International J. of Pure & Engg. Mathematics (IJPEM)
ISSN 2348-3881, Vol. 5 No. III (December, 2017), pp. 111-119

MEAN SUM SQUARE PRIME LABELING OF SOME PATH RELATED GRAPHS

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Abstract

Mean sum square prime labeling of a graph is the labeling of the vertices with $\{0, 1, 2, \cdots, p-1\}$ and the edges with mean of the square of the sum of the labels of the incident vertices or mean of the square of the sum of the labels of the incident vertices and one, depending on the sum is even or odd. The greatest common incidence number of a vertex (*gcin*) of degree greater than one is defined as the greatest common divisor of the labels of the incident edges. If the *gcin* of each vertex of degree greater than one is one, then the graph admits mean sum square prime labeling. Here we identify some path related graphs for mean sum square prime labeling.

1. Introduction

All graphs in this paper are simple, finite and undirected. The symbol $V(G)$ and $E(G)$ denotes the vertex set and edge set of a graph G. The graph whose cardinality of the

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Key Words : *Graph labeling, Sum square, Greatest common incidence number, Prime labeling, Paths.*

vertex set is called the order of G, denoted by p and the cardinality of the edge set is called the size of the graph G, denoted by q. A graph with p vertices and q edges is called a (p, q)-graph.

A graph labeling is an assignment of integers to the vertices or edges. Some basic notations and definitions are taken from [2], [3] and [4]. Some basic concepts are taken from [1] and [2]. In [5], we introduced the concept of sum square prime labeling and proved the result for some cycle related graphs. In [6], [7], [8], [9], we proved the result for some path related graphs, some snake related graphs, some tree graphs, triangular belt, jelly fish graph, some star related graphs. In this paper we introduced mean sum square prime labeling using the concept greatest common incidence number of a vertex. We proved that some path related graphs admit mean sum square prime labeling.

Definition: 1.1 : Let G be a graph with p vertices and q edges. The greatest common incidence number ($gcin$) of a vertex of degree greater than or equal to 2, is the greatest common divisor (gcd) of the labels of the incident edges.

2. Main Results

Definition 2.1 : Let $G = (V, E)$ be a graph with p vertices and q edges. Define a bijection $f : V(G) \to \{0, 1, 2, 3, \cdots, p-1\}$ by $f(v_i) = i - 1, 1 \leq i \leq p$. Define a 1-1 mapping

$f^*_{mssp} : E(G) \to$ set of natural numbers N by

$f^*_{mssp}(uv) = \frac{\{f(u)+f(v)\}^2}{2}$ when $f(u) + f(v)$ is even.

$f^*_{mssp}(uv) = \frac{\{f(u)+f(v)\}^2+1}{2}$, when $f(u) + f(v)$ is odd. The induced function f^*_{mssp} is said to be a mean square sum prime labeling, if the gcin of each vertex of degree at least 2, is 1.

Definition 2.2 : A graph which admits mean sum square prime labeling is called a mean sum square prime graph.

Theorem 2.1 : Path P_n admits mean sum square prime labeling.

Proof : Let $G = P_n$ and let $v_1, v_2, \cdots, v_n$ are the vertices of G.

Here $|V(G)| = n$ and $|E(G)| = n - 1$.

Define a function $f : V \to \{0, 1, 2, 3, \cdots, n-1\}$ by

$$f(v_i) = i - 1, \quad i = 1, 2, \cdots, n.$$

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{mssp} is defined as follows

$$f^*_{mssp}(v_i\ v_{i+1}) = 2i^2 - 2i + 1, \qquad i = 1, 2, \cdots, n-1.$$

Clearly f^*_{mssp} is an injection.

$$\begin{aligned} gcin\ \text{ of } (v_{i+1}) &= \text{gcd of } \{f^*_{mssp}(v_i\ v_{i+1}), f^*_{mssp}(v_{i+1}\ v_{i+2}) \\ &= \text{gcd of } \{2i^2 - 2i + 1, 2i^2 + 2i + 1\} \\ &= \text{gcd of } \{4i, 2i^2 - 2i + 1\}, \\ &= \text{gcd of } \{i, 2i^2 - 2i + 1\} = 1, \qquad i = 1, 2, \cdots, n-2. \end{aligned}$$

So, $gcin$ of each vertex of degree greater than one is 1.

Hence P_n, admits mean sum square prime labeling.

Theorem 2.2 : $(P_n)^2$ admits mean sum square prime labeling.

Here $|V(G)| = n$ and $|E(G)| = 2n - 3$.

Define a function $f : V \to \{0, 1, 2, 3, \cdots, n-1\}$ by

$$f(v_i) = i - 1, \quad i = 1, 2, \cdots, n.$$

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{mssp} is defined as follows

$$f^*_{mssp}(v_i\ v_{i+1}) = 2i^2 - 2i + 1, \qquad i = 1, 2, \cdots, n-1$$

$$f^*_{mssp}(v_i\ v_{i+2}) = 2i^2, \qquad i = 1, 2, \cdots, n-2.$$

Clearly f^*_{mssp} is an injection.

$$\begin{aligned} gcin \text{ of } (vi + 1) &= 1, \qquad i = 1, 2, \cdots, n-2 \\ gcin \text{ of } (v_1) &= \text{gcd of } \{f^*_{mssp}(v_1\ v_2), f^*_{mssp}(v_1\ v_3)\} \\ &= \text{gcd of } \{1, 2\} = 1. \end{aligned}$$

$$\begin{aligned} gcin \text{ of } (v_n) &= \text{gcd of } \{f^*_{mssp}(v_n\ v_{n-1}), f^*_{mssp}(v_n\ v_{n-2})\} \\ &= \text{gcd of } \{2n^2 - 6n + 5, 2n^2 - 8n - 8\} \\ &= \text{gcd of } \{2n - 3, 2n^2 - 8n + 8\} \\ &= \text{gcd of } \{2n - 3, n - 1\} \\ &= \text{gcd of } \{n - 1, n - 2\} = 1. \end{aligned}$$

So, $gcin$ of each vertex of degree greater than one is 1.

Hence $(P_n)^2$, admits mean sum square prime labeling. □

Theorem 2.3 : Middle graph of path P_n admits mean sum square prime labeling.

Proof : Let $G = M(P_n)$ and let $v_1, v_2, \cdots, v_{2n-1}$ are the vertices of G.

Here $|V(G)| = 2n - 1$ and $|E(G)| = 3n - 4$.

Define a function $f : V \to \{0, 1, 2, 3, \cdots, 2n - 2\}$ by

$$f(v_i) = i - 1, \quad i = 1, 2, \cdots, 2n - 1.$$

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{mssp} is defined as follows

$$\begin{aligned} f^*_{mssp}(v_i\ v_{i+1}) &= 2i^2 - 2i + 1, \quad i = 1, 2, \cdots, 2n - 2 \\ f^*_{mssp}(v_{2i}\ v_{2i+2}) &= 8i^2, \quad i = 1, 1, \cdots, n - 2 \end{aligned}$$

Clearly f^*_{mssp} is an injection.

$$gcin \text{ of } (v_{i+1}) = 1, i = 1, 2, \cdots, 2n - 3.$$

So, $gcin$ of each vertex of degree greater than one is 1.

Hence $M(P_n)$, admits mean sum square prime labeling. □

Example 2.1 : Let $G = M(P_4)$.

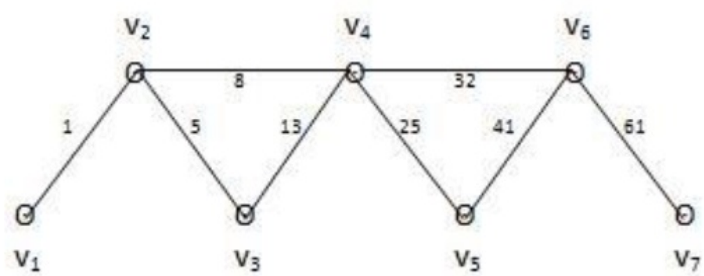

Fig – 2.1

Theorem 2.4 : Total graph of path P_n admits mean sum square prime labeling.

Proof : Let $G = T(P_n)$ and let $v_1, v_2, \cdots, v_{2n-1}$ are the vertices of G.

Here $|V(G)| = 2n - 1$ and $|E(G)| = 4n - 5$.

Define a function $f : V \to \{0, 1, 2, 3, \cdots, 2n-2\}$ by

$$f(v_i) = i-1, \quad i = 1, 2, \cdots, 2n-1.$$

Clearly f is a bijection. f, the induced edge labeling f^*_{mssp} is defined as follows

$$\begin{aligned} f^*_{mssp}(v_i\ v_{i+1}) &= 2i^2 - 2i + 1, \quad i = 1, 2, \cdots, 2n-2 \\ f^*_{mssp}(v_{2i-1}\ v_{2i+1}) &= 8i^2 - 8i + 2, \quad i = 1, 2, \cdots, n-1 \\ F^*_{mssp}(v_{2i}\ v_{2i+2} &= 8i^2, \quad i = 1, 2, \cdots, n-2 \end{aligned}$$

Clearly f^*_{mssp} is an injection.

$$gcin \text{ of } (v_{i+1}) = 1, \qquad i = 1, 2, \cdots, 2n-3.$$

$$\begin{aligned} gcin \text{ of } (v_1) &= \text{gcd of } \{f^*_{mssp}(v_1\ v_2), f^*_{mssp}(v_1\ v_3)\} \\ &= \text{gcd of } \{1, 2\} \end{aligned}$$

$$\begin{aligned} gcin \text{ of } (v_{2n-1}) &= \text{gcd of } \{f^*_{mssp}(v_{2n-3}\ v_{2n-1}), f^*_{mssp}(v_{2n-1}, v_{2n-2})\} \\ &= \text{gcd of } \{8n^2 - 20n + 13, 8n^2 - 24n + 18\} \\ &= \text{gcd of } \{4n-5, 8n^2 - 24n + 18\}, \\ &= \text{gcd of } \{2n-2, 4n-5\}, \\ &= \text{gcd of } \{2n-2, 2n-3\} = 1. \end{aligned}$$

So, $gcin$ of each vertex of degree greater than one is 1.

Hence $T(P_n)$, admits mean sum square prime labeling. □

Example 2.2 : Let $G = T(P_4)$.

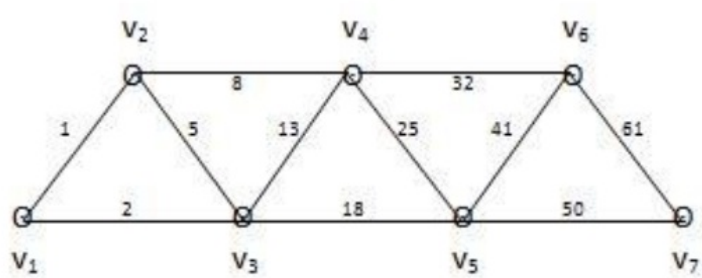

Fig – 2.2

Theorem 2.5 : Z graph of path P_n admits mean sum square prime labeling.

Proof : Let $G = Z(P_n)$ and let $v_1, v_2, \cdots, v_{2n}$ are the vertices of G.
Here $|V(G)| = 2n$ and $|E(G)| = 3n - 3$.
Define a function $f : V \to \{0, 1, 2, 3, \cdots, 2n - 1\}$ by

$$f(v_i) = i - 1, \quad i = 1, 2, \cdots, 2n.$$

Clearly f is a bijection.
For the vertex labeling f, the induced edge labeling f^*_{mssp} is defined as follows.

$$\begin{aligned} f^*_{mssp}(v_{2i-1}\ v_{2i+1}) &= 8i^2 - 8i + 2, \quad i = 1, 2, \cdots, n-1 \\ f^*_{mssp}(v_{2i}\ v_{2i+2}) &= 16i^2, \quad i = 1, 2, \cdots, n-1 \\ f^*_{mssp}(v_{2i}\ v_{2i+1}) &= 8i^2 - 4i + 1, \quad i = 1, 2, \cdots, n-1 \end{aligned}$$

Clearly f^*_{mssp} is an injection.

$$\begin{aligned} gcin \text{ of } (v_{2i+1}) &= \text{gcd of } \{f^*_{mssp}(v_{2i-1}\ v_{2i+1}), f^*_{mssp}(v_{2i}\ v_{2i+1})\} \\ &= \text{gcd of } \{8i^2 - 8i + 2, 8i^2 + 4i + 1\} \\ &= \text{gcd of } \{4i - 1, 8i^2 - 8i + 2\}, \\ &= \text{gcd of } \{4i - 1, 4i^2 - 4i + 1\}, \\ &= \text{gcd of } \{i, 4i - 1\} \\ &= 1. \qquad i = 1, 2, \cdots, n-1. \end{aligned}$$

$$\begin{aligned} gcin \text{ of } (v_{2i}) &= \text{gcd of } \{f^*_{mssp}(v_{2i}\ v_{2i+2}), f^*_{mssp}(v_{2i}\ v_{2i+1})\} \\ &= \text{gcd of } \{16i^2, 8i^2 - 4i + 1\} \\ &= \text{gcd of } \{4i, 8i^2 - 4i + 1\}, \\ &= 1. \qquad i = 1, 2, \cdots, n-1. \end{aligned}$$

So, $gcin$ of each vertex of degree greater than one is 1.
Hence $Z(P_n)$, admits mean sum square prime labeling. □

Theorem 2.6 : Let G be the graph obtained by duplicating each vertex of path P_n by an edge.

G admits mean sum square prime labeling.

Proof : Let G be the graph and let $v_1, v_2, \cdots, , v_{3n}$ are the vertices of G.
Here $|V(G)| = 3n$ and $|E(G)| = 4n - 1$.
Define a function $f : V \to \{0, 1, 2, 3, \cdots, 3n - 1\}$ by

$$f(v_i) = i - 1, \qquad i = 1, 2, \cdots, 3n.$$

Clearly f is a bijection.
For the vertex labeling f, the induced edge labeling f^*_{mssp} is defined as follows.

$$\begin{aligned} f^*_{mssp}(v_{3i-2}\ v_{3i-1}) &= 18i^2 - 30i + 13, \quad i = 1, 2, \cdots, n \\ f^*_{mssp}(v_{3i-1}\ v_{3i}) &= 18i^2 - 18i + 5, \quad i = 1, 2, \cdots, n \\ f^*_{mssp}(v_{3i-2}\ v_{3i}) &= 18i^2 - 24i + 8, \quad i = 1, 2, \cdots, n \\ f^*_{mssp}(v_{3i-1}\ v_{3i+2}) &= 18i^2 - 6i + 1, \quad i = 1, 2, \cdots, n - 1. \end{aligned}$$

Clearly f^*_{mssp} is an injection.

$$\begin{aligned} gcin \text{ of } (v_{3i-2}) &= \text{gcd of } \{f^*_{mssp}(v_{3i-2}\ v_{3i-1}), f^*_{mssp}(v_{3i-2}\ v_{3i})\} \\ &= \text{gcd of } \{18i^2 - 30i + 13, 18i^2 - 24i + 8\} \\ &= \text{gcd of } \{6i - 5, 18i^2 - 30i + 13\}, \\ &= \text{gcd of } \{3i - 2, 6i - 5\}, \\ &= \text{gcd of } \{3i - 2, 3i - 3\} \\ &= 1. \qquad i = 1, 2, \cdots, n. \end{aligned}$$

$$\begin{aligned} gcin \text{ of } (v_{3i}) &= \text{gcd of } \{f^*_{mssp}(v_{3i}\ v_{3i-1}), f^*_{mssp}(v_{3i-2}\ v_{3i})\} \\ &= \text{gcd of } \{18i^2 - 24i + 8, 18i^2 - 18i + 5\} \\ &= \text{gcd of } \{6i - 3, 18i^2 - 24i + 8\}, \\ &= \text{gcd of } \{3i - 1, 6i - 3\}, \\ &= \text{gcd of } \{3i - 1, 3i - 2\}, \\ &= 1. \qquad i = 1, 2, \cdots, n. \end{aligned}$$

$$\begin{aligned} gcin \text{ of } (v_{3i-1}) &= \text{gcd of } \{f^*_{mssp}(v_{3i-2}\ v_{3i-1}), f^*_{mssp}(v_{3i-1}\ v_{3i})\} \\ &= \text{gcd of } \{18i^2-30i+13, 18i^2-18i+5\} \\ &= \text{gcd of } \{12i-8, 18i^2-30i+13\}, \\ &= \text{gcd of } \{3i-2, (3i-2)(6i-6)+1\}, \\ &= 1. \qquad\qquad i = 1,2,\cdots,n. \end{aligned}$$

So, *gcin* of each vertex of degree greater than one is 1.

Hence G, admits mean sum square prime labeling. □

Theorem 2.7 : Splitting graph of path P_n admits mean sum square prime labeling, if n is odd.

Proof : Let $G = S'(P_n)$ and let $v_1, v_2, \cdots, v_{2n}$ are the vertices of G.

Here $|V(G)| = 2n$ and $|E(G)| = 3n-3$.

Define a function $f : V \to \{0,1,2,3,\cdots,2n-1\}$ by

$$f(v_i) = i-1, \quad i = 1,2,---,2n.$$

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{mssp} is defined as follows.

$$\begin{aligned} f^*_{mssp}(v_i\ v_{i+1}) &= 2i^2-2i+1, \quad i = 1,2,\cdots,n-1 \\ f^*_{mssp}(v_{2i}\ v_{2i+4}) &= 8i^2+8i+2, \quad i = 1,2,\cdots,\frac{n-1}{2} \\ f^*_{mssp}(v_{2i}\ v_{2i+6}) &= 8i^2+16i+8, \quad i = 1,2,\cdots,\frac{n-1}{2} \\ f^*_{mssp}(v_{n+i}\ v_{n+i+1}) &= 2(n+i)^2-2(n+i)+1, \quad i = 1,2,\cdots,n-1. \end{aligned}$$

Clearly f^*_{mssp} is an injection.

$$\begin{aligned} gcin \text{ of } (v_{i+1}) &= 1, \qquad i = 1,2,\cdots,n-2 \\ gcin \text{ of } (v_{n+i+1}) &= 1, \qquad i = 1,2,\cdots,n-2 \\ gcin \text{ of } (v_{n+1}) &= \text{gcd of } \{f^*_{mssp}(v_{n+1}\ v_{n+2}), f^*_{mssp}(v_2\ v_{n+1})\} \\ &= \text{gcd of } \{2n^2+2n+1, \frac{n^2+2n+1}{2}\} \\ &= 1. \end{aligned}$$

$$\begin{aligned} gcin \text{ of } (v_{2n}) &= \text{gcd of } \{f^*_{mssp}(v_{2n}\ v_{2n-1}), f^*_{mssp}(v_{n-1}\ v_{2n})\} \\ &= \text{gcd of } \{8n^2 - 12n + 5, \frac{9n^2 - 18n + 9}{2}\} \\ &= 1. \end{aligned}$$

So, *gcin* of each vertex of degree greater than one is 1.

Hence $S'(P_n)$, admits mean sum square prime labeling. □

Example : Let $G = S'(P_5)$.

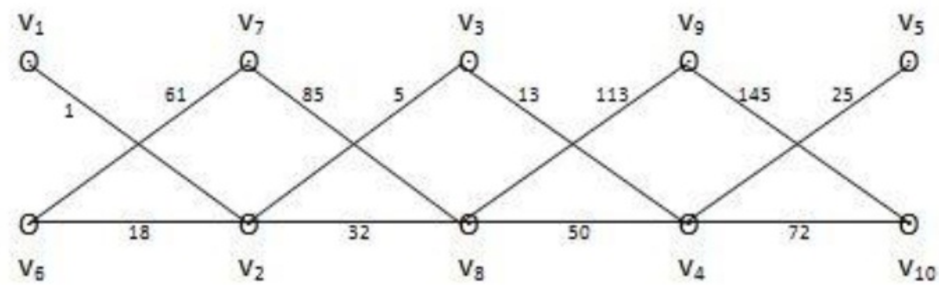

Fig – 2.3

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