment.
That is obsebed data $x_1, x_2, \cdots, x_k$ and $Y_i = 1, 2, \cdots, n$.
Given as $L(\theta, \emptyset, x, y) \prod_{i=1}^{k} f(x_{i,j}) \prod_{i=1}^{n} A_k g(y_i)$.

3. Simulation Study and Data Analysis

3.1. Simulation Study

The average bias and average MSE decrease as sample size increases for both methods of estimation in reliability. Also the bias is negative in both situations of (s, k). It verifies the consistency property of the MLE of $R_{s,k}$. Whereas, among the parameters the absolute bias and MSE decrease as a increases for a fixed β in both cases of (s, k) and the absolute bias and MSE increase as β increases for a fixed a in both the cases of (s, k). The length of the confidence interval also decreases as the sample size increases. The coverage probability is close to the nominal value in all cases but slightly less than 0.95. Overall, the performance of the confidence interval is quite good for all combinations of parameters. Whereas, among the parameters we observed the same phenomenon for average length and average coverage probability that we observed in the case of average bias and MSE.

3.2. Data Analysis

In this subsection we analyze two real data sets and demonstrate how the proposed methods can be used in practice. The first data set is reported by Lawless (1982) and the second one is given by Linhardt & Zucchini (1986). Both are analyzed and fitted for various lifetime distributions. We fit the generalized exponential distribution to the two data sets separately. The first data set (Lawless 1982, p. 228) presented here arose in tests on endurance of deep groove ball bearings.

Table 1: Average bias of the simulated estimates of $R_{s,k}$.

(n,m)	$]b$	R	$\hat{R}_{ML}$	$Bias$	MSE
(10,10)	0.25	0.2000	0.2101	0.0101	0.1061
	0.50	0.3333	0.3411	0.0078	0.0872
	1.00	0.5000	0.5005	0.0005	0.0843
	2.00	0.6667	0.6599	-0.0068	0.0626
	3.00	0.7500	0.7410	-0.0090	0/0527
(10,15)	0.25	0.2000	0.2087	0.0087	0.0485
	0.50	0.3333	0.3432	0.0099	0.0365
	1.00	0.5000	0.5019	0.0019 Y	0.0640
	2.00	0.6667	0.6641	-0.0026	00401
	3.00	0.7500	0.7457	-0.0043	0.0337
(15, 15)	0.25	0.2000	0.2044	0.0044	0.0232
	0.50	0.3333	0.3357	0.0024	0.0204
	1.00	0.5000	0.4977	-0.0023	0.0373
	2.00	0.6667	0.6603	-0.0064	0.0218
	3.00	0.7500	0.7427	-0.0073	0.0183
(25,25)	0.25	0.2000	0.2063	0.0063	0.0176
	0.50	0.3333	0.3403	0.0073	0.0148
	1.00	0.5000	0.5001	0.0001	0.0246
	2.00	0.6667	0.6600	-0.0070	0.0148
	3.00	0.7500	0.7427	-0.0073	0.0056
(25, 50)	0.25	0.2000	0.2033	0.0033	0.0119
	0.50	0.3333	0.3345	0.0015	0.0098
	1.00	0.5000	0.5035	0.0035	0.0091
	2.00	0.6667	0.6664	-0.0003	0.0056
	3.00	0.7500	0.7546	-0.0044	0.0042
(50, 50)	0.25	0.2000	0.2049	0.0049	0.0084
	0.50	0.3333	0.3350	0.0020	0.0077
	1.00	0.5000	0.4996	-0.0004	0.0063
	2.00	0.6667	0.6647	-0.0023	0.0035
	3.00	0.7500	0.7476	-0.0024	0.0028

Table 2 : Exact and asymptotic confidence intervals of R based on MLEs and at significance level 0.05 and scale parameter is known ($\lambda = 1$)

(n, m)	R	CI_{EX}	CI_{A1}
	0.2000	(0.1113), (0.3610)	(0.1317, 0.2885)
	0.3333	(0.1960, 0.5237)	(0.2666, 0.43156)
(10,10)	0.5000	(0.3525, 0.6803)	(0.4279, 0.5730)
	0.6667	(0.4774, 0.8047)	(0.5952, 0.7246)
	0.7500	(0.5739, 0.8587)	(0.6822, 0.7998)
	0.2000	(0.1201, 0.3497)	(0.1401, 0.2773)
0.3333	(0.2129, 0.5159)	(0.2824, 0.4039)	
(10,15)	0.5000	(0.3428, 0.6726)	(0.4451, 0.5587)
	0.6667	(0.5058, 0.8013)	(0.6131, 0.7151)
	0.7500	(0.6029, 0.8567)	(0.7026, 0.7888)
	0.2000	(0.1225, 0.3211)	(0.1456, 0.2632)
	0.3333	(0.2154, 0.4819)	(0.2808, 0.3906)
(15, 15)	0.5000	(0.3499, 0.6459)	(0.4487, 0.5467)
	0.6667	(0.5136, 0.7816)	(0.6172, 0.7034)
	0.7500	(0.6106, 0.8416)	(0.7035, 0.7819)

4. Conclusions

In this paper, we have studied the multicomponent stress-strength reliability for generalized exponential distribution when both stress, strength variates follow the same population. Also, we have estimated asymptotic confidence interval for the multicomponent stress-strength reliability. The simulation results indicate that the average bias and average the MSE decrease as sample size increases for both situations of (s, k). Among the parameters the absolute bias and MSE decrease (increase) as a increases (increases) in both the cases of (s, k). the reliability of the system is estimated by using ML and Bayesian approaches .we have also considered two real data set to illustrate the applicability of the considered model in real life scenario and it is found that the considered model provides better fit to the given data set.

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